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# Managing a Profitable Interactive Email Marketing Program: Modeling and Analysis

Xi Zhang

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**Managing a Profitable Interactive Email Marketing Program:  
Modeling and Analysis**

BY

*XI ZHANG*

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree

Of

Doctor of Philosophy

In the Robinson College of Business

Of

Georgia State University

GEORGIA STATE UNIVERSITY  
ROBINSON COLLEGE OF BUSINESS  
2015

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## ACCEPTANCE

This dissertation was prepared under the direction of the *Xi Zhang's* Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Business Administration in the J. Mack Robinson College of Business of Georgia State University.

Richard Phillips, Dean

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## ABSTRACT

### **Managing a Profitable Interactive Email Marketing Program: Modeling and Analysis**

BY

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July 13, 2015

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Despite the popularity of mobile and social media, email continues to be the marketing tool that brings the highest ROI, according to the Direct Marketing Association's "Power of Direct" (2011) study. An important reason for email marketing's success is the application of an idea—"Permission Marketing," which asks marketers to seek consent from customers before sending them messages. Permission-based email marketing seeks to build a two-way interactive communication channel through which customers can engage with firms by expressing their interests, responding to firms' email messages and making purchases. This thesis consists of two essays that address several key questions that are related to the management of a profitable interactive permission-based email marketing program.

Existing research has examined the drivers of customers' opt-in and opt-out decisions, but it has investigated neither the timings of two decisions nor the influence of transactional activity on the length of time a customer stays with an email program. In the first essay, we adopt a multivariate copula model using a pair-copula construction method to jointly model opt-in time (from a customer's first purchase to opt-in), opt-out time (from customer opt-in to opt-out) and average transaction amount. Through such multivariate dependences, this model significantly improves the predictive performance of the opt-out time in comparison with several benchmark models. The study offers several important findings (1) marketing intensity affects opt-in and opt-out times (2)

customers with certain characteristics are more or less likely to opt-in or opt-out (3) firms can extend customer opt-out time and increase customer spending level by strategically allocating resources.

Firms are using email marketing to engage with customers and encourage active transactional behavior. Extant research either focuses only on how customers respond to email messages or looks at the “average” effect of email on transactional behavior. In the second essay, we consider not only customers’ response to emails and their correlated transactional behavior, but also the dynamics that govern the evolving of the two types of customer relationship: email-response and purchase relationships. We model the email open count with a Binomial distribution and the purchase count with a zero-inflated negative binomial model. We capture the dependence between the two discrete distributions using a copula approach. In addition, we develop a hidden Markov model to model the effects of email contacts on purchase behavior. We also allow the relationship that represents customers’ responsiveness to email marketing to evolve flexibly along with the relationship of purchase.

In the second essay, we apply the proposed model in a non-contractual context where a retailer operates a large-scale email marketing program. Through the empirical study, we capture a positive dependence between the opening of emails and purchase behavior. We identify three purchase-behavior states along with three email-response states. The empirical finding suggests that the customers who are in the medium relationship state have the highest intrinsic propensity to open an email, followed by the customers in the lowest and highest relationship state. Furthermore, we derive a dynamic email marketing resource allocation policy using the hidden Markov model, the purchase and email open model estimates. We demonstrate that a forward-looking agent could maximize the long-term profits of its existing email subscribers.

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## **Introduction**

Since the advent of Internet, email marketing has been a valuable marketing tool for firms. Now with the rise of smartphones, tablets and social media, email marketing is expected to make more impact on firms and consumers, as said by 83% of CMOs who participated in a benchmark survey (MarketingSherpa 2011). ExactTarget (2012), an email service provider, conducted a series of studies, showed that among consumers surveyed, 77% preferred to receive permission-based promotional messages through email as opposed to 9% through direct mail, 5% through text messages, and 6% through social media (Facebook, Twitter and Mobile App). In addition, 66% of consumers surveyed have made a purchase as a result of a marketing message received through email. According to the Direct Marketing Association's "Power of Direct" (2011) study, email brought in an average of \$40.56 for every dollar spent in 2011, compared to a dollar-return rate of \$7.30 for catalogs, \$10.51 for mobile, \$12.71 for social networking, \$19.72 for Internet display advertising, and \$22.24 for search engine marketing. These staggering statistics prove that email is still one of the most effective channels for companies to communicate with consumers and generate profits.

Despite the superior ROI of email marketing, not every email reaches the consumers' inbox due to junk mail blocking technology. Permission marketing, or invitational marketing, proposed by Godin (1999), says that marketer should seek consent from customers in advance before sending them promotional messages. Typically, customers express their potential needs or interests at sign-up (opt-in) and marketers will fulfill customers' requests by sending relevant information regularly. Customers are given full rights to unsubscribe (opt-out) if the information marketers provide fails to meet their expectations. Research has shown that, compared to unsolicited emails, permission-based emails have better click-through rates, more precise

segmentation and targeting, and can significantly improve customer brand loyalty (Krishnamurthy 2001). Gartner (2002) reported that unsolicited direct mail or email has a response rate of 1% while the average click-through rate of permission-based email is between 6% and 8%, respectively.

The importance of permission marketing can be addressed from the following aspects. First, permission marketing has been considered “the way to make advertising work again” (Godin 1999). Permission marketing offers marketers a way to communicate and interact with customers without intruding their privacy or causing advertising irritation. It is closely related to “relationship marketing” and “one-to-one marketing” (Krishnamurthy 2000) and intends to help companies build and sustain a long-term relationship with customers. It has created a channel for active customer engagement with firms, which has been deemed crucial for firm value creation.

The two essays of this thesis address several key questions under the principles of rigor and relevance. The first essay examines the timings of opt-in and opt-out and the influence of transactional activity on the length of time a customer stays with an email program. It proposes the use of a multivariate copula model using a pair-copula construction method to jointly model opt-in time, opt-out time, and the average transaction amount. The second essay investigates the evolving of the two types of customer relationships—email-response and purchase relationships using a hidden Markov model. It also adopts the dynamic programming approach to derive the optimal email marketing resource allocation policy.

Although the two essays are interconnected under the umbrella of permission-based email marketing, each essay is written in a way that each can be read as an independent paper. Therefore, in the following sections, I begin with the first essay including the main text, references, tables and the appendix. Subsequently, I present the second essay with full details.

## **ESSAY 1**

### **Modeling Customer Opt-In and Opt-Out in a Permission-Based Marketing Context**

#### **1.1 Introduction**

Conventional wisdom suggests that customers do not welcome communications from marketers, and consider their messages unwanted interruptions that are to be avoided by registering for do-not-mail or do-not-call lists. However, in today's digital age, it is increasingly apparent that customers in fact, enthusiastically interact with firms by joining their email programs voluntarily, proactively downloading their mobile applications, and following their social media accounts. We therefore argue that customers are not reluctant to receive marketing materials if they are first asked for consent. In 1999, Seth Godin proposed an idea, called "permission marketing," and advised marketers to seek permission from customers before sending them promotional messages. Permission marketing creates a channel for two-way interaction and engagement, which is seen as crucial for firm value creation. Thus, permission marketing emerges as a solution to the challenge faced by conventional marketing.

Permission marketing typically relies on the use of "new media" channels, such as web, email, mobile and social media, which are well suited for interactive marketing (e.g., Winer 2009). Forrester (2011) forecasted that marketers in the US will spend \$77 billion on interactive marketing by 2016, the same amount as that is being spent on TV advertising today. Among the channels of new media, email and mobile have gained much attention due to their interactive, digital and cost-effective features (e.g., Shankar and Balasubramanian 2009; Shankar et al. 2010). Forrester forecasts that mobile marketing spending will increase by nearly three times from \$2.8 billion in 2012 to \$8.2 billion in 2016. The Direct Marketing Association (DMA 2011)

forecasts that commercial email will drive up sales by \$82.2 billion in 2016.

Previous literature has shown that various factors such as trust and previous experience can affect customers' willingness to accept permission-based marketing (e.g., Tezinde, Smith, and Murphy 2002; Jayawardhena et al. 2009), that trust is an important determinant of online and offline buyer-seller relationships (e.g., Ganesan 1994; Bart et al. 2005), that online habits and socio-demographics affect customers' interest in permission-based web or mobile-marketing programs (e.g., Brey et al. 2007; Barnes and Scornavacca 2008), and that a wrongly- designed message can decrease the response rate and increase the unsubscribe rate (e.g., Marinova, Murphy, and Massey 2002). However, these research studies were typically conducted in experimental settings and only examined the process of opt-in and opt-out separately, neglecting the possibility that the same customer's opt-in and opt-out behavior could be interdependent. In addition, although some prior studies have discovered that permission marketing can increase customer brand loyalty and purchase intention (e.g., DuFrene et al. 2005; Jolley et al. 2013), they have not investigated the possibility that the changes of customer loyalty could adversely affect the length of time a customer is willing to stay in a permission-based marketing program. Thus, it is imperative for us to ask whether customers' opt-in and opt-out behavior can be modeled jointly, how to incorporate the influence of transactional behavior into the modeling of opt-in and opt-out decisions, and how to quantify the influence of a firm's marketing activities on customers' opt-in, opt-out and purchases.

We attempt to bridge the gap in the permission marketing literature by addressing five research questions. (1) What types of customers are more likely to opt-in in a permission-based marketing program? (2) How do firms' marketing activities influence the timing of customers' opt-in and opt-out decisions? (3) Is there a dependence between the opt-in and opt-out times?

and (4) How do transactional behavior and customers' willingness to stay in the marketing program influence each other? (5) How can firms optimize their marketing contacting strategy to both, extend the length of time customers stay in the marketing program and increase their spending level?

To answer these research questions, we analyze a unique data set from a U.S. retailer spanning 47 months. This database records the time when a customer opts in and out of the firm's email program, the transactions made by the customer, the email open and click histories, and the retailer's marketing activities. To obtain each individual customer's online habits and socio-demographics, we merge the sampled data from the retailer's database using key identifier information to an external database provided by a marketing research firm, Acxiom. The methodological challenge of the research is to jointly model three variables: the opt-in timing, the opt-out timing and the purchase behavior. We use a multivariate copula model, called *vine copulas* (e.g., Aas et al. 2009; Smith et al. 2010), to capture the dependence structure of the three variables. For the marginal distributions, we model the opt-in and opt-out times using Weibull hazard models and account for the unobserved heterogeneity by incorporating a gamma random effect term. We model the average transaction amount using a random effect log-normal model.

To the best of our knowledge, this is the first empirical study that examines the timing of customers' opt-in and opt-out decisions while accounting for their purchase behavior. In addition, we extend the bivariate copula model into a multivariate copula model by introducing 'vine copula' to the marketing literature for the first time. Therefore, our study contributes to the existing literature substantially and methodologically.

In the following sections, we first review the literature on (1) permission-based marketing, (2) linkage between the opt-in and opt-out decisions, (3) capturing the dependence between the

durations, and (4) incorporation of the purchase behavior. Second, we describe our data and present descriptive statistics. Third, we discuss the proposed modeling framework. Fourth, we present the model results and model validation. Finally, we discuss the managerial implications and conclusions.

## **1.2 Literature Review**

### **1.2.1 Permission-Based Marketing**

Permission marketing, coined by Godin (1999), proposes that marketers should seek their customers' permission to send them marketing messages. There are two types of permission marketing, namely opt-in and opt-out marketing. Opt-in marketing refers to firms explicitly asking customers for permission, usually when an online account is created. Customers can opt-out any time after they opt-in. Opt-out marketing refers to firms sending promotional messages to customers without seeking their permission, including for the first message, but providing customers an option to opt-out on each occasion. As most marketers adopt the former, we focus on this type of permission marketing and directly examine the behaviors of opt-in and opt-out in this study.

The three main characteristics of permission marketing are “anticipated, personal, and relevant” (Godin 1999). Contrary to spam, a permission-based message is anticipated and its sender is trusted by customers (we believe that customers will not join the firm's email program in the first place if they do not trust the firm). Firms can personalize the marketing messages according to customers' specific interests, which customers can indicate at the time of opt-in. To improve targeting precision, marketers also can tailor the promotional information included in the message, based on the customer's past purchase behavior. Gartner (2002) reported that

unsolicited direct mail or email has a response rate of 1% while the average click-through rate of permission-based emails is between 6% and 8%. Jolley et al. (2013) showed that a permission-based email marketing program can extend a customer's lifetime value.

There are two critical aspects that firms need to manage in order to ensure the success of a permission-based marketing program: the customer's opt-in and his/her opt-out. Research on permission marketing has explored that several factors including brand equity, previous relationship (Tezinde, Smith and Murphy 2002), income, gender, advertising message volume, previous experience with mobile ads (Barnes and Scornavacca 2008), and brand image and trust (Jayawardhena et al. 2009) will influence a customers' willingness to give permission to marketers. While customers' opt-in decisions are influenced by the aforementioned factors, it is also important to identify the drivers of customer opt-out so that firms can make efforts to retain the existing subscribers. Previous research related to customer opt-out has discovered that message relevance and monetary benefit positively influence customers' interest in a permission marketing program (Krishnamurthy 2001), that highly personalized messages (e.g., using the customer's name in the email subject line) would make customers opt-out (Marinova, Murphy, and Massey 2002), and that the more lengthy an email is and the fewer links it contains, the higher the 'unsubscribe' rate (Chittenden and Rettie 2003).

### **1.2.2 Linkage between Opt-in and Opt-out**

Although previous research has identified many factors that could influence customer opt-in and opt-out behavior, it has mainly focused only on the incidence of opt-in and opt-out, and has not studied the timing of the two decisions nor the possible linkage between the two. The timing of customers' opt-in and opt-out decisions depends on who they are (socio-

demographics), how they live (lifestyle, online habits), how they are influenced (marketing contacts), and how satisfied they are (relevant messages). Some customers may opt-in the first time they have an interaction with the firm, i.e., made a purchase, and opt-out at end of their customer lifecycle. Some customers may need more time trying and testing with the firm before they opt-in, and they may only stay with the email program for a limited period of time and withdraw as soon as they feel that the program fails to meet their expectations. While there is so much heterogeneity in customer opt-in and opt-out behavior, we argue that there might be a dependence between the two variables and ignoring this dependence may lead to biased inferences which could adversely affect the marketer's decision making.

Broadly speaking, customers' opt-in and opt-out times may be positively or negatively correlated. The nature and extent of their dependence could be determined by the following factors. First, opt-in and opt-out have some drivers such as marketing activities in common. For example, if direct mails substitute emails before a customer opts-in but complements emails after the customer opts-in, direct mails would extend both his/her opt-in and opt-out time, leading to a positive dependence between the two. In contrast, if direct mails are always substitutes or complements of emails, customers' opt-in and opt-out times would be negatively correlated. Second, the observed heterogeneity, such as customer characteristics, affects a person's decision to opt-in and opt-out. For example, customer "inertia" makes customers delay their decision to opt-in and once they have opted in, they tend to stay for a long time and do not bother to opt-out. In this case, opt-in and opt-out times are positively correlated. In contrast, "variety-seeking" customers are reluctant to stick to one company so they need more time to sign up but once they have opted in, they will quickly switch to another program for a better offer. In such cases, customers will demonstrate a negative dependence between the opt-in and the opt-out time.

Third, the effectiveness of the email program, such as the number of email programs a customer has already subscribed to and the relevance of the email messages of the focal firm may influence his/her opt-in and opt-out likelihood. Customers who have already subscribed to a large number of email programs are less likely to opt-in to another one and once they have opted in and are able to receive personalized relevant messages, they tend to stay for a long time. In this case, their opt-in and opt-out time are positively correlated. In contrast, if the same customers receive many non-relevant messages after opt-in, they will opt-out very quickly to release the pressure of information overload. In such cases, their opt-in and opt-out times will be negatively correlated. While the dependence between opt-in and opt-out times may vary across firms and industries, researchers should empirically test the true dependence between them, based on their data. The scope of this study is not to generalize whether the dependence should be positive or negative and offer explanations for such phenomena, but simply to capture the dependence through an empirical model.

### **1.2.3 Capturing the Dependence between Durations**

Accounting for dependence between two durations such as the dependence between acquisition and retention, and between email open and click is not uncommon in the marketing literature. Chintagunta and Haldar (1998) adopted the Farlie-Gumbel-Morgenstern (FGM) family of bivariate distributions to capture the dependence between customer purchase of products in two related categories, such as pasta and pasta sauce. Park and Fader (2004) adopted the Sarmanov bivariate distributions to investigate customer co-visit timing behavior between the websites of two competing retailers. Schweidel, Fader and Bradlow (2008) used the Sarmanov family to model the dependence between the time to customer acquisition and the subsequent

duration of being “alive”. Bonfrer and Drèze (2009) developed hazard models of email open and click time with the Sarmanov family to capture the dependence between open and click rate.

Noticeably, Schweidel et al. (2008)’s model is developed and applied to a context similar to that of this study— by jointly modeling the timings of when a customer starts to engage and disengage with a firm. However, the model proposed in this study distinguishes from that of Schweidel et al. (2008) in several aspects. First, the Sarmanov families are limited in the dependence ranges they can account for (Danaher and Smith 2011). Schubina and Lee (2004) discuss that the dependence range for Sarmanov family depends on the specification of marginal distributions. They calculated the exact maximum dependence ranges that can be attained for several marginal distribution specifications, for example, the range of uniform is  $[-3/4, 3/4]$  and of normal is  $[-2/\pi, 2/\pi]$ . While the Sarmanov family may well be applied in some context such as Park and Fader (2004) and Schweidel et al. (2008), we prefer to use copulas that can accommodate a wider range of dependence. In the empirical application of this study, we test both the Gaussian and Frank copulas, two copulas that can account for nearly the full  $(-1, 1)$  range of dependence (Trivedi and Zimmer 2005).

Second, although a bivariate model has its advantages in solving marketing problems, real-world applications may require a model that can capture complex and high-dimensional dependence structures. The Sarmanov families do not easily capture the dependence structure of three or more dimensions (Danaher and Smith 2011). In this study, we propose to use a vine copula (e.g., Aas et al. 2009), recently popularized in the statistics literature, to jointly model the opt-in time, the opt-out time and the purchase behavior. We will discuss the method of vine copula in the proposed model framework section.

Third, Schweidel et al. (2008) develop their model in a contractual telecommunication

services context but does not consider the possibility that service subscription time is dependent on the types (low/medium/high margin) of service that customers choose to subscribe to. In this study, we investigate in a non-contractual, retailing context where the duration of a customer staying in a marketing program and his/her purchases are two separate but interdependent behaviors (e.g., Ascarza and Hardie 2013; Netzer, Lattin and Srinivasan 2008). While one may argue that the effect of a customer's purchase behavior on the length of time he/she stays in a marketing program could be estimated by including it as a covariate in the marginal model, we determine that it would suffer from endogeneity because unobserved factors such as customer loyalty are highly likely to affect both variables. The vine copulas model proposed in this study avoids the potential endogeneity issue by modeling the purchase behavior, the opt-out and the opt-in simultaneously. In the next section, we discuss the substantive importance of jointly examining the opt-in, the opt-out and the purchase behavior.

#### **1.2.4 Incorporation of Purchase Behavior**

Krishnamurthy (2001) discusses that customer interest in a permission marketing program is positively related to the customer's level of participation in the program. Krishnamurthy states that customers opt-in in a marketing program to obtain information related to the products and promotions that add value to their lives by reducing the cost of information search and by providing monetary benefits. Most permission-based marketing programs allow customers to opt-out or unsubscribe at any time if they are no longer willing to receive messages from the firm. The length of time a customer is willing to stay in a marketing program may depend on the relevance of the message, the intensity of marketing activities and customer loyalty. We argue that customers who receive a higher proportion of relevant messages and/or who have a higher

level of spending with the firm are more likely to stay longer in the marketing program. Firms can identify those short-life customers at their earlier stage by analyzing their buying patterns (Reinartz and Kumar 2000).

From another angle, participation in a permission marketing campaign can change the attitudes and behaviors of customers by increasing their purchase intention (DuFrene et al. 2005), making them spend more money (Jolley et al. 2013), and being more responsive to firms' marketing messages (Marinova, Murphy, and Massey 2002). The longer the customers stay in a marketing program, the more familiar they would be with the firm and the more likely to shop with the firm. In summary, we argue that staying in a marketing program and actually making purchases are two interdependent processes (e.g., Danaher 2002) which should be jointly studied to avoid potential endogeneity issues. Firms should not only invest resources to make customers stay longer in the marketing program but also make them spend more money while they are still subscribed to the program. Since the timings of joining and withdrawing the marketing program are also interdependent, we model the three processes jointly in an integrated model framework.

### **1.3 Data Description**

Our database comprises information from a U.S. retailer that sells multiple categories of home improvement products. The data set consists of information on the time when a customer opts-in and opts-out of the firm's email program, the transactions made by the customer, the email open and click histories, and the marketing activities of the firm. We construct a calibration data set by sampling at random, a cohort of 9,180 customers who made their first purchases from the firm between February 2007 and July 2007. We construct a holdout data set by sampling another cohort of 9,180 customers to validate our proposed model.

To obtain information on customers' online habits and socio-demographics, a multinational marketing technology and services firm, named Acxiom, appends the data we sampled with one of their databases using several key identifiers with a 100%, one-to-one match rate. The database provided by Acxiom, trademarked as "PersonicX Digital", assigns people to one of the 13 segments based on how they use the internet, how they shop online, when and where they access the internet and their demographic attributes (see Table 1 for a description of each cluster). We include this external segmentation to account for the customer characteristics that are useful in explaining the customer opt-in and opt-out propensities (Brey et al. 2007).

Insert Table 1 about here

The retailer that provides us with the data currently operates a large-scale email program with a large number of subscribers. The email program is permission-based in the sense that people need to subscribe first to receive any emails from the retailer. Although purchase is not required to subscribe to the email program, the majority of the existing email subscribers have purchase histories with the firm before opt-in, according to the management team of the retailer. The number of email subscribers who opt-in on the same day as their first purchase is insignificant (about 0.03% of the sample). We argue that customers need a period of time to develop trust with the firm to let it send messages to their email inbox.

We consider purchasing and subscribing to be two separate decisions for the customers of the focal retailer. The retailer does not have any policy to encourage customers to opt-in its email program when they purchase from its physical stores. In addition, although customers can create an online account to manage their orders with convenience when they purchase online, they are considered opt-in only when they click the checkbox "willing to receive further email marketing messages". After the opt-in, the subscribers receive emails which contain instructions for opt-out

at the bottom of the message. Customers can opt-out at any time by clicking the “unsubscribe” link, calling the customer service center, or writing to the retailer’s office.

There are two characteristics of this study we need to clarify. First, this study focuses on the opt-in and opt-out behaviors of existing customers. We agree that other firms may have some proportion of email subscribers who have no purchase history before opt-in. While it may be worthwhile to examine the opt-in behavior of prospects, the managerial implications we draw from this study apply to existing customers. Second, we focus this study on customers’ first opt-in and opt-out decisions. We observe a very small number of customers who have multiple opt-in and opt-out records (about 0.1% of the sample).

## **1.4 Descriptive Statistics**

The key variables of interest in this study are the timings of opt-in and opt-out. Opt-in time is computed as the number of days elapsed between a customer’s first purchase and opt-in. Opt-out time is computed as the number of days elapsed between opt-in and opt-out. We observe that both opt-in and opt-out times may be right-censored. In the calibration sample, 22.8% of the customers didn’t opt-in, 18.5% of the customers opted in but opted out before the end of the observation window, and 58.7% of the customers opted in and stayed till the end of the observation window. Of the customers who did opt-in, the mean opt-in time is 597 days and the median opt-in time is 611 days. Of the customers who have opted in but opted out, the mean opt-out time is 410 days and median opt-out time is 343 days.

To illustrate the differences in purchase behavior of email subscribers and non-subscribers, we randomly select two samples of equal size (subscribers and non-subscribers) and report the descriptive statistics of several variables which are computed for the same period of time (see

Table 2). As compared to nonsubscribers, email subscribers spend more money, make purchases more frequently, redeem more coupons, receive more direct mails, and make more returns. The results are consistent with the previous findings that permission-based marketing programs reinforce customer loyalty and induce more active customer engagement.

Insert Table 2 about here

To further illustrate the importance of the study on opt-out time, we conduct a preliminary analysis to explore the relationship between the length of time a customer stays in an email program and his/her purchase behavior. We randomly select 103 customers who started their relationship with the retailer (first purchase) at the same time (February 2007), opted in the email program at the same time (June 2008) but opted out at the different times. We split these customers into the following three cohorts based on the length of time the customer had been with the retailer: 1 to 6 months (cohort 1), 7 to 12 months (cohort 2) and 13 to 18 months (cohort 3). We summarize their purchase behavior for the same time-window from June 2008 to December 2010 (see Table 3). To ensure that the three cohorts of customers are comparable, we ensure that the customers selected have a similar purchase pattern before opt-in, such as making a purchase every 1.4 to 1.8 months.

Insert Table 3 about here

Table 3 shows that, on average, the customers who stayed in the email program for a longer period of time tend to purchase more frequently and spend more money. The statistics can be interpreted from two perspectives. From one point of view, the customers who choose to stay longer in the program demonstrate stronger interests in the product category and the brand, have a higher chance to be exposed to the firm's email marketing, and are more active in purchases. From another point of view, the customers who have longer lasting interests in home

improvement products, who are more accustomed to read emails to obtain information and who have a higher intention of purchase are more likely to stay in the email program for a longer period of time.

## **1.5 Proposed Modeling Framework**

### **1.5.1 Modeling Challenges**

In this study, we jointly model three variables, the customer's opt-in time, opt-out time and average transaction amount. We need a multivariate copula model that can capture the three-dimensional dependence structure. Research on multivariate copula models have received attention from the fields of statistics, finance, insurance (e.g., Smith et al. 2010; Zimmer and Trivedi 2006) and marketing (e.g., Danaher and Smith 2011; Stephen and Galak 2012; Kushwaha and Shankar 2013). However, the number of multivariate distributions that are readily applicable to three or higher dimensional problems is limited. Multivariate Gaussian copula, an example of the elliptical copula, has been used to model inter-magazine exposures and page views of multiple websites (Danaher and Smith 2011) and model multivariate count data (Stephen and Galak 2012). In addition to the elliptical copula, there are several studies that attempt to extend the bivariate Archimedean copula to higher dimensions (e.g., Zimmer and Trivedi 2006; Savu and Trede 2010). Most commonly used Archimedean copulas include Clayton, Gumbel and Frank (Trivedi and Zimmer 2005). However, these extensions are developed at the expense of dependence measures. A flexible  $n$ -variate copula should be able to accommodate  $n(n - 1)/2$  dependence parameters for each pair of the marginal distributions. However, for example, a trivariate extension of a bivariate Frank copula which is proposed by Zimmer and Trivedi (2006) only allows for two (instead of three) dependence parameters which

also need to be positive. This restriction limits its application to many practical problems. Thus, a flexible multivariate copula model is needed.

Based on the work of Joe (1997) and Bedford and Cooke (2002), Aas et al. (2009) shows that multivariate data can be decomposed into a cascade of bivariate copulas, called “pair-copula constructions”. A pair-copula decomposition offers a highly flexible way to construct multivariate distributions and has been the focus of many studies recently (e.g., Smith et al. 2010; Min and Czado 2010; Kurowicka and Joe 2011; Panagiotelis et al. 2012; Hobæk Haff 2013). It has no restrictions on the dependence parameters. It allows the selection of any copulas to build bivariate copulas, such as Gaussian,  $t$ , Gumbel and Frank. Compared with that of multivariate Gaussian copula, the estimation of pair-copula construction is relatively easy as the parameters of each pair-copula can be estimated sequentially. But a multivariate Gaussian copula requires the evaluation of multiple integral without a closed-form solution, which can only be approximated in a numerical way. In a simulation study, Smith et al. (2010) compares and shows that the vine copula outperforms a multivariate Gaussian copula in forecasting. In this study, we construct a trivariate pair-copula model and test it in an empirical application with Gaussian and Frank copulas as pair-copulas. We choose the “best-fitting” copula among the two models using model selection criteria such as the Bayesian Information Criterion (BIC). In the next section, we discuss the marginal models for opt-in time, opt-out time and average transaction amount and the modeling of the dependence with pair-copula construction.

### **1.5.2 Modeling the Opt-In Time and Opt-Out Time**

Since the opt-in and opt-out times are both continuous survival data, we model these variables using the conditional hazard model (e.g., Jain and Vilcassim 1991), which is well

suited for censored observations. Let  $(T_{i1}, T_{i2})$  and  $(C_{i1}, C_{i2})$  denote the paired opt-in and opt-out times and censoring times for customer  $i = 1, \dots, n$ . Let  $t_{ij} = \min(T_{ij}, C_{ij})$  denote the actual observed durations,  $\delta_{ij} = I(t_{ij} = T_{ij})$  and  $\mathbf{Z}_{ij}$  be a vector of covariates for customer  $i$ , where the subscript  $j$  denotes the opt-in time ( $j = 1$ ) or the opt-out time ( $j = 2$ ). Note that  $\delta_{i1} = 0$  denotes the case where customer  $i$  didn't opt-in during the observation period<sup>1</sup>. The opt-in or opt-out time  $t_j$  is assumed to follow the Weibull distribution, characterized by the distribution function  $F_j(t_j)$ . We use the Weibull distribution because it is highly flexible that it can accommodate flat, monotonically increasing or decreasing hazard functions and has been proved useful in marketing applications (e.g., Chintagunta and Halder 1998; Seetharaman and Chintagunta 2003).

The density function of the Weibull distribution is  $f_j(t_j) = \alpha_j \lambda_{ij} t_j^{\alpha_j - 1} \exp(-\lambda_{ij} t_j^{\alpha_j})$  where  $\alpha_j$  represents the shape parameter and  $\lambda_{ij}$  controls the scale parameters. We allow the scale parameter to be customer-specific,  $\lambda_{ij}$ , which is specified as a function of the corresponding vector of covariates  $\mathbf{Z}_{ij}$  and parameter sets  $\boldsymbol{\beta}_j$ . To ensure that the scales are positive, we use exponential specifications as

$$\begin{aligned} \lambda_{ij} = \exp(&\beta_{0j} + \sum_{d=1}^{12} \beta_{dj} \text{PERSONICX}_{id} + \beta_{13j} \text{COUPON}_{ij} \\ &+ \beta_{14j} \text{DMAIL}_{ij} + \beta_{15j} \text{RETURN}_{ij} \\ &+ \beta_{16,j=2} \text{EMAIL}_{i,j=2} + \beta_{17,j=2} \text{OPEN}_{i,j=2} \\ &+ \beta_{18,j=2} \text{CLICK}_{i,j=2}) \end{aligned} \quad (1)$$

for every  $i = 1, \dots, n$ ,  $j = 1, 2$ , and  $d = 1, \dots, 12$ . Here,  $\beta_{0j}$  captures a customer  $i$ 's intrinsic probability to opt-in or opt-out. The variables EMAIL, OPEN, and CLICK are related to activities

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<sup>1</sup> In this study, we assume that every customer will eventually opt-in the retailer's email program given a long enough period of time. A split-hazard model can be used to account for the opt-in probability if the assumption that a proportion of customers will never opt-in is made (e.g., Sinha and Chandrashekar 1992; Schweidel, Fader and Bradlow 2008).

that can only occur after a customer has opted in a permission-based email program, so  $\beta_{16,j=2}$ ,  $\beta_{17,j=2}$  and  $\beta_{18,j=2}$  are specified only in the opt-out model. All the variables in Equation (1) are explained in the following sections.

PERSONICX represents a vector of binary variables that indicate the segment to which a customer is assigned according to PersoniX Digital, the database from Acxiom. The database assigns customers to one of the 13 segments based on their demographics and online behaviors (see Table 1). We use Group 1, labeled as “Superhighway Superusers”, as the reference group to create 12 dummy variables. We expect these variables to provide some explanatory power on the opt-in and opt-out model because online habits and socio-demographics affect customers’ interest in permission-based web or mobile-marketing programs (e.g., Brey et al. 2007).

COUPON<sub>ij</sub> is operationalized as the total number of coupons that customer  $i$  redeemed before opt-in ( $j = 1$ ) or between the opt-in and opt-out or the censoring time ( $j = 2$ ). In addition to direct mail, company website or referral, email program is another option customers can utilize to obtain saving opportunities such as coupon code or price discount information. We expect that customers who are already active in coupon redemption have smaller probabilities to opt-in due to high information processing cost but low incremental saving benefit. Meanwhile, for email subscribers, we expect that the coupon redemption activities could indicate the relevance of emails to their purchase needs which could subsequently affect their interests in the email program (Krishnamurthy 2001).

DMAIL<sub>ij</sub> is operationalized as the average number of direct mail customer  $i$  received per month before the opt-in ( $j = 1$ ) or between the opt-in and the opt-out or the censoring time ( $j = 2$ ). Direct mail usually uses product information and coupons to attract customers to visit the stores. While direct mail and email serve similar marketing purposes, it is uncertain how they

may affect each other. It is likely that customers who receive substantial direct mails are less motivated to participate in an email marketing program due to the increase of information burden (Krishnamurthy 2001). We test the non-linear forms (logarithmic and quadratic) of  $DMAIL_{ij}$  in both models because there could be an optimal level of marketing communications (Nash 1993).

$RETURN_{ij}$  is operationalized as the total number of product return occasions customer  $i$  made before the opt-in ( $j = 1$ ) or between the opt-in and the opt-out or the censoring time ( $j = 2$ ). The customers who have a medium level of returns are found to have the highest customer lifetime value (e.g., Petersen and Kumar 2009). The product return frequency signifies the relationship between customers and firms which is important for the customer opt-in and opt-out decisions (e.g., Jayawardhena et al. 2009). We test the non-linear effect (logarithmic and quadratic) of  $RETURN_{ij}$  and expect to find an optimal level of product return frequency.

$EMAIL_{i,j=2}$  is operationalized as the average number of emails customer  $i$  received per month while  $OPEN_{i,j=2}$  is computed by dividing the number of emails opened by the total number of emails received and  $CLICK_{i,j=2}$  is computed by dividing the number of emails clicked by the total number of emails opened, between the opt-in and the opt-out or the censoring time. Krishnamurthy (2001) discusses that message processing costs and message relevance are the two important factors that could affect customers' interest in permission marketing programs. Due to the intrusive nature of email promotions, Ha (1996) argues that customers' attitudes towards email marketing decrease as firms' emailing frequency increases. We expect to find a U-shape effect of the email quantity on the customer opt-out probability. In addition, we use the email open and click rates as a measure of message relevance which could indicate the category-message fit and the perceived attractiveness of advertisers (Krishnamurthy 2001). Firms that can consistently send messages relevant to customers' needs will be more appreciated and customers

will be less likely to opt-out. But we expect that the utilities derived from relevant messages increase up to a threshold as customers usually have a spending limit or a share-of-wallet for a certain firm. Thus, we use the linear and quadratic form of  $EMAIL_{i,j=2}$  and the logarithmic forms of  $OPEN_{i,j=2}$  and  $CLICK_{i,j=2}$ .

### *Heterogeneity*

In addition, there is unobserved heterogeneity in terms of customers' hazard of the opt-in and the opt-out. For example, some customers may be more likely to opt-in or opt-out, but this heterogeneity is not directly measured. To account for this unobserved heterogeneity in both the opt-in and the opt-out models, we incorporate an unobservable multiplicative effect  $v_{ij}$ , called “a frailty term”, on the Weibull hazard functions (e.g., Han and Hausman 1990; Schmittlein and Morrison 1983). The conditional hazard function is specified as  $h(t_{ij}|v_{ij}) = \alpha_j \lambda_{ij} t_{ij}^{\alpha_j-1} v_{ij}$ .

Following Sahu et al. (1997), we assume that the random variable  $v_{ij}$  follows a gamma distribution with a mean of 1 (for identification purpose) and a variance of  $1/\gamma_j$ , where  $\gamma_j$  is a parameter to be estimated. By integrating  $v_{ij}$ , we obtain the closed-form solutions for the unconditional Weibull survival function (e.g., Gutierrez 2002; Meade and Islam 2010)

$$S(t_{ij}) = \left[ 1 + \gamma_j \lambda_{ij} t_{ij}^{\alpha_j} \right]^{-1/\gamma_j} \quad (2)$$

and the unconditional Weibull density function

$$f(t_{ij}) = S(t_{ij})^{1+\gamma_j} \alpha_j \lambda_{ij} t_{ij}^{\alpha_j-1}. \quad (3)$$

### **1.5.3 Modeling the Average Transaction Amount**

We assume that the average transaction amount (in U.S. dollars)  $AMT_i$  that customer  $i$  spent during the time he/she stayed with the email program, follows a log-normal distribution

(e.g., Borle, Singh and Jain 2008)

$$\log AMT_i \sim \text{Normal}(\mu_i, \sigma^2) \quad (4)$$

where  $\mu_i$  is the mean and  $\sigma^2$  is the variance of the normal distribution. We assume that the mean parameter  $\mu_i$  is a function of the individual-level covariates as shown below:

$$\begin{aligned} \mu_i = & \mu_{0i} + \mu_1 \text{Avg\_Coupon}_i + \mu_2 \text{Avg\_Dmail}_i + \mu_3 \text{Avg\_Email}_i \\ & + \mu_4 \text{Avg\_Return}_i + \mu_5 \text{Avg\_CrossBuy}_i \\ & + \mu_6 \text{Avg\_Open}_i + \mu_7 \text{Avg\_Click}_i + \mu_8 \text{Avg\_IPT}_i \end{aligned} \quad (5)$$

To account for the unobserved heterogeneity, we allow the intrinsic average transaction amount  $\mu_{0i}$  to be customer-specific. We assume that this heterogeneous parameter is normally distributed across customers as  $\mu_{0i} = \mu_0 + \Delta\mu_{0i}$ , where  $\Delta\mu_{0i} \sim N(0, \sigma_{\mu_0}^2)$  and  $\sigma_{\mu_0}^2$  is the variance parameter. Thus,  $\mu_0$ - $\mu_8$ ,  $\sigma^2$  and  $\sigma_{\mu_0}^2$  are the parameters to be estimated from the data.

Note that  $\log AMT$  is actually a mixture of two normals, one for the idiosyncratic variation and one for the random effect. Here, we use average transaction amount instead of total amount spent because total amount spent is a cumulative measurement which is likely to be a function of time elapsed. The joint modeling of opt-out time and total amount spent would create a positive dependence because of the shared effect of time elapsed. Since average transaction amount is calculated by dividing the total amount spent by the total number of purchase trips, the joint modeling of average transaction amount and opt-out time can capture the dependence that has teased out the shared effect of time elapsed. We explain all the variables specified in Equation (5) next.

$\text{Avg\_Coupon}_i$  is operationalized as the average number of coupons customer  $i$  redeemed in every transaction. The use of coupons can lead to unplanned purchases and increase the amount of money a customer typically spends (e.g, Heilman, Nakamoto and Rao 2002). However, highly price conscious or deal-prone customers typically have budget constraints and tend to pay

reduced prices (e.g., Völckner 2008). We expect that customers who redeem a medium level of coupons have the biggest shopping basket.

Avg\_Dmail<sub>*i*</sub> and Avg\_Email<sub>*i*</sub> are operationalized as the average number of direct mails or emails customer *i* received between two transactions. Marketing communications can retain existing customers and increase brand loyalty. But excessive marketing contacts could be detrimental to the firm-customer relationship (e.g., Venkatesan and Kumar 2004). We expect to identify an optimal level of marketing contacts such as direct mail and email.

Avg\_Return<sub>*i*</sub> is computed as the average number of product return occasions customer *i* made for each transaction. Avg\_CrossBuy<sub>*i*</sub> is computed as the average number of product categories customer *i* purchased in each transaction. The product return frequency has an inverted-U shape effect on the firm-customer relationship (Petersen and Kumar 2009). Customers who buy from multiple categories tend to shop from a wider range of products in a purchase occasion (Venkatesan and Kumar 2004). Similarly, we expect to find an inverted-U shape effect of product return and a positive effect of cross-buy on the average transaction amount.

Avg\_Open<sub>*i*</sub> and Avg\_Click<sub>*i*</sub> are operationalized as the number of emails customer *i* opened or clicked between two transactions. Permission-based email messages can increase customers' trust with the firm, their purchase intentions and their lifetime values (e.g., DuFrene et al. 2005; Jolley et al. 2013). We expect that customers with higher email open and click rates are more interested in the firm and spend more money on the firm.

Avg\_IPT<sub>*i*</sub> is operationalized as customer *i*'s average inter-purchase time which is computed across the customer's purchase history between the opt-in and the opt-out or the censoring time. We use the average inter-purchase time as a control variable and expect that customers who have

shorter inter-purchase time spend less money on each transaction.

#### 1.5.4 Modeling the Dependence using Pair-Copula Construction

Let  $\mathbf{X} = (X_1, X_2, X_3)$  be a vector of random variables with a joint density function as  $f(x_1, x_2, x_3)$ . We demonstrate how to decompose the joint density into a cascade of pair-copulas.

First, based on Sklar's theorem (Sklar 1959), the bivariate joint density can be expressed as

$$f(x_1, x_2) = c_{12}(F_1(x_1), F_2(x_2)) \cdot f_1(x_1) \cdot f_2(x_2) \quad (6)$$

where  $F_1(x_1)$  and  $F_2(x_2)$  are continuous marginal distributions and  $c_{12}(\cdot)$  is the bivariate pair-copula density. Based on basic probability theory, we can obtain the conditional density as

$$f(x_2|x_1) = c_{12}(F_1(x_1), F_2(x_2)) \cdot f_2(x_2) \quad (7)$$

The conditional density in a three-dimensional case is given by

$$\begin{aligned} f(x_2|x_1, x_3) &= \frac{f(x_2, x_3|x_1)}{f(x_3|x_1)} \\ &= \frac{c_{23|1}(F(x_2|x_1), F(x_3|x_1)) \cdot f(x_2|x_1) \cdot f(x_3|x_1)}{f(x_3|x_1)} \quad (8) \\ &= c_{23|1}(F(x_2|x_1), F(x_3|x_1)) \cdot f(x_2|x_1) \end{aligned}$$

where  $c_{23|1}(\cdot)$  is the suitable bivariate pair-copula density for  $F(x_2|x_1)$  and  $F(x_3|x_1)$ . Note that  $c_{23|1}(\cdot)$  captures the dependence between two reduced conditional distributions.

Based on Equations (6-8), we can write down the joint density  $f(x_1, x_2, x_3)$  using pair-copulas and the marginal densities. Applying the same logic to our empirical problem, we can construct the joint density function of opt-in time, opt-out time and average transaction amount. Since the opt-in and the opt-out times are data of lifetimes, we use survival copulas in our specification (e.g., Shih and Louis 1995; Nelsen 2006)

$$\begin{aligned}
f(t_{i1}, t_{i2}, AMT_i) = & c_{23|1}(S(t_{i2}|t_{i1}), S(AMT_i|t_{i1}); \Omega_{23|1}) \\
& \cdot c_{12}(S_1(t_{i1}), S_2(t_{i2}); \Omega_{12}) \cdot c_{13}(S_1(t_{i1}), S_3(AMT_i); \Omega_{13}) \\
& \cdot f_1(t_{i1}) \cdot f_2(t_{i2}) \cdot f_3(AMT_i)
\end{aligned} \tag{9}$$

where  $c_{23|1}(\cdot)$ ,  $c_{12}(\cdot)$  and  $c_{13}(\cdot)$ <sup>2</sup> are the density functions of the associated survival copulas;  $S_1(t_{i1})$ ,  $S_2(t_{i2})$  and  $S_3(AMT_i)$  are the marginal survival functions defined in Equations (2) and (4);  $S(t_{i2}|t_{i1})$  and  $S(AMT_i|t_{i1})$  are the conditional survival functions with a common conditioning variable  $t_{i1}$ ;  $f_1(t_{i1})$ ,  $f_2(t_{i2})$  and  $f_3(AMT_i)$  are the marginal densities defined in Equations (3-4);  $\Omega_{23|1}$ ,  $\Omega_{12}$  and  $\Omega_{13}$  are the pair-copula parameters (see also Panagiotelis, Smith and Danaher 2013).

Equation (9) only applies to the situation where the opt-in and opt-out times are both observed. We need to consider the cases when the opt-in or opt-out times are censored. First, when the opt-in time is observed but the opt-out time is censored, we need to evaluate the conditional survival function  $S(t_{i2}|t_{i1})$  which gives the probability the customer  $i$  stays in the email program for at least time  $t_{i2}$  given that the customer's the opt-in time is  $t_{i1}$ . The conditional survival function is given by the first partial derivative of the bivariate copula function (He and Lawless 2003),

$$S(t_{i2}|t_{i1}) = \frac{\partial C_{t_{i1}, t_{i2}}(S_1(t_{i1}), S_2(t_{i2}); \Omega_{12})}{\partial S_1(t_{i1})} \tag{10}$$

where  $C_{t_{i1}, t_{i2}}(\cdot)$  is the bivariate copula function and  $\Omega_{12}$  is the bivariate copula parameter defined in Equation (9). Second, when both the opt-in and opt-out times are censored, the customer-level likelihood is given by the marginal survival function  $S_1(t_{i1})$  (He and Lawless 2003).

The bivariate pair-copulas can be specified as Gaussian,  $t$ , Gumbel, Clayton, Frank, etc.

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<sup>2</sup> For distributions of high dimensions, the number of unique pair-copula decompositions increases significantly. Vine copulas, initially introduced by Joe (1997) and Bedford and Cooke (2002), provide a graphical way to organize the pair-copula construction conveniently. For a general way to construct a high-dimensional distribution through vine representation, readers can refer to the Appendix.

We empirically test Gaussian and Frank copulas in this study. We provide the copula distribution function, density, first partial derivative and its inverse function for the Gaussian and Frank bivariate copulas in the Appendix.

### 1.5.5 Model Estimation

We have two sets of parameters to estimate, one of the marginal models and the other of the pair-copulas. Following Shih and Louis (1995) and Danaher and Smith (2011), we use a two-step procedure<sup>3</sup> which yields consistent estimates for all parameters. In the first step, we estimate the parameters of each marginal model using maximum likelihood estimation (MLE). The marginal likelihoods are specified previously in Equations (1-5).

In the second step, we estimate the set of pair-copula parameters assuming the parameters estimated from the first step as fixed. We maximize the log-likelihood function given by

$$\begin{aligned}
 LL(\boldsymbol{\Omega}) = \sum_{i=1}^N \{ & \delta_{i1}\delta_{i2} \log(c_{12}(S_1(t_{i1}), S_2(t_{i2}); \Omega_{12})) + \delta_{i1}(1 \\
 & - \delta_{i2}) \log(S(t_{i2}|t_{i1}); \Omega_{12}) \\
 & + \delta_{i1} \log(c_{13}(S_1(t_{i1}), S_3(AMT_i)); \Omega_{13}) \\
 & + \delta_{i1}\delta_{i2} \log(c_{23|1}(S(t_{i2}|t_{i1}), S(AMT_i|t_{i1})); \Omega_{23|1})) \}
 \end{aligned} \tag{11}$$

where  $\delta_{i1}$  and  $\delta_{i2}$  are the indicator variables that equal 1 when  $t_{1i}$  and  $t_{2i}$  are observed and 0 otherwise;  $\boldsymbol{\Omega} = \{\Omega_{12}, \Omega_{13}, \Omega_{23|1}\}$  are the pair-copula parameters to be estimated. Note that for the customers who did not opt-in ( $\delta_{i1} = 0$ ), we use these observations in estimating the marginal opt-in model. However, we do not use them in the estimation of the copula dependence parameters because the dependence relies on the observed opt-in time.

Following Aas et al. (2009), we estimate these parameters sequentially. We first estimate

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<sup>3</sup> Pair-copula constructions can also be estimated using a Bayesian method (Min and Czado 2010).

$\Omega_{12}$  and  $\Omega_{13}$  by maximizing the first three terms of Equation (11). Second, we calculate  $S(t_{i2}|t_{i1})$  and  $S(AMT_i|t_{i1})$  in a way analogous to that in Equation (10), based on the estimates from the first step. Third, we estimate  $\Omega_{23|1}$  by maximizing the fourth term of Equation (11). Lastly, we use the estimates from the previous three steps as our starting point and maximize the full log-likelihood specified in Equation (11). We recover all the parameters in the simulation. We provide the data generating algorithm for simulation study in the Appendix. We also provide estimation details of the proposed model with Frank pair-copula in the Appendix.

## 1.6 Results

### 1.6.1 Main Findings

We estimate the model specified in Equations (1-11) with Gaussian and Frank as pair-copulas using a maximum likelihood estimation in GAUSS. We compare the log-likelihood and the BIC of the two models to choose the “best-fitting” model. Table 4 gives the in-sample log-likelihood and the BIC. Based on the log-likelihood and the BIC, we determine that the proposed model with pair-copulas specified as Frank copula provides a better fit to the calibration data. Thus, we choose the Frank copula specification in this study. Table 5 reports the estimates of the marginal models of the opt-in time, the opt-out time and the average transaction amount. We discuss the results of each model in the following sections.

Insert Tables 4 & 5 about here

*Opt-in Time Model Estimates.* We examine how a customer’s opt-in decision is affected. As the logarithm of  $\gamma_1$  is -0.600, we calculate that the variance of the gamma frailty term equals  $1/\exp(-0.600) = 1.82$ , indicating a strong degree of heterogeneity in customers’ opt-in decisions. Some customers are more prone to opt-in than others with the same covariates value.

After controlling for the unobserved heterogeneity, we explore some interesting findings.

Coupon redemption frequency has a negative effect on customer's opt-in probability. One of the main benefits of email program is the saving opportunities delivered through emails. If a customer has already been active in redeeming coupons which may be distributed through other channels such as direct mail, company website or referral, it is unlikely that the customer will turn to another marketing program as the marginal benefit would not be high enough.

We find that the number of direct mails a customer receives has a negative effect on opt-in probability but this effect diminishes with an increase in the quantity of direct mails. Consistent with Barnes and Scornavacca (2008), this finding suggests that the marketing exposure a customer receives affects his/her decision to opt-in. Since direct mail and email share similar marketing functions, the customers who are already contacted via many direct mails are less likely to join another marketing program, which could increase their information processing burden (Krishnamurthy 2001). In addition, the product return frequency has a negative but diminishing effect on the opt-in probability. This finding is consistent with the previous literature (e.g., Petersen and Kumar 2009) that customers with a moderate amount of product returns are the ones that firms should invest resources in to build a relationship with.

Furthermore, customers with different characteristics and online habits have different opt-in propensities, according to the estimates of the PersoniX variables. Customers who belong to the groups labeled "Second Nature Surfers" and "Voluminous Variety" are statistically significantly different in opt-in likelihood from those who belong to the reference group labeled "Superhighway Superusers"; while the rest of the customers do not show significant differences from the reference group. "Second Nature Surfers" customers are heavy online users, age from 24 to 39, either have no children or have just started a family, and prefer youth-oriented activities

such as music, social networking and online auctions (see Table 1). Customers of this group are less likely to opt-in in the retailer's email program probably because they have less interest in or limited use for home improvement products. "Voluminous Variety" customers are familiar with and tend to use the internet to obtain information on a daily basis (see Table 1). Customers of this group wouldn't join the retailer's email program easily probably because they have better ways or alternatives to obtain product and promotion information. These findings are also consistent with the previous studies (e.g., Brey et al. 2007) that socio-demographics and information search behavior affect customers' interest in permission marketing.

*Opt-out Time Model Estimates.* We discuss how a customer's opt-out decision is affected. Based on the estimate of  $\gamma_2$ , we calculate the variance of the gamma frailty term as  $1/\exp(1.994) = 0.14$ , indicating a moderate level of heterogeneity in customers' opt-out decisions. After controlling for the unobserved heterogeneity, we discover that coupon redemption frequency has a strong negative effect on customer's opt-out probability. If the saving opportunities delivered through emails are relevant to customer's needs, customers would be more responsive by making more purchases with coupons. In such a case, customers have no reason to opt-out to turn away from an effective email program. In addition, the number of direct mails or emails a customer receives has a U-shape effect on the opt-out probability. In line with previous research (e.g., Krishnamurthy 2001; Nash 1993), we find that too much communication is harmful to the firm-customer relationship and makes customers less interested in the participation of the permission marketing program. Firms should plan an appropriate level of marketing intensity and avoid over-marketing to customers.

Furthermore, we find that email open or click rate has a negative but diminishing effect on opt-out probability. It indicates that customers who open email or click the links included in the

emails more often are less likely to end their email subscription. Consistent with Krishnamurthy (2001), the finding suggests that message relevance in terms of the category fit or incentive size is an important factor for customers to stay in a permission email program. Firms may customize their email messages based on customers' past purchases to increase the open and click rate. In line with Venkatesan and Kumar (2004), we also find that a moderate amount of product returns is healthy for the firm-customer relationship as it implies that customers are less likely to opt-out.

From the estimates of the PersoniX variables, we discover some interesting results. Compared with customers of the reference and the other groups, customers who belong to the groups labeled "Second Nature Surfers", "Affluent Aficionados", "My Internet, My Way" and "Functional Frequency" are statistically significantly less likely to opt-out. Noticeably, customers of these four groups stay in the email program probably for different reasons. For example, "Functional Frequency" customers subscribe to the email program to obtain promotional information because they are at an average age of 39, with low to middle income and need to raise a family. But, "Affluent Aficionados" customers opt-in probably for new product information or gardening workshops because they typically are well-educated and wealthy, at the age of retiring, and have disposable time to pursue their personal hobbies.

*Average Transaction Amount Model Estimates.* We discuss the factors that determine the dollar amount a customer spends per transaction. The findings related to the average transaction amount are all consistent with previous research (e.g., Venkatesan and Kumar 2004; Völckner 2008; Petersen and Kumar 2009). Customers with a moderate level of coupon redemption or product return history are expected to spend the most. A moderate level of marketing such as direct mail and email is healthy for the firm-customer relationship and over-marketing would decrease a customer's purchase spending. Customers who buy across multiple product categories

tend to spend more in each transaction. In addition, we find that the number of emails opened between two transactions has a positive but diminishing effect while the number of email links clicked between two transactions has no statistically significant effect on the spending size. Although the current available information does not allow us to link the email open or click rate directly to purchases, we suspect that the retailer's current email strategy does not result in a good conversion rate. The email messages sent probably have more advertising effect than instantaneous promotional effect (e.g., Li, Sun and Montgomery 2011). If so, we suggest the retailer customize their email messages which could help improve the conversion rate.

*Pair-Copula Dependences.* Table 5 reports both the Frank copula parameter estimates and the transformed Spearman's rho coefficients (in parentheses). Spearman's rho measures the rank-order correlation coefficient which is not affected by the specification of the marginal distributions of the raw data (see Danaher and Smith 2011). The bivariate copula parameter  $\Omega_{12}$ , which has an estimate of 0.204 of Spearman's rho, shows a moderate dependence between opt-in and opt-out times. Customers who take a longer time to opt-in tend to stay in the email program for a longer time. The bivariate copula parameter,  $\Omega_{23|1}$ , which has an estimate of 0.094 of Spearman's rho, measures the dependence between opt-out time and average transaction amount, conditional on opt-in time. Caution should be exercised for the interpretation of  $\Omega_{23|1}$ , as it measures the dependence between two conditional distributions. However, if the interest is in the unconditional dependence such as  $\Omega_{23}$ , it can be obtained by permuting the variables specified in the vine structure in Equation (9) and re-estimate the model.

### 1.6.2 Opt-Out Time Prediction

The key questions the manager of a permission-based email program faces are: when do

email subscribers opt-out, and how to prevent them from leaving. In this section, we attempt to use the pair-copula model proposed in this study to predict the customer opt-out time at an individual level. Because the opt-out time is dependent on the opt-in time and the average transaction amount, in order to predict the mean opt-out time, we need to evaluate the conditional expectation which can be expressed as (e.g., Yeo and Valdez 2006)

$$E(t_{i2}|t_{i1}, AMT_i) = \int_0^{\infty} t_{i2} \cdot f_{2|13}(t_{i2}|t_{i1}, AMT_i) dt_{i2} \quad (12)$$

where  $f_{2|13}(\cdot)$  is given by  $c_{23|1}(S(t_{i2}|t_{i1}), S(AMT_i|t_{i1}))c_{12}(S_1(t_{i1}), S_2(t_{i2}))f_2(t_{i2})$  (see Equation 8). Since the integral in Equation (12) does not have a closed-form, we solve the integral in a numerical way and obtain the prediction of mean opt-out time at the individual customer level.

We also compare the proposed model with four benchmark models for validation on the holdout sample. For comparison purposes, we only predict the opt-out time for customers who have completely observed opt-in and opt-out data. The four benchmark models are (1) univariate model of opt-out time (here, univariate means the model does not consider any dependence with other variable(s)), (2) univariate model of the opt-out time with the observed opt-in time as a covariate, (3) bivariate model of the opt-out time and the average transaction amount with the observed opt-in time as a covariate (here, bivariate means a bivariate frank copula model with the marginal models specified as in Equations 1-5), and (4) bivariate model of opt-in and opt-out time (see Table 6).

Insert Table 6 about here

Armstrong, Morwitz and Kumar (2000) define Relative Absolute Error (RAE) as the mean absolute deviation of the error values of a model relative to that of the benchmark model. So a higher RAE indicates better predictive performance. As Table 6 shows, the proposed vine copula

model gives the best prediction of mean opt-out time as all the RAEs of the benchmark models are less than 1. If we compare the RAEs among the benchmark models, we find that by including the observed opt-in time as a covariate in the univariate model of opt-out time (RAE=0.58), we do not improve the prediction accuracy, as compared to the univariate model without opt-in time as a covariate (RAE=0.65). In addition, we discover that modeling the opt-in and opt-out times together (RAE=0.70) slightly improves the predictive performance over the univariate model of opt-out time (RAE=0.65). More importantly, the model that considers the dependence among the opt-in time, the opt-out time and the average transaction amount improves the predictive performance significantly. In the next section, we demonstrate the changes of customer opt-in and opt-out times by simulating different levels of marketing activities.

## **1.7 Managerial Implications**

### **1.7.1 Marketing Policy Simulation**

Using the parameter estimates (in Table 5), we can assess how a firm's marketing policy affects the opt-in and opt-out behavior of its customers. We conduct several simulations to show that firms can adjust their marketing contact frequency to strategically manage customers' opt-ins and opt-outs. To do so, we select the customers who have observed data of opt-in and opt-out from the holdout sample, constructing a sample of 1,696 customers. The average opt-in time of the sample is 15.9 months and the average opt-out time is 13.4 months. By doing the simulations, we attempt to answer two questions: (1) What is the impact on customers' opt-in time if the retailer changes the direct mail marketing intensity before customers opt-in? (2) What is the impact on customers' opt-out time if the retailer changes the direct mail and email marketing intensity after customers opt-in?

To answer the first question (see Scenario 1 in Table 7), we vary the average number of direct mail a customer receives per month before opt-in and predict the changes in customer opt-in time using the hazard model specified in Equations (1-3). As Table 7 shows, the increase in the frequency of direct mail contact could increase the time a customer takes to opt-in the email program. For example, one more direct mail per month would prolong the customer opt-in time for an average of 13.6 months while 4 more direct mails per month would make customers need an average of 47.6 more months to opt-in. This suggests that firms should be cautious of the cannibalizing effect that direct mail contact can have on their email program subscription rate. If a firm has already marketed its customers with a massive number of direct mails every month, its customers probably would not want to join the firm's email program. While most firms may treat direct mail and email as two separate marketing activities in practice, we suggest that firms should coordinate well between the two. Firms should closely monitor the ROI of their marketing activities and allocate resources accordingly among different programs, such as direct mail, email, web and mobile to maximize company profits.

Insert Table 7 about here

To address the second question (see Scenario 2 in Table 7), we vary the marketing contacts (direct mail and email) the retailer sends out to its customers after they opt-in. As the marketing contacts also influence the purchase behavior, we predict the opt-out time using our proposed vine copula model which considers the influence of the opt-in time and purchase (see Equation 12). As Table 7 shows, the increase of direct mail contact initially could make customers stay with the email program for longer time. For example, 1 more direct mail per month would extend the customer opt-out time for an average of 7.9 months. However, this positive effect diminishes and in fact, reverses after the increase reaches a threshold. Table 7 shows that 3 more direct

mails per month can extend the opt-out time for a shorter length of time than 1 or 2 more direct mails can. Similarly, the increase in the number of emails sent to customers also has an inverted-U shape effect on extending the opt-out time. For example, 10 more emails sent per month can extend the opt-out time for an average of 3.6 months while 25 more emails sent per month can only extend the opt-out time for an average of 2.6 months. These findings suggest that firms should not focus on increasing marketing intensity but on the contents of the marketing messages they deliver to customers. Customers are willing to receive marketing materials that can match their needs, including category fit and monetary incentives (e.g., Krishnamurthy 2001). Firms should customize their marketing efforts based on customers' interests, preferences and past purchase histories. Undifferentiated mass marketing not only results in poor targeting but also generate negative feelings and leads to email opt-out and customer churn.

### **1.7.2 Optimal Resource Allocation**

The parameter estimates in Table 5, show that a firm's marketing contact policy influences both the length of time a customer stays in an email program and the average amount a customer spends on a transaction while he/she is subscribing to the email program. In this section, we address the question that under the current budget constraint, how the firm can optimally reallocate budget to different customers and across different marketing channels (direct mail and email) to maximize both customer's email subscription time and the sales revenue. We randomly select four customers who are predicted to opt-out between 1 and 2 years after they opt-in and simulate the optimal marketing contact decisions. Our simulation is based on the assumption that customers will demonstrate a similar purchase frequency to that during their subscription to the email program if the length of subscription time were extended. We argue that the assumption is

reasonable because customers who subscribe to the email program are exposed to persuasive email messages aimed at retaining them; cross-selling and up-selling to them; and customers in such a relationship are therefore more likely be more active purchasers than those who are not enrolled in the email program.

The retailer provides us with the approximate cost estimates such that one direct mail costs \$0.67 and one email costs \$0.25<sup>4</sup>. So we calculate that the firm spent an average of \$12.36 in total on the four customers in each month during the period of time they stayed in the email program. Under the firm's current marketing contact policy, Customers 1, 2, 3 and 4 are expected to stay in the email program for about 17, 17, 16 and 24 months, respectively, and opt-out of the email program. During their email subscription time, Customers 1, 2, 3 and 4 contribute an average of \$25.35, \$6.05, \$34.28 and \$12.77 in profit every month, respectively (see Table 8). Here, although we acknowledge that the gross margin varies across product categories, we apply a constant 30% gross margin to calculate profit and the potential profit mentioned afterwards with the consent from the retailer.

To find the optimal marketing contact decisions, we keep the current budget constraint \$12.36/month unchanged and set the marketing contacts as changing cells to be optimized using Excel Solver. We calculate the expected length of email subscription time and the average transaction amount using our proposed model (Equations 1-11) and the estimated parameters (Table 5). In the optimization process, we set the objective as to maximize the total profit generated from the four customers. Under the current budget constraint, we find the optimal contacting strategies as increasing the marketing contact for Customers 1 and 4 through both

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<sup>4</sup> We acknowledge that the average unit cost of an email is higher than average industry standard. The cost is estimated by the retailer who generally runs email marketing campaigns. Each campaign may include the costs of initial content selection and design, marketing research, pre-test operation, modification and redesign, mailing and post-evaluation.

direct mail and email while decreasing the spending on Customers 2 and 3 (see Table 8). For example, the retailer should target Customers 1 and 2 with both one direct mail every other week and about one email every other day or every week, respectively. The expected benefits include extending Customers 1 and 2's email subscription time to the maximum of about 30 and 39 months and increasing their potential profit to \$50.07 and \$41.43 per month, respectively.

Insert Table 8 about here

In summary, the two additional analyses discussed in this section indicate that marketing intensity has a significant influence on customers' opt-in and opt-out time. By strategically reallocating resources across different communication channels, firms can extend the length of time their email subscribers stay with them and maximize customers' spending.

## **1.8 Conclusions and Future Research**

The objective of this paper is to explore the factors that are critical for managing an effective permission-based marketing program, for example, email marketing. To maximize ROI, firms always desire to increase customer's opt-in rate and decrease their opt-out rate. To achieve these goals, marketers may want to understand what makes customers willing to grant permission to firms, what triggers them to withdraw, and how to influence their decisions.

Once customers have joined the permission marketing program, firms can send marketing messages to customers' email inbox or mobile device to influence their purchase behavior. A customer's decision to stay in the marketing program is associated with his/her purchase behavior and capturing such dependence helps predict when the customer is likely to opt-out. This paper proposes a trivariate copula model that can jointly model a customer's opt-in time, the opt-out time and the average transaction amount. The empirical study discovers a positive

dependence between the opt-in and the opt-out time and a positive dependence between the opt-out time and the average transaction amount conditional on the opt-in time. By capturing such dependence, the proposed model improves the predictive performance of the opt-out time over several benchmark models.

In addition, this paper discovers several important findings of managerial relevance. It finds that customers with certain characteristics are more likely to opt-in or opt-out. It finds that customers under a high marketing intensity are less likely to opt-in. After customers have joined the marketing program, over-marketing could make them withdraw more quickly. Through a simulation study, this paper demonstrates how to optimally allocate resources to different channels such as direct mail and email under the current budget constraint. Furthermore, this paper also finds that higher email open rate leads to higher spending levels, suggesting firms deliver marketing messages that are relevant to their email subscribers.

To the best of our knowledge, this is the first study that models customer opt-in and opt-out time while incorporating the purchase behavior. Nevertheless, there are some limitations we would like to address. First, because permission marketing concerns customer interests and needs, our findings may be constrained by the industry of analysis. For example, if we are to analyze a permission-based email program for another industry like music, the people who are most likely to have active interest in the category are probably those who are younger, in the low-to medium-income bracket, and heavy mobile and online users who are fans of social media, online shopping, and so on. As the music industry is more digitalized, traditional marketing channels like direct mail may not be as influential as digital channels like online search or social media or email marketing. Thus, to generalize our findings from this study, future research could apply our model to other product categories or industries.

Second, in this study, our main objective is neither to generalize under what conditions opt-in and opt-out times are positively or negatively correlated nor offer explanations for such phenomena. Our main focus is to capture such dependence through an empirical model and provide a better prediction of opt-out time. While we recognize the importance of identifying the causes of possible dependence, we leave this question for future study.

Third, in the optimization process, while we capture the effects of marketing contact on purchase amount through the log-normal model, we keep customer's purchase frequency and email open and click rate as constant. Nevertheless, it is possible that customers may increase or decrease their purchase frequency and email response rate if the marketing intensity changes. Future research could take these factors into consideration.

Fourth, to apply our proposed model to other permission-based contexts, such as mobile or social media, there are more factors to consider. Mobile-based permission marketing may depend on factors such as the design of mobile website, cell phone screen size and resolution, and the ease of mobile payment. Social-networking-based permission marketing could depend on the number of "friends", their profile and activeness and privacy concerns. Nowadays, email providers such as Google have redesigned the email inbox to allow users to categorize their emails using "tabs". For example, different tabs can be created to organize emails from different sources, such as 'Work', 'Social networks' and 'Promotions'. Such a feature could make customers less likely to open an email from a less important tab such as 'Promotions'. It would be interesting for future research to study how such environmental factors and changes in interfaces impact the participation of customers in permission-based marketing programs.

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**TABLE 1 DATABASE SEGMENTATION DESCRIPTIONS**

<b>Variables</b>	<b>Segments Label</b>	<b>Average Age</b>	<b>Income /Wealth</b>	<b>Sociographics and Online Behavior</b>
Group1	Superhighway Superusers	25~55	Medium to High	Extremely comfortable online user, like sports, music, social network, shopping or investment
Group2	Second Nature Surfers	24~39	Low to High	Frequent mobile user, no children or just start a family, online shopper, like music, job search, online auction and social network
Group3	High-Speed Checkout	41~42	High	Online shopper, preferring either apparel, toys, games or travel
Group4	Affluent Aficionados	56~68	High	Heavy online user, working, shopping and investment
Group5	Voluminous Variety	38~39	Medium to High	Heavy online user, either child-centric or pursuing personal hobbies like news, sports and travel
Group6	My Internet, My Way	24~40	Medium to High	Fans of online social networking, job searches, and personal entertainment
Group7	ECommerce Experts	55~70	Low to High	Heavy online shopper and online search like automobile category
Group8	Selective Surfers	54~58	Medium to High	Moderate online user, focusing on relaxation, social network, investment, and shopping
Group9	Rural Connections	41~58	Medium to High	Below average online user, focusing on insurance quotes, sports apparel or phone call
Group10	Senior Investors	67~78	High	Fans of online shopping and investment
Group11	Functional Frequency	38~40	Low to Medium	Home-centric, online usage mostly for job searches and some social network
Group12	Limited Logons	56~58	Low to Medium	Low online usage, mostly evenings or weekends
Group13	Sans Surfers	67~78	Low to Medium	Very low online activities, preferring traditional channels like direct mail and telephone

Source: Acxiom PersoniX Digital

**TABLE 2 SUMMARY STATISTICS OF BEHAVIORAL VARIABLES FOR SUBSCRIBERS AND NON-SUBSCRIBERS**

Variables	Operationalization *	Email Non-Subscribers Sample		Email Subscribers Sample**	
		Mean	Standard Deviation	Mean	Standard Deviation
Total_Money (\$)	The total amount of money spent	4,440	5,755	5,430	6,503
Total_Freq	The total number of purchase occasions	41.50	52.65	51.05	52.37
Total_Coupon	The total number of coupons redeemed	1.17	2.57	1.77	3.25
Total_Dmail	The total number of direct mail campaigns targeted	4.44	5.31	4.81	5.30
Total_Return_Freq	The total number of return occasions	4.36	7.67	5.58	8.35
Avg_Cross_Buy	The average number of product categories purchased per transaction	2.66	1.00	2.73	0.93

\*The variables are computed for the time period from February, 2007 to December, 2010.

\*\*For email non-subscribers, they didn't opt-in from February, 2007 to December, 2010. For email subscribers, they opted in on February, 2007 and have stayed in the email program till December, 2010.

**TABLE 3 PURCHASE COMPARISONS OF CUSTOMERS WITH DIFFERENT OPT-OUT TIMES**

	Sample Size (N=103)	Average Inter-purchase Time before Opt-In (Months)	Time Elapsed between Opt-In and Opt-Out (Months)	Total Purchase Amount between Opt-In and the End of Observation	Total Number of Purchase Occasions between Opt-in and the End of Observation
Cohort 1	27%		1~6	\$2,262	20
Cohort 2	43%	1.4~1.8	7~12	\$2,584	23
Cohort 3	30%		13~18	\$3,022	34

**TABLE 4 MODEL FIT COMPARISON**

	The Proposed Model with Pair-Copulas Specified as	
	Gaussian	Frank
Log-likelihood	-1281	-1265
BIC	2590	2558

**TABLE 5 PARAMETER ESTIMATES**

Covariates	Estimates	Standard Error	T-Value
<i>Opt-in Time Hazard Function</i>			
Intercept	0.914	0.069	13.192
COUPON <sub>1</sub> × 10 <sup>-1</sup>	-0.273	0.091	-2.992
DMAIL <sub>1</sub> (log)	-1.294	0.052	-24.691
RETURN <sub>1</sub> × 10 <sup>-1</sup> (log)	-0.770	0.062	-12.486
PERSONICX DUMMY 1 (Second Nature Surfers)	-0.194	0.088	-2.206
PERSONICX DUMMY 2 (High-Speed Checkout)	-0.059	0.076	-0.776
PERSONICX DUMMY 3 (Affluent Aficionados)	0.080	0.068	1.172
PERSONICX DUMMY 4 (Voluminous Variety)	-0.179	0.070	-2.566
PERSONICX DUMMY 5 (My Internet, My Way)	-0.123	0.080	-1.533
PERSONICX DUMMY 6 (ECommerce Experts)	-0.008	0.065	-0.121
PERSONICX DUMMY 7 (Selective Surfers)	-0.047	0.074	-0.641
PERSONICX DUMMY 8 (Rural Connections)	-0.091	0.075	-1.204
PERSONICX DUMMY 9 (Senior Investors)	0.081	0.080	1.013
PERSONICX DUMMY 10 (Functional Frequency)	-0.038	0.082	-0.465
PERSONICX DUMMY 11 (Limited Logons)	-0.111	0.088	-1.260
PERSONICX DUMMY 12 (Sans Surfers)	0.023	0.087	0.261
Weibull Shape $\alpha_1$ (log)	0.517	0.016	33.017
$\gamma_1$ (log)	-0.600	0.080	-7.467
<i>Opt-out Time Hazard Function</i>			
Intercept	3.676	0.425	8.642
COUPON <sub>2</sub> × 10 <sup>-2</sup>	-6.825	2.406	-2.837
DMAIL <sub>2</sub>	-2.224	0.185	-12.052
(DMAIL <sub>2</sub> ) <sup>2</sup>	0.488	0.046	10.656
RETURN <sub>2</sub> × 10 <sup>-1</sup>	-1.713	0.227	-7.562
(RETURN <sub>2</sub> × 10 <sup>-1</sup> ) <sup>2</sup>	0.265	0.065	4.046
EMAIL <sub>2</sub> × 10 <sup>-1</sup>	-1.194	0.331	-3.602
(EMAIL <sub>2</sub> × 10 <sup>-1</sup> ) <sup>2</sup>	0.283	0.136	2.081

OPEN <sub>2</sub>	-1.256	0.197	-6.374
CLICK <sub>2</sub>	-2.413	0.270	-8.933
PERSONICX DUMMY 1 (Second Nature Surfers)	-0.552	0.311	-1.777
PERSONICX DUMMY 2 (High-Speed Checkout)	-0.238	0.271	-0.878
PERSONICX DUMMY 3 (Affluent Aficionados)	-0.450	0.255	-1.769
PERSONICX DUMMY 4 (Voluminous Variety)	-0.381	0.260	-1.468
PERSONICX DUMMY 5 (My Internet, My Way)	-0.642	0.281	-2.283
PERSONICX DUMMY 6 (ECommerce Experts)	-0.027	0.236	-0.113
PERSONICX DUMMY 7 (Selective Surfers)	0.106	0.269	0.395
PERSONICX DUMMY 8 (Rural Connections)	-0.112	0.272	-0.413
PERSONICX DUMMY 9 (Senior Investors)	0.533	0.263	2.028
PERSONICX DUMMY 10 (Functional Frequency)	-0.735	0.290	-2.532
PERSONICX DUMMY 11 (Limited Logons)	-0.229	0.297	-0.770
PERSONICX DUMMY 12 (Sans Surfers)	0.209	0.301	0.695
Weibull Shape $\alpha_2$ (log)	0.670	0.050	13.483
$\gamma_2$ (log)	1.994	0.099	20.160
<i>Average Transaction Amount*</i>			
Intercept	-4.021	0.052	-77.157
Avg_Coupon $\times 10^{-1}$	17.379	1.399	12.423
(Avg_Coupon $\times 10^{-1}$ ) <sup>2</sup>	-82.609	17.021	-4.853
Avg_Dmail (log)	0.177	0.020	8.729
Avg_Email $\times 10^{-3}$	11.040	1.128	9.786
(Avg_Email $\times 10^{-3}$ ) <sup>2</sup>	-74.068	8.070	-9.179
Avg_Return $\times 10^{-1}$	9.460	0.906	10.437
(Avg_Return $\times 10^{-1}$ ) <sup>2</sup>	-46.566	9.284	-5.016
Avg_CrossBuy (log)	0.829	0.036	22.880
Avg_Open $\times 10^{-3}$ (log)	4.690	1.545	3.035
Avg_Click $\times 10^{-3}$ (log)	-6.444	6.326	-1.019
Avg_IPT (months)	0.071	0.006	11.557
$\sigma^2$ (log)	-0.219	0.009	-24.377
$\sigma_{\mu_0}^2$ (log)	-4.620	0.850	-5.435
<i>Pair-Copula Dependences</i>			
$\Omega_{12}^{**}$	1.081(0.204)	0.135	7.993
$\Omega_{13}$	-0.261(-0.043)	0.153	-1.701
$\Omega_{23 1}$	0.567(0.094)	0.128	4.444

\*Average transaction amount is scaled by  $10^{-3}$ .

\*\*The corresponding Spearman's rho in the parentheses (see Trivedi and Zimmer 2005 for the transformation of the dependence measures).

**TABLE 6 COMPARISON OF THE PROPOSED MODEL WITH BENCHMARK MODELS**

	RAE* Relative to the Benchmark Model (with Respect to Opt-out Time)
<i>Proposed Model</i>	
Trivariate model of opt-in, opt-out and average transaction amount	—
<i>Benchmark Models</i>	
1. Univariate model of opt-out time	0.65
2. Univariate model of opt-out time with observed opt-in time as a covariate	0.58
3. Bivariate model of opt-out time and average transaction amount with observed opt-in time as a covariate	0.61
4. Bivariate model of opt-in and opt-out time	0.70

\*Relative absolute error is defined as the mean absolute deviation in the opt-out time (in months) prediction of the proposed model relative to that of the benchmark model (see Armstrong, Morwitz and Kumar 2000).

**TABLE 7 MARKETING POLICY SIMULATION RESULTS**

	Scenario 1— Email Program Opt-in Time (in Months)		Scenario 2— Email Program Opt-out Time (in Months)	
	Prediction	Changes	Prediction	Changes
No change on marketing intensity	23.0 (9.2)*	—	17.5 (12.8)	—
Average number of direct mail per month +1	36.6 (10.2)	13.6	25.4 (12.1)	7.9
Average number of direct mail per month +2	48.7 (11.4)	25.7	27.0 (12.7)	9.5
Average number of direct mail per month +3	59.9 (12.8)	36.9	21.9 (13.4)	4.4
Average number of direct mail per month +4	70.6 (14.2)	47.6	12.1 (10.2)	-5.4
Average number of email per month +5			19.7 (13.0)	2.2
Average number of email per month +10			21.1 (12.9)	3.6
Average number of email per month +15			21.7 (12.7)	4.2
Average number of email per month +20			21.4 (12.6)	3.9
Average number of email per month +25			20.1 (12.4)	2.6
Average number of email per month +30			17.9 (21.1)	0.4

\*Standard deviation in the parentheses.

**TABLE 8 COMPARISON OF CURRENT AND OPTIMAL CONTACTING STRATEGY**

	<b>Current Resource Allocation (per month)</b>				<b>Optimal Resource Allocation (per month)</b>			
	Direct Mail	Email	Time expected to stay in the email program (in months)	Current Profit	Direct Mail	Email	Time expected to stay in the email program (in months)	Potential Profit
Customer 1	1	8	17	\$25.35	2	13	30	\$50.07
Customer 2	4	2	17	\$6.05	2	4	39	\$41.43
Customer 3	2	12	16	\$34.28	1	6	40	\$47.80
Customer 4	1	6	24	\$12.77	2	7	35	\$26.83

## Appendix A: Estimation of the Proposed Pair-Copula Model

We show the estimation of the trivariate copula model with Frank pair-copulas. Assume we have a three-dimensional data simulated using the algorithm (see the Appendix C) with  $N$  observations. Let  $(T_{i1}, T_{i2})$  denote the simulated paired opt-in and opt-out times. Since opt-in and opt-out are sequentially observed events, if the observation window is of a fixed length  $C_i$  and the opt-in time is observed as  $T_{i1}$ , the maximum (censoring) lifetime that can be observed for opt-out time  $T_{i2}$  is  $C_{i2}^{max} = C_i - T_{i1}$ . Let  $(t_{i1}, t_{i2}) = (\min(T_{i1}, C_i), \min(T_{i2}, C_i - T_{i1}))$  denote the observed opt-in and opt-out times. Let  $x_{i3}$  denote the simulated log-normally distributed variable that is only observed when the opt-in time is observed.

Using the two-step procedure, we first estimate the parameters of each marginal model (see Equations 1-5 in the main text) using maximum likelihood estimation. Second, we plug in these estimates to estimate the pair-copula parameters by maximizing the log-likelihood function specified in Equation (11) in the main text. Following Aas et al. (2009), we estimate the parameters sequentially. First, we estimate the parameter  $\Omega_{12}$  from the original data by maximizing the first two terms of Equation (11) given by

$$l_1(\Omega_{12}) = \sum_{i=1}^N \{ \delta_{i1} \delta_{i2} \log(c_{12}(S_1(t_{i1}), S_2(t_{i2}); \Omega_{12})) + \delta_{i1} (1 - \delta_{i2}) \log(S(t_{i2}|t_{i1}); \Omega_{12}) \} \quad (13)$$

where the bivariate copula density  $c_{12}(\cdot)$  is given by

$$c_{12}(S_1(t_{i1}), S_2(t_{i2}); \Omega_{12}) = \frac{e^{-\Omega_{12}(u_{i1}+u_{i2})}(1 - e^{-\Omega_{12}})\Omega_{12}}{[e^{-\Omega_{12}} - 1 + (e^{-\Omega_{12}u_{i1}} - 1)(e^{-\Omega_{12}u_{i2}} - 1)]^2} \quad (14)$$

and the conditional survival function  $S(t_{i2}|t_{i1})$  is given by

$$S(t_{i2}|t_{i1}; \Omega_{12}) = \frac{e^{-\Omega_{12}u_{i1}}(e^{-\Omega_{12}u_{i2}} - 1)}{e^{-\Omega_{12}} - 1 + (e^{-\Omega_{12}u_{i1}} - 1)(e^{-\Omega_{12}u_{i2}} - 1)} \quad (15)$$

We also estimate the parameter  $\Omega_{13}$  from the original data by maximizing the third term of Equation (11) given by

$$l_2(\Omega_{13}) = \sum_{i=1}^N \delta_{i1} \log(c_{13}(S_1(t_{i1}), S_3(x_{i3}); \Omega_{13})) \quad (16)$$

where the bivariate copula density  $c_{13}(\cdot)$  is given by

$$c_{13}(S_1(t_{i1}), S_3(x_{i3}); \Omega_{13}) = \frac{e^{-\Omega_{13}(u_{i1}+u_{i3})}(1 - e^{-\Omega_{13}})\Omega_{13}}{[e^{-\Omega_{13}} - 1 + (e^{-\Omega_{13}u_{i1}} - 1)(e^{-\Omega_{13}u_{i3}} - 1)]^2} \quad (17)$$

Here, we use  $u_{i1}$ ,  $u_{i2}$  and  $u_{i3}$  to denote the marginal survival functions  $S_1(t_{i1})$ ,  $S_2(t_{i2})$  and  $S_3(x_{i3})$  for variables  $t_{i1}$ ,  $t_{i2}$  and  $x_{i3}$ .

Second, we estimate the parameter  $\Omega_{23|1}$  by maximizing the fourth term of Equation (11)

$$l_3(\Omega_{23|1}) = \sum_{i=1}^N \delta_{i1} \delta_{i2} \log(c_{23|1}(S(t_{i2}|t_{i1}), S(x_{i3}|t_{i1}); \Omega_{23|1})) \quad (18)$$

where the bivariate copula density  $c_{23|1}(\cdot)$  is given by

$$\begin{aligned} c_{23|1}(S(t_{i2}|t_{i1}), S(x_{i3}|t_{i1}); \Omega_{23|1}) \\ = \frac{e^{-\Omega_{23|1}(u_{i2|1}+u_{i3|1})}(1 - e^{-\Omega_{23|1}})\Omega_{23|1}}{[e^{-\Omega_{23|1}} - 1 + (e^{-\Omega_{23|1}u_{i2|1}} - 1)(e^{-\Omega_{23|1}u_{i3|1}} - 1)]^2} \end{aligned} \quad (19)$$

where  $u_{i2|1}$  and  $u_{i3|1}$  denotes the conditional survival functions  $S(t_{i2}|t_{i1})$  and  $S(x_{i3}|t_{i1})$ , which are computed as

$$S(t_{i2}|t_{i1}) = \frac{e^{-\Omega_{12}u_{i1}}(e^{-\Omega_{12}u_{i2}} - 1)}{e^{-\Omega_{12}} - 1 + (e^{-\Omega_{12}u_{i1}} - 1)(e^{-\Omega_{12}u_{i2}} - 1)} \quad (20)$$

and

$$S(x_{i3}|t_{i1}) = \frac{e^{-\Omega_{13}u_{i1}}(e^{-\Omega_{13}u_{i3}} - 1)}{e^{-\Omega_{13}} - 1 + (e^{-\Omega_{13}u_{i1}} - 1)(e^{-\Omega_{13}u_{i3}} - 1)} \quad (21)$$

where  $\Omega_{12}$  and  $\Omega_{13}$  are estimated from the previous steps.

Third, using the estimates of  $\Omega_{12}$ ,  $\Omega_{13}$  and  $\Omega_{23|1}$  from the previous steps as the starting value, we maximize the full log-likelihood specified in Equation (11) to obtain the final

estimates  $\hat{\Omega}_{12}$ ,  $\hat{\Omega}_{13}$  and  $\hat{\Omega}_{23|1}$ .

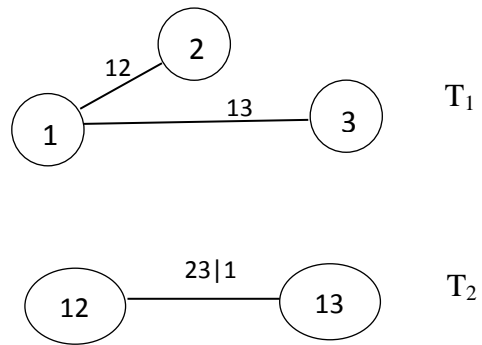
## Appendix B: Vine Copulas

Joe (1996) is the first one to construct a multivariate copula using pair-copulas. Since the possible ways of pair-copula constructions are not unique and can increase significantly for high dimensional distributions, Bedford and Cooke (2001, 2002) develop a graphical model, referred as *regular vine*, to organize the decompositions. Research thereafter focuses mainly on two specific, but important ways of decomposition, referred as *canonical vine* and *D-vine* (Kurowicka and Cooke 2004). We briefly introduce these two methods of pair-copula constructions.

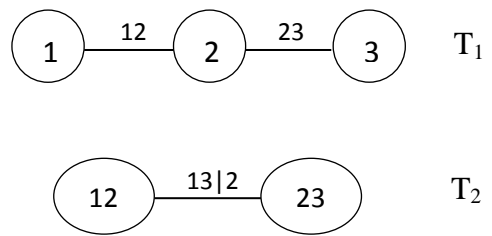
Both the canonical vine and the D-vine can be specified as a nested set of trees. A  $d$ -dimensional vine consists of  $d - 1$  trees,  $T_1, \dots, T_{d-1}$  for  $i = 1, \dots, d - 1$ . Each tree  $T_i$  consists of nodes,  $N_i$ , which are connected by edges  $E_i$ . In Figures 1 and 2, we show a possible decomposition corresponding to a three-dimensional canonical vine and D-vine, respectively. For the canonical vine, in tree  $T_1$  there is one key variable, called *root*, that connects to other variables with  $d - 1$  edges. For the D-vine, there is no node that has more than two edges. Figure 2 shows that in tree  $T_1$ , the marginal densities are lined up in order and the edges that connect them represent the unconditional pair-copula densities. For example, edge 12 represents the pair-copula density denoted  $c_{12}(\cdot)$  as in the main text. In trees  $T_2$ , the conditional distributions are connected by the edge which corresponds to the conditional bivariate copula density. For example, edge 13|2 represents the pair-copula density which can be denoted as  $c_{13|2}(\cdot)$ .

Aas et al. (2009) discuss that the use of a tree structure to represent vine copulas is not strictly required to construct a multivariate copula. But the tree structure helps researchers

visualize the possible ways of pair-copula decompositions.



**Figure 1 A Canonical Vine Representation for  $d = 3$**



**Figure 2 A D-Vine Representation for  $d = 3$**

## Appendix C: Simulation Study

Following Aas et al. (2009), we demonstrate how to simulate data using pair-copula constructions with three random variables. Since the construction relates to the marginal conditional distributions, it is useful to first define the  $h$ -function and the  $h^{-1}$ -function. Joe (1996) shows that, for every  $j$

$$F(x|\mathbf{v}) = \frac{\partial C_{x,v_j|\mathbf{v}_{-j}}(F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j}))}{\partial F(v_j|\mathbf{v}_{-j})} \quad (22)$$

where  $v_j$  can be any element chosen from the vector  $\mathbf{v}$  and  $\mathbf{v}_{-j}$  denotes the vector  $\mathbf{v}$  excluding element  $v_j$ .  $C_{x,v_j|\mathbf{v}_{-j}}$  is a bivariate copula distribution function. For the special case where  $\mathbf{v}$  has only one component, Equation (WB1) reduces to

$$F(x|v) = \frac{\partial C_{xv}(F(x), F(v); \theta)}{\partial F(v)} \quad (23)$$

where  $F(x|v)$  is referred by Aas et al. (2009) as the  $h$ -function  $h(x|v; \theta)$  and is given by the first partial derivative of the bivariate copula function  $C_{xv}(\cdot)$  with the dependence parameter  $\theta$ . The inverse of the conditional distribution  $F(x|v)$  is defined as the  $h^{-1}$ -function  $h^{-1}(x|v; \theta)$ , the inverse of the  $h$ -function.

In this study, we need to simulate three random variables with three-dimensional dependence structure. Two variables are both simulated from a Weibull hazard model and the other variable is simulated from a log-normal model. Following Aas et al. (2009), we simulate the data with the following algorithm:

- Step 1: Generate independent uniform (0, 1) random variables  $z_1, z_2, z_3$ ;
- Step 2: Make  $z_1 = u_1$ ;
- Step 3: Make  $u_2 = h^{-1}(z_2|u_1; \theta_{12})$  and calculate  $F(u_2|u_1) = h(u_2|u_1; \theta_{12})$ ;
- Step 4: Make  $u_3 = h^{-1}\{h^{-1}(z_3|h(u_2|u_1; \theta_{12}); \theta_{23|1})|u_1; \theta_{13}\}$ ;

Step 5: Invert the Weibull distribution functions  $u_1$  and  $u_2$  to obtain the survival times  $T_1$  and  $T_2$ . For example,  $T_{i1} = \left[ -\frac{\ln(1-u_{i1})}{\lambda_{i1}} \right]^{\frac{1}{\alpha_1}}$ , where  $\alpha_1$  and  $\lambda_{i1}$  are the shape and scale parameters of the Weibull distribution of time,  $T_{i1}$ ;  $\lambda_{i1}$  is specified as a function of the covariates as Equation (1) in the main text with a multiplicative gamma random effect;

Step 6: Since there is no closed-form for the inverse of normal distribution, we follow Atkinson and Pearce (1976) to approximate the standard normal distribution by setting  $u_4 = \{u_3^{0.135} - (1 - u_3)^{0.135}\}/0.1975$  and obtain the log-normal distributed variable  $x_{i3} = e^{(\mu_i + \sigma u_4)}$  where  $\mu_i$  and  $\sigma$  are the mean and standard deviation.  $\mu_i$  is specified as a function of covariates as Equation (5) in the main text.

Here,  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23|1}$  are the pair-copula parameters. The  $h$ -function and  $h^{-1}$ -function differ with specification of different copulas. We provide the copula distribution functions, densities,  $h$ -functions and  $h^{-1}$ -functions for the Gaussian and Frank copula in Appendix D.

## Appendix D: Gaussian and Frank Copula

### *Gaussian Copula*

The copula distribution function for the bivariate Gaussian copula is given by

$$C(u_1, u_2; \theta) = \Phi_2(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \theta) \quad (\text{WC1})$$

where  $\Phi$  denotes the standard normal distribution function and  $\Phi_2$  denotes the standard bivariate normal distribution with a dependence parameter  $\theta$  ( $-1 \leq \theta \leq 1$ ).

The density of the copula function is given by

$$c(u_1, u_2; \theta) = \frac{1}{\sqrt{1 - \theta^2}} \exp \left\{ -\frac{\theta^2(\omega_1^2 + \omega_2^2) - 2\theta\omega_1\omega_2}{2(1 - \theta^2)} \right\} \quad (\text{WC2})$$

where  $\omega_1 = \Phi^{-1}(u_1)$ ,  $\omega_2 = \Phi^{-1}(u_2)$  and  $\Phi^{-1}$  is the inverse of the standard normal distribution.

The  $h$ -function for the Gaussian copula is given by

$$h(u_1|u_2; \theta) = \Phi \left( \frac{\Phi^{-1}(u_1) - \theta\Phi^{-1}(u_2)}{\sqrt{1 - \theta^2}} \right) \quad (\text{WC3})$$

and the  $h^{-1}$ -function is given by

$$h^{-1}(u_1|u_2; \theta) = \Phi \left( \Phi^{-1}(u_1)\sqrt{1 - \theta^2} + \theta\Phi^{-1}(u_2) \right) \quad (\text{WC4})$$

### *Frank Copula*

The copula distribution function for the bivariate Frank copula is given by

$$C(u_1, u_2; \theta) = -\frac{1}{\theta} \log \left\{ 1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{e^{-\theta} - 1} \right\} \quad (\text{WC5})$$

where the dependence parameter  $\theta \in (-\infty, \infty)$ .

The density of the copula function is given by

$$c(u_1, u_2; \theta) = \frac{e^{-\theta u_1} e^{-\theta u_2} (1 - e^{-\theta}) \theta}{[e^{-\theta} - 1 + (e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)]^2} \quad (\text{WC6})$$

The  $h$ -function for the Gaussian copula is given by

$$h(u_1|u_2; \theta) = \frac{e^{-\theta u_2}(e^{-\theta u_1} - 1)}{e^{-\theta} - 1 + (e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)} \quad (\text{WC7})$$

and the  $h^{-1}$ -function is given by

$$h^{-1}(u_1|u_2; \theta) = -\frac{1}{\theta} \ln \left[ \frac{1 + (e^{\theta(u_2-1)} - 1)u_1}{1 + (e^{\theta u_2} - 1)u_1} \right] \quad (\text{WC8})$$

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## **ESSAY 2**

### **Dynamically Managing a Profitable Email Marketing Program**

#### **2.1 Introduction**

Email marketing is a widely-used marketing tool for firms to communicate with customers, handle customer complaints, and cross-sell and up-sell products. The ultimate goal of maintaining an email marketing program is to generate profit by making customers become more active in purchases. However, such a positive linkage between emails and purchases is based on the assumption that customers who are responsive to emails are also the ones who are active in purchases. Our observation shows that customers' responsiveness to emails may not indicate an active purchase relationship.

Previous research has shown that the dynamics of the customer relationship can be captured with several discrete latent states, such as dormant, occasional, and frequent (Netzer, Lattin and Srinivasan 2008). This stream of research primarily focuses on the purchase behavior, ignoring the fact that other activities can also reveal customer-firm relationship (except the study by Schweidel et al. 2014). In the context of email marketing, we argue that customers' responsiveness to emails should also indicate their relationship with the firm. Email marketers should incorporate the evolving state of email response into the study of the evolving state of purchase. Note that we define the email response as the opening of an email a customer receives.

One of the major concerns for the email marketing industry is that customers receive too many emails. Practitioners tend to treat each email campaign as an independent solicitation process and fail to consider its long-term impact on both the email response and purchase behavior. We argue that such practice does not retain the subscribers in the email program but

also does not keep them active. In this study, we seek to address the following questions: (1) whether the latent states that characterize customers' responsiveness to emails evolve along with the latent states that characterize customers' purchases, (2) whether there is a correlation between customers' email open<sup>5</sup> and purchase behavior, (3) the effect of email contacts on customers' purchase behavior, and (4) what is the optimal email marketing policy that can maximize the long-term profit of a firm.

In this study, we capture the dynamics in customers' purchase behavior using a hidden Markov model (HMM). We capture the dynamics in customers' email open behavior by allowing it to evolve with the purchase relationship states. We model the email open count using a Binomial distribution and the purchase count using a zero-inflated negative binomial distribution model (ZNIB). We use a bivariate Frank copula to investigate the linkage between email open and purchase behavior. We capture the unobserved heterogeneity in the email open and purchase model using a random effect specification.

The empirical study identifies three purchase relationship states—low, medium and high states. It shows that customers who are in the medium state have the highest intrinsic propensity to open an email, followed by the customers in the lowest state and those in the highest state. We also identify a positive correlation between email open and purchase behavior. We derive a dynamic email marketing resource allocation policy using the hidden Markov model, the purchase and email open model estimates. We demonstrate that a forward-looking company could maximize the long-term profits of its existing email subscribers.

To the best of our knowledge, this is the first empirical study that investigates both the email open and purchase behavior using a hidden Markov model. This research provides

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<sup>5</sup> For the purpose of this study, we define “email open” as the action of opening an email message.

important implications for firms to understand customers' behavioral attitudes towards email marketing. This study provides managerial guidelines for resource reallocation to maximize long-term profitability.

In the following sections, we first review the literature on email marketing and customer relationship dynamics. Second, we describe our data and present descriptive statistics. Third, we discuss the modeling framework. Fourth, we present the empirical results and derive the optimal email marketing policy. Finally, we discuss the conclusions and future studies.

## **2.2 Literature Review**

### **2.2.1 Email Marketing**

There are several reasons for email marketing's popularity. First, email enables marketers to send messages to its customers at a very low cost. Chittenden and Rettie (2003) demonstrated the total cost for acquisition and retention campaigns of email to be \$26,500 per 5,000 customers, as compared to that of direct mail at \$69,600 per 5,000 customers. Second, email messaging requires less time to prepare and execute. Industry practice shows that an email marketing campaign targeting 50,000 customers needs only 6 hours to prepare and send, while direct mail needs 17 days before it can reach a customer's mailbox. Third, email can generate faster response and create interactive communication. Using a computer or mobile phone, a customer can respond to a promotional email the moment he/she receives it by clicking the hyperlinks that direct the customer to the company's website.

In this study, we focus on permission-based email marketing. Permission-based email marketing requires marketers to seek the customers' permission before sending them email

messages (Godin 1999; Kumar, Zhang and Luo 2014). This type of email marketing intends to maintain a long-term relationship with the customers, rather than getting customers to buy once without return for future purchases. In line with this idea, previous research has found that email marketing has a positive effect on customer loyalty. Tezinde et al. (2002) discovered that email advertisements were useful to consumers by inducing them to visit the physical store. Merisavo and Raulas (2004) found that email marketing can enhance consumer attitudinal loyalty towards the brand. Their study found that customers would recommend the email messages to their friends if they found them interesting and useful.

Although the overall influence of email marketing is positive, we argue that researchers and practitioners should examine the customers' response to email-based marketing messages from two perspectives. First, customers open and read an email simply to keep track of the firm's products and offerings. This behavior does not necessarily indicate that they are actively looking for the information to assist their purchase decisions. Bonfrer and Dreze (2009) studied a series of email marketing campaigns and proposed a bivariate hazard model to predict when customers open or click an email. Kumar, Zhang and Luo (2014) looked at the total number of emails that were opened and clicked and investigated their impact on the time the customers are subscribed to the email program. Second, customers make purchases as a result of the email marketing messages. Sahni, Zou and Chintagunta (2014) analyzed 70 randomized field-experiments and found that email promotions not only increase customers' average purchase spending during the promotion window, but also carry over to a week later after the promotion expires. Kumar, Zhang and Luo (2014) found that the average email open-rate has a positive effect on average purchase spending while the effect of the average email click-rate is not significant.

Although there are studies that look into each of the two perspectives separately—email

open and purchase, it is surprising that there is no study that investigates both behaviors together. Bonfrer and Dreze (2009) did not consider the possible link between email open/click rate and purchase behavior due to the limitation of data. Kumar, Zhang and Luo (2014) capture the “average” effect of the customers’ response to emails on purchase, but they did not consider the dynamics and heterogeneity in both email open and purchase behavior. Sahni, Zou and Chintagunta (2014) conduct a post-hoc analysis of the experiments to show the aggregate-level of effects of the emails on customer expenditure. They do not quantify how email open behavior affects customer purchase nor do they consider the dynamic effects.

### **2.2.2 Customer Relationship Dynamics**

Previous research has shown that the customers’ relationship with the firm evolves over their lifetime. Netzer et al. (2008) proposed a hidden Markov model to model the transitions among latent relationship states. Montoya et al. (2010) incorporated a partially observed Markov decision process into a dynamic marketing resource allocation policy across physicians. Kumar et al. (2011) found that customers in the highest relationship state may not yield the highest lifetime value to the firm. Li et al. (2011) derived an optimal cross-selling policy by considering customers’ hidden financial states. Luo and Kumar (2013) used a forward-backward Gibbs sampler method to recover the hidden buyer-seller relationship states precisely to capture the effect of marketing contacts in business-to-business markets.

This stream of research primarily examines customer-firm relationships by looking at purchase behavior; we argue that other activities can also reveal customer-firm relationships. For example, Schweidel et al. (2014) developed a flexible model to empirically study the dependence of two activities and the associated relationship evolution. In the context of email marketing,

customers' responsiveness to emails should also indicate their relationship with the firm. We argue that the relationship states that characterize email open and purchase behavior evolve for the following reasons. First, customers may have different interests and needs at different time points, but companies may not be able to adapt to these changes by tailoring their email messages. Second, customers are in a learning mode for the emails they receive. In one type of learning mode, customers frequently open the emails in the first few months and decrease the frequency gradually due to the loss of interest. In another type of learning mode, customers open the emails in a consistent manner and remain constant over time. Because customers continue to learn the effectiveness of the email program in which they are enrolled, their relationship with the focal firm is constantly evolving based on their learning. Third, companies may adopt different email marketing strategies at different times. It is possible that companies may not tailor the email strategies by individual or by segment based on email open and purchase behavior. Customers may respond to emails in a different manner, showing different levels of engagement at different times.

To the best of our knowledge, this is the first empirical study that investigates these two relationships in the same modeling framework. We incorporate the evolving states of email open into the evolving states of purchase. We allow email open and purchase behavior to be correlated. In the following sections, we first describe the data and present the modeling framework subsequently.

### ***2.3 Data Description***

Our database comprises information from a U.S. retailer that sells multiple categories of products. The dataset consists of information on the transactions made by the customers, the

number of emails the firm sent to the customers, and the email open histories. We randomly select a cohort of 300 customers who opted-in to the retailer's email program in February 2007. Thus, we have data comprising customers' email open and purchase activities over a 39-month period of time. In Table 1, we report the descriptive statistics of the data. On average, the retailer sent 6.90 emails to its customers per month. The customers opened 1.64 emails and made 0.69 purchases on average per month. Note that we only count unique email opens because the majority of the emails were only opened once, if they were opened.

### **Insert Table 1**

To demonstrate the complexity of customers' purchase and email open behavior, we randomly select three customers and plot the count of their purchase and email open frequency over the 39 month (see Figure 1). We can see that Customer 1 is not active in both purchase and opening emails. Customer 1 only made purchases in three months and opened emails in two months. The time elapsed between the first and second purchase is 21 months. Customer 1 ceased to either purchase or open emails after month 26. In comparison, Customer 2 has more active purchase behavior in the total of 39 months. However, Customer 2 is not equally active in opening emails as he or she only opened emails in two months—month 10 and 11. In addition, we find that Customer 2 decreased his/her purchase activity after month 19, demonstrating a dynamic purchase behavior. Customer 3 is moderately active in both purchase and opening emails. We observe that the average inter-purchase time of Customer 3 is approximately 4 months.

### **Insert Figure 1 here**

Figure 1 shows that customers' purchase and email open behavior is both heterogeneous

and dynamic, and that purchase and email open do not perfectly align with one another. It indicates that the two behaviors may correlate with each other but their relationships may evolve differently. In the subsequent section of modeling framework, we discuss how we model both the heterogeneity and dynamics in customer purchase and email open behavior. We also demonstrate how the rate of email responsiveness evolves along with the development of purchase activity.

In addition, to understand the process of customer purchase and email open frequency, we plot the distributions of both behaviors (see Figure 2). The distribution of purchase count shows that a discrete distribution such as Poisson process may be able to capture the data generating process of purchase. The mean (0.68) and the variance (2.64) of the purchase count variable suggest overdispersion, which violates the assumption of Poisson distribution. Furthermore, we observe an excess of zero purchases (71%) which can affect the estimation of the Poisson model. To account for both the overdispersion and excess of zeros, we use the zero-inflated negative binomial model (ZINB) to model the purchase count variable.

The distribution of email open count also suggests a discrete distribution. By definition, the (unique) email open count is the number of unique emails that were opened conditional on the total number of emails the customer receives in a given month. We observe that the maximum number of the emails the customers received in our data set is 20 emails. Thus, the maximum number of unique emails a customer can open will not exceed 20. To capture this process, we use the typical binomial distribution which captures the number of success (email open) in a sequence of event (email received).

**Insert Figure 2 here**

## 2.4 Modeling Framework

In line with previous research, we consider the hidden Markov model to identify customer relationship states and the transition. An HMM describes a Markov process with the unobserved states. HMM is a stochastic model that can be used to capture the transition between these states and translate these latent states to the observed behavior. HMMs have been applied in the marketing field to study customer-firm relationships (e.g., Netzer et al. 2008; Montoya et al. 2010; Kumar et al. 2011; Luo and Kumar 2013).

In the context of email marketing, the two customer actions, purchase and email open, could be governed by two separate Markov process. If the focus of the study is to examine the relationship and the possible transition between two hidden Markov chains, a coupled hidden Markov model can be used (e.g., Brand, Oliver and Pentland 1997). In this context, since the focus of the firm is on the customer purchase behavior, we use a hidden Markov chain to capture the evolvement of the purchase relationship state. We put the restriction on the conditional purchase model so that the intrinsic utility of making purchases for a higher state is higher than that for a lower state. We allow the email open behavior to depend on the purchase relationship state but we do not put any restrictions as we do on the purchase behavior. Thus, we not only can capture the dynamics in email open behavior but also allow it to be flexible that, for example, customers in a higher purchase state are less likely to respond to email messages.

### 2.4.1 Overview of the Model

Let  $O_{it}$  be the number of emails customer  $i$  opens in month  $t$ . Let  $Y_{it}$  be the number of times customer  $i$  purchases (online or offline) in month  $t$ . We allow the decisions of  $Y_{it}$  and  $O_{it}$  to be correlated. We model the sequence of observations  $\{(Y_{i1} = y_{i1}, O_{i1} = o_{i1}), \dots, (Y_{it} =$

$y_{it}, O_{it} = o_{it})\}$  using a HMM characterized by (1) the initial state distribution ( $\boldsymbol{\pi}_i$ ), (2) a sequence of transition probabilities ( $\mathbf{Q}_{it}$ ), and (3) a vector of probabilities that relate the latent states to the observed purchases and email opens ( $\mathbf{H}_{it}$ ).

## 2.4.2 The Initial State Distribution

At any given time  $t$ , let  $s$  denote the strength of the purchase relationship between the customer  $i$  and the firm at time  $t$ . Let  $\pi_{is}$  be the probability that customer  $i$  is initially in state  $s$ , where  $\pi_{is} \geq 0$  and  $\sum_{s=1}^S \pi_{is} = 1$ . In this study, we assume that all customers start at the lowest purchase state in the first month. Therefore,  $\boldsymbol{\pi}'_i = [\pi_{i1}, \pi_{i2}, \dots, \pi_{iS}] = [1, 0, \dots, 0]$ .

## 2.4.3 The Markov Chain Transition Matrix

In our proposed HMM framework, we allow that customers can transit to a lower or higher purchase state or stay in the same purchase state. We model customers' transition probability using their previous experience, including the time elapsed since previous purchase and the time elapsed since previous email open.

Following Kumar et al. (2011), we use a multinomial logit model to formulate the transition matrix, in order to allow transitions to all possible purchase states. We define the transition matrix as

$$\mathbf{Q}_{i,t-1 \rightarrow t} = \begin{array}{c} \text{State at } t-1 \\ \begin{array}{c} 1 \\ 2 \\ \vdots \\ NS \end{array} \end{array} \begin{array}{c} \text{State at } t \\ \begin{array}{c} 1 \quad 2 \quad \dots \quad NS \\ \hline q_{it11} \quad q_{it12} \quad \dots \quad q_{it1NS} \\ q_{it21} \quad q_{it22} \quad \dots \quad q_{it2NS} \\ \vdots \quad \vdots \quad \ddots \quad \vdots \\ q_{itNS1} \quad q_{itNS2} \quad \dots \quad q_{itNSNS} \end{array} \end{array} \quad (1)$$

$q_{itss'}$  is the conditional probability that customer  $i$  moves from state  $s$  at time  $t - 1$  to state  $s'$  at time  $t$ , where  $0 \leq q_{itss'} \leq 1 \forall s, s'$ , and  $\sum_{s'} q_{itss'} = 1$ . We specify that the transition utility from purchase state  $s$  at period  $t - 1$  to state  $s'$  at time  $t$  as follows:

$$u_{itss'} = \alpha_{ss'} + \gamma_{1ss'} LO_{it} + \gamma_{2ss'} LY_{it} + \gamma_{3ss'} G(EM_{it-1}) + e_{itss'} \quad (2)$$

where  $\alpha_{ss'}$  is the intrinsic utility value to transition from purchase state  $s$  to  $s'$ ,  $LO_{it}$  is the time elapsed since last time customer  $i$  opened an email, and  $LY_{it}$  is the time elapsed since last time customer  $i$  made a purchase. We use the logarithm of  $LO_{it}$  and  $LY_{it}$  to capture the diminishing effects.  $G(EM_{it-1})$  is a function of the number of emails customer  $i$  received at time  $t - 1$ . We use the linear and quadratic specification.  $\gamma_{1ss'}$ ,  $\gamma_{2ss'}$  and  $\gamma_{3ss'}$  are the corresponding parameters to move a customer from state  $s$  to  $s'$ .  $e_{itss'}$  denotes the random utility of the transition propensity which follows a Type 1 extreme value distribution. We restrict the utility for customer  $i$  at time  $t$  to transition to the lowest state to be zero for the purpose of identification.

#### 2.4.4 Conditional Purchase Count Model

Conditioned on being in purchase state  $s$  at time  $t$ , we assume that the number of purchases of customer  $i$  follows a zero-inflated negative binomial model with parameters  $\varphi_{ist}$ ,  $\lambda_{ist}$  and  $r$ . For each observation  $y_{it}$ , ZINB assumes that there are two data generating processes which are defined as:

$$P(Y_{it} = y_{it}) = \begin{cases} \varphi_{ist} + (1 - \varphi_{ist}) \left(1 + \frac{\lambda_{ist}}{r}\right)^{-r} & \text{if } y_{it} = 0 \\ (1 - \varphi_{ist}) \frac{\Gamma(y_{it} + r)}{y_{it}! \Gamma(r)} \left(1 + \frac{\lambda_{ist}}{r}\right)^{-r} \left(1 + \frac{r}{\lambda_{ist}}\right)^{-y_{it}} & \text{if } y_{it} > 0 \end{cases} \quad (3)$$

where  $r$  is a shape parameter which quantifies the amount of overdispersion. The mean and variance of the ZINB distribution are  $E(y_{it}) = (1 - \varphi_{ist})\lambda_{ist}$  and  $var(y_{it}) = (1 -$

$\varphi_{ist})\lambda_{ist}(1 + \varphi_{ist}\lambda_{ist} + \lambda_{ist}/r)$ , respectively.  $\varphi_{ist}$  captures the zero-inflated probabilities and  $\lambda_{ist}$  is the expected purchase count for customer  $i$  at time  $t$ . We allow both  $\varphi_{ist}$  and  $\lambda_{ist}$  to be state-dependent.

To account for the process of excess of zeros, we model  $\varphi_{ist}$  using a logit function as

$$\text{logit}(1 - \varphi_{ist}) = \delta_{0s} + \delta_{1s}LY_{it} \quad (4)$$

where  $\delta_{0s}$  captures the intrinsic utility of making a purchase, and  $\delta_{1s}$  captures the effects of duration dependence, given state  $s$ .  $LY_{it}$  is the time elapsed since last time customer  $i$  made a purchase. We take the logarithm of  $LY_{it}$  to capture the diminishing effect.

In addition, we model  $\lambda_{ist}$  as a function of the number of emails sent by the retailer and the time since last purchase, given by

$$\lambda_{ist} = \exp(\alpha_{ips} + \beta_{1ps}K_{it} + \beta_{2ps}K_{it}^2 + \beta_{3ps}LY_{it}) \quad (5)$$

Where  $K_{it}$  is the total number of emails customer  $i$  received in month  $t$ , and  $K_{it}^2$  is the square term of  $K_{it}$ . We also include the time since last purchase variable (logarithm of  $LY_{it}$ ) to capture the effect of duration dependence. Conditional on state  $s$ ,  $\alpha_{ips}$  is the intrinsic propensity to make purchases,  $\beta_{1ps}$ ,  $\beta_{2ps}$  and  $\beta_{3ps}$  are the corresponding response parameters. For the purpose of identification, we impose the restriction that  $\alpha_{ips+1} = \alpha_{ips} + \exp(\Delta\alpha_{ips+1})$ , where  $\Delta\alpha_{ips+1}$  is a parameter to estimate from the data. Thus, customers in a higher purchase relationship state have a higher propensity to make purchases than those in a lower state. In addition, to account for the unobserved heterogeneity, we allow  $\alpha_{ips}$  to be customer-specific. We assume that  $\alpha_{ips}$  are normally distributed across customers as follows:

$$\alpha_{ip} = \alpha_p + \Delta\alpha_{ip} \quad (6)$$

where  $\Delta\alpha_{ip} \sim N(0, \sigma_{\alpha_p}^2)$ , and  $\sigma_{\alpha_p}^2$  is the variance of the corresponding parameter.

#### 2.4.5 Email Open Count Model

We assume that the number of emails customer  $i$  opens at time  $t$  follows a binomial distribution with parameters  $K_{it}$  and  $p_{ist}$ , given by

$$P(O_{it} = o_{it}) = \binom{K_{it}}{o_{it}} p_{ist}^{o_{it}} (1 - p_{ist})^{K_{it} - o_{it}} \quad (7)$$

Where  $K_{it}$  is the total number of emails customer  $i$  received in month  $t$ , and  $p_{ist}$  is the probability that customer  $i$  opens an email in month  $t$ .

We model  $p_{ist}$  as a function of customers' past email open behavior as

$$p_{ist} = \frac{\exp(\alpha_{ios} + \beta_{os} LO_{it})}{1 + \exp(\alpha_{ios} + \beta_{os} LO_{it})} \quad (8)$$

Where  $\alpha_{ios}$  is the intrinsic probability to open an email given purchase state  $s$ . We include the time since last email open (logarithm of  $LO_{it}$ ) to capture the effect of duration dependence.  $\beta_{os}$  are the corresponding response parameters that capture the short-term effect of past experience. Note that while we allow  $\alpha_{ios}$  to vary with the purchase states, we do not impose any restriction on  $\alpha_{ios}$  such that  $\alpha_{io1} \leq \alpha_{io2} \leq \dots \leq \alpha_{ios}$ . We estimate  $\alpha_{ios}$  based on the empirical data. This flexible structure allows us to examine the possibility that customers in a higher purchase state have lower probability to open emails. Similarly, we allow  $\alpha_{ios}$  to be customer-specific to control for the unobserved heterogeneity. We assume that  $\alpha_{ios}$  follows a normal distribution with mean  $\alpha_o$  and variance  $\sigma_{\alpha_o}^2$  which can be estimated from the data.

#### 2.4.6 The Dependence between Purchase and Email Open Behavior

At any given time  $t$ , customers need to decide on two actions: the number of purchases

and the number of emails to open. Since both actions indicate the interests and interaction the customers have with the firm, we argue that the purchase and email open behavior may be correlated. Note that both the purchase count  $Y_{it}$  and the email open count  $O_{it}$  follow a discrete distribution. It is not easy to find a bivariate distribution that can capture the dependence between the ZINB and binomial distribution.

Danaher and Smith (2011) suggest the copula approach to link two marginal distributions which are not from the same family (see also Kumar, Zhang and Luo 2014). Following Nikoloulopoulos and Karlis (2010), we use a copula to construct a bivariate distribution of  $O_{it}$  and  $Y_{it}$ . Since we are dealing with bivariate count data, we cannot obtain the bivariate density by deriving the bivariate distribution function as in a continuous case. As Nikoloulopoulos and Karlis (2010) suggest, we obtain the bivariate probability mass function using finite differences of the copula function,

$$\begin{aligned} h(o_{it}, y_{it}) = & C(F_1(o_{it}), F_2(y_{it})) - C(F_1(o_{it} - 1), F_2(y_{it})) \\ & - C(F_1(o_{it}), F_2(y_{it} - 1)) + C(F_1(o_{it} - 1), F_2(y_{it} - 1)) \end{aligned} \quad (9)$$

where  $C(\cdot)$  is the copula function,  $F_1(o_{it})$  and  $F_2(y_{it})$  are the distribution function of  $O_{it}$  and  $Y_{it}$ , respectively. We use Frank copula (e.g., Frank 1979; Genest 1987) in this context because of its flexibility to capture the full range of correlation. The Frank copula function is given by

$$C(u_1, u_2; \theta) = -\frac{1}{\theta} \log \left\{ 1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{e^{-\theta} - 1} \right\} \quad (10)$$

where  $u_1$  and  $u_2$  are the distribution functions and  $\theta$  is the Frank copula correlation parameter.

#### 2.4.7 Model Estimation

There are two sets of parameters to be estimated from our model. The first set of parameters  $\{\alpha_{ss'}, \gamma_{1ss'}, \gamma_{2ss'}, \gamma_{3ss'}\}$  for  $s$ , are the parameters in the transition matrix. The second set of parameters includes all the parameters from Equations (3-10). Following Netzer et al. (2008), we write the vector of the bivariate probability mass function as a diagonal matrix  $\mathbf{H}_{it}$ . Given the proposed HMM structure, the likelihood function for a sequence of observations given by  $\{(Y_{i1} = y_{i1}, O_{i1} = o_{i1}), \dots, (Y_{it} = y_{it}, O_{it} = o_{it})\}$  can be expressed as

$$\begin{aligned}
 L &= \prod_{i=1}^N P((Y_{i1} = y_{i1}, O_{i1} = o_{i1}), \dots, (Y_{it} = y_{it}, O_{it} = o_{it})) \\
 &= \prod_{i=1}^N \boldsymbol{\pi}_i' \mathbf{H}_{i1} \prod_{t=2}^T \mathbf{Q}_{it} \mathbf{H}_{it} \mathbf{1}
 \end{aligned} \tag{11}$$

Where  $\mathbf{1}$  is an  $S \times 1$  vector of ones.

## 2.5 Empirical Results

### 2.5.1 Selecting the Number of States

We estimate the HMM model using the maximum likelihood estimation (MLE) method. We use the simulated MLE to estimate the variance of the random intercepts. We first select the number of HMM states based on the log-likelihood and Bayesian information criterion (BIC). We compare the performance of HMM models of up to 4 states (see Table 2). We find that the HMM model with 3 states provide the best-fit to the data as it gives the lowest BIC value. Thus, we choose the 3-state HMM model.

**Insert Table 2 here**

### 2.5.2 Parameter Estimates

Table 3 reports the parameter estimates for the 3-state HMM model. Note that we have put the restriction that customers in a higher relationship state purchase more frequently, conditional on purchase. Thus, we can label the three states as “low”, “medium”, and “high” relationship states that govern the frequency of purchase in each month. An interesting specification of our model is that customers’ responsiveness to emails does not need to align with their purchase activeness. The empirical finding suggests that the customers who are in the medium state have the highest intrinsic ( $\alpha_{o2}=0.428$ ) propensity to open an email, followed by the customers in the low state ( $\alpha_{o1}=-2.255$ ) and those in the high state ( $\alpha_{o3}=-6.024$ ). Surprisingly, the customers in the highest relationship state (State 3) are those who are the most reluctant to open an email. It is possible that the heavy buyers are already very familiar with the firm’s products and promotions so that they are less motivated to keep track with the firm’s offerings through emails.

**Insert Table 3 here**

Table 3 shows that customers in different states vary in purchase duration dependence. The customers who are in the lowest state have positive duration dependence. The longer they have not made a purchase, the more frequently they purchase. However, the customers who are in the medium or high state have negative duration dependence. The longer they have not made a purchase, the less frequently they will purchase. This is an interesting finding as it reveals the diversity in customer purchase dynamics. In addition, Table 3 also shows that the email contact has a nonlinear effect on purchase count for the customers in all three states. For the customers who are in the low state, the email contact initially has a positive effect (1.129) on purchase count but the effect becomes negative (-0.095) when the quantity of emails increases and passes

a certain threshold. In contrary, for the customers who are in the medium or high state, the email contact starts with a negative effect (-1.173, -0.102) on purchase count and such effect increases (0.752, -0.006) with the increase of email intensity. This finding shows that customers respond in a diverse way to email contacts depending on their purchase activeness.

The estimates for the binomial model show that the customers who are in the different purchase states respond to emails in a diverse way. First, there is strong heterogeneity ( $\sigma_{\alpha_o}^2=5$ ) in the intrinsic propensity of opening emails across customers. Second, the customers in all three states demonstrate negative duration dependence in responding to emails. The longer they have not opened an email, the less likely they will open an email. It is interesting to see that such negative effect of duration dependence is the strongest (-1.056) for the customers in the highest relationship state, compared to those in the low (-0.275) and medium (-0.274) state.

### **2.5.3 Transition Probability Matrix**

The transition matrix from the HMM model shows how customers evolve across different relationship states. We calculate the transition probabilities of a “typical” customer with explanatory variables (time since last purchase and time since last email open) at their mean level using Equations (1-2). We vary the number of emails the customers receive in the previous period and check the effect of email contacts on the state transitions (see Table 4).

**Insert Table 4 here**

When there is no email contact, the customers tend to stay (from State 1) or switch (from States 2 or 3) to the lowest state (State 1), compared to the cases where there is some email contacts. This finding is consistent with Luo and Kumar (2013). Therefore, it shows that it is

important to invest in email marketing to maintain the customers at a higher relationship. Table 4 also shows that the customers do not tend to move and will stick to where they are from. Email contacts can move these customers away from their status quo. For example, one email contact increases the likelihood that a customer in State 1 will move to State 3 from 6.1% to 14.3%. However, five email contacts only marginally increase such likelihood to 14.9%. In addition, we find that it is not true that the more email contacts the better. For example, when the customers are from State 2, one email contact increases the likelihood that they remain in State 2 from 70.8% to 83.6% while five email contacts increase such likelihood to 77.6% and ten email contacts increase such likelihood to 78.3%.

In Figure 3, we plot the average probabilities of customers residing in the three states over time. We calculate the state membership distribution of each customer using the filtering approach (Montgomery et al. 2004; Netzer et al. 2008). Since we assume that each customer started from the lowest state, we drop the first five periods as initialization periods and plot the states evolvement over the rest of the time. We find that, on average, the customers started from State 1 with a probability of 0.50, from State 2 with a probability of 0.14, and from State 3 with a probability of 0.36. It took approximately 12 months for the aggregate distribution of the States 1 and 3' membership to stabilize to within the range of 0.3 and 0.4. The aggregate distribution of the State 2's membership evolves slowly and reaches the stabilized range (0.3 ~0.4) after 26 months.

**Insert Figure 3 here**

## **2.6 Optimal Email Marketing**

At any given time  $t$ , the firm has to make the decision of how many emails to send. Given our demand specification, this email contact decision has both short-term and long-term implications on customer behavior. Short term effect comes due to the direct effect of emails on customer purchase and email open behaviors at time  $t$  (see Equations 4-7). Long-term effect comes from two sources: (1) customers' purchase and email open behavior at time  $t$  will impact the evolution of customer relationship state from time  $t$  to  $t+1$  (see Equations 1-2); (2) the email contacts at time  $t$  will directly influence the relationship state transition from time  $t$  to  $t+1$  (see Equations 1-2). Due to these long term effects, determining the optimal number of emails to send by the firm requires one to solve a dynamic programming problem.

From the firm perspective, the variable of focus is the number of times a customer purchases from the store (online and/or offline) in any given month. Under the assumption of constant purchase amount per purchase and fixed gross margin for the retailer, the purchase count can be directly translated into the firm's profit at each time period. We make these assumptions for the following reasons. First, our setting is a non-contractual setting where customer churn is not easy to predict. Second, the firm is struggling to engage with the customers to increase the contact frequency the customers initiate. Previous experience shows that the more frequently the customers purchase, either online or offline, the stickier the customers will be and less likely they will churn and switch to competitors.

For our dynamic optimization problem, the payoff relevant state variables become: (1) the probabilities that the customer exists in each of the purchase relationship states, (2) the time since last purchase, and (3) the time since last email open. Following Kumar et al. (2011), we assume the timing of the email contact decisions as follows. At the beginning of each month  $t$ , the firm predicts the probability that the customer exists in each of the three relationship states,

$p_{1t}$ ,  $p_{2t}$  and  $p_{3t}$ . We use a multinomial logit function to capture these three probabilities and relate them to two parameters  $\omega_{1t}$  and  $\omega_{2t}$  as follows:

$$\begin{aligned} p_{1t} &= \frac{\exp(\omega_{1t})}{1 + \exp(\omega_{1t}) + \exp(\omega_{2t})} \\ p_{2t} &= \frac{\exp(\omega_{2t})}{1 + \exp(\omega_{1t}) + \exp(\omega_{2t})} \\ p_{3t} &= 1 - p_{1t} - p_{2t} \end{aligned} \tag{12}$$

Let  $S_t = (\omega_{1t}, \omega_{2t}, \theta_{1t}, \theta_{2t})$  denote the state vector at time  $t$ .  $\theta_{1t}$  and  $\theta_{2t}$  are states for the time since last purchase and time since last email open, respectively. Time from last purchase and open states,  $\theta_{1t}$  and  $\theta_{2t}$ , evolve based on whether the consumer makes a purchase and opens an email at time  $t$ . If the customer makes a purchase at time  $t$ , the corresponding state  $\theta_{1,t+1}$  becomes 1, and if she doesn't make a purchase  $\theta_{1,t+1}$  becomes  $\theta_{1t} + 1$ . Similarly, if the customer opens an email at time  $t$ , the corresponding state  $\theta_{2,t+1}$  becomes 1, and if she doesn't open an email  $\theta_{2,t+1}$  becomes  $\theta_{2t} + 1$ . Since the purchase and email open decisions of customers are modeled with zero inflated negative binomial and binomial distributions, the time from last purchase and open states,  $\theta_{1t}$  and  $\theta_{2t}$ , evolves in a stochastic manner as follows

$$\begin{aligned} \theta_{1,t+1} &= \begin{cases} 1, & \text{with } Pr(Y_{it} > 0) \\ \theta_{1t} + 1, & \text{with } Pr(Y_{it} = 0) \end{cases} \\ \theta_{2,t+1} &= \begin{cases} 1, & \text{with } Pr(O_{it} > 0) \\ \theta_{2t} + 1, & \text{with } Pr(O_{it} = 0) \end{cases} \end{aligned} \tag{13}$$

The evolution of the first two state variables  $\omega_{1t}$  and  $\omega_{2t}$  conditional on the firm's email contact decision  $EM_t$  are given as

$$\begin{aligned}
\omega_{s,t+1}|EM_t = & Pr(Y_{it} > 0, O_{it} > 0) \sum_{k=1}^S \omega_{kt} q_{t,k \rightarrow s} (1, 1, EM_t) \\
& + Pr(Y_{it} = 0, O_{it} > 0) \sum_{k=1}^S \omega_{kt} q_{t,k \rightarrow s} (\theta_{1t} + 1, 1, EM_t) \\
& + Pr(Y_{it} > 0, O_{it} = 0) \sum_{k=1}^S \omega_{kt} q_{t,k \rightarrow s} (1, \theta_{2t} + 1, EM_t) \\
& + Pr(Y_{it} = 0, O_{it} = 0) \sum_{k=1}^S \omega_{kt} q_{t,k \rightarrow s} (\theta_{1t} + 1, \theta_{2t} + 1, EM_t)
\end{aligned} \tag{14}$$

where  $q_{t,k \rightarrow s}(\theta_{1,t+1}, \theta_{2,t+1}, EM_t)$  is the transition function from Equations 1-2 which is used to calculate the probability of transitioning customers from state  $k$  to  $s$  at time  $t$  conditional on the email contacts at time  $t$ , and the variables of duration dependence.

At each time  $t$  conditional on the state vector  $S_t$ , the objective of the firm is to determine the optimal number of email contacts to maximize the sum of discounted expected future profits. Under some regularity conditions this objective can be written in following form of the following Bellman equation

$$V(S_t) = \max_{EM_t} \{ \pi_t(EM_t, S_t) + \rho EV(S_{t+1}|S_t, EM_t) \} \tag{15}$$

where  $\rho$  is the discount factor and the expectation is over all the future states and actions of the firm.

For this dynamic optimization problem, we discretize the state space for our first two dimensions,  $\omega_{1t}, \omega_{2t}$ , with 13 levels each. For our second two dimensions,  $\theta_{1t}$  and  $\theta_{2t}$ , we use the range of 1 to 12 periods. This gives us a state space of 24,336 state combinations. We use the value iteration algorithm (Rust 1987) to find the optimal mappings of firm's email contacts to our state combinations. Due to the discretization of the first two state dimensions, the value functions for

the other points in the state space are computed via interpolation (Keane and Wolpin 1994).

After we get the vector of optimal mapping of email contacts, we estimate a multinomial logit model to predict optimal email contacts for any state combination out of the chosen state combinations.

Based on the optimal mapping of email contacts to states from the value iteration algorithm, we find that the optimal email contact number ranges from 0 to 6. We see a lot of heterogeneity in the ranges of optimal number of emails sent based on whether the customers are in the low, medium or the high purchase states. For instance, if the firm has a belief that with more than 16% probability that the customer is in the lowest purchase state, the optimal number of emails to send ranges between 5 and 6. Sending a large number of emails to the customers at the lowest purchase state is beneficial for the firm due to 1) more emails increase the firm's profitability in the short-term; 2) more emails help the firm shift these customers from the low purchase state to the higher purchase states. As opposed to the customers in the low purchase state, the customers in the high purchase state should receive much less emails. For instance, if the firm has a belief that the customer belongs to the high purchase state with more than 77% probability, the optimal action becomes sending no emails. This is mainly due to the fact that these customers tend to purchase less frequently as they receive more emails.

In subsequent, we use the estimated policy function to simulate the firm's email contact decisions over a long horizon to reach the steady-state distributions of email contacts and purchase state distributions. We find that in the steady-state 50.7% of the customers become the members of the low purchase state, 10.6% of the customers become the members of the medium purchase state, and finally 38.7% of the customers become the members of the high purchase

state. We find that the optimal number of emails to send becomes 6 emails per month, and the lifetime purchase count and email open count become 71 and 109 per customer, respectively.

Furthermore, we conduct a what-if simulation study to test how much profit the firm can generate if it deviates from the optimal email policy obtained from the dynamic programming study (see Figure 4). We use the steady-state distribution as the starting state combinations and test four alternative scenarios: 1) two scenarios in which the firm sends lower than the optimal level of 6 emails (4, and 5 emails per period) , 2) two scenarios in which the firm sends higher than the optimal level of 6 emails (7, and 8 emails per period). Figure 4 shows that sending sub-optimal number of emails might cause the firm to lose significant amount of profit. For instance, sending 4 or 8 emails instead of the optimal level of 6 emails could cause firm to lose 27% or 39% of its monthly profit per customer.

**Insert Figure 4 here**

## **2.7 Conclusions, Limitations and Future research**

The email marketing program has been used extensively in various industries to engage customers. The general practice in the industry in measuring the effectiveness of an email marketing program is to examine customer responsiveness to emails such as email open rate. However, this study shows that considering only the email open rate could be misleading. Our empirical study shows that customers who are in an active purchase state have the lowest intrinsic propensity to open an email. If firms solely focus on email open rate to allocate resources, they could potentially overlook a pool of customers who are inactive in responding to

emails but are active in purchases.

The purpose of this study is not trying to divert firms' attention from email open rate. This study also shows that there is a positive relationship between email open behavior and purchase behavior, on average. Instead, firms should look at customers' responsiveness to emails and their purchase behavior together. If the end goal is to maximize long-term profit, firms should not over-market to customers with too many emails but to keep them in a moderate relationship state. We present the optimal email marketing policy using the dynamic programming approach. We offer a method to study an important substantive problem that can save retailers millions of dollars.

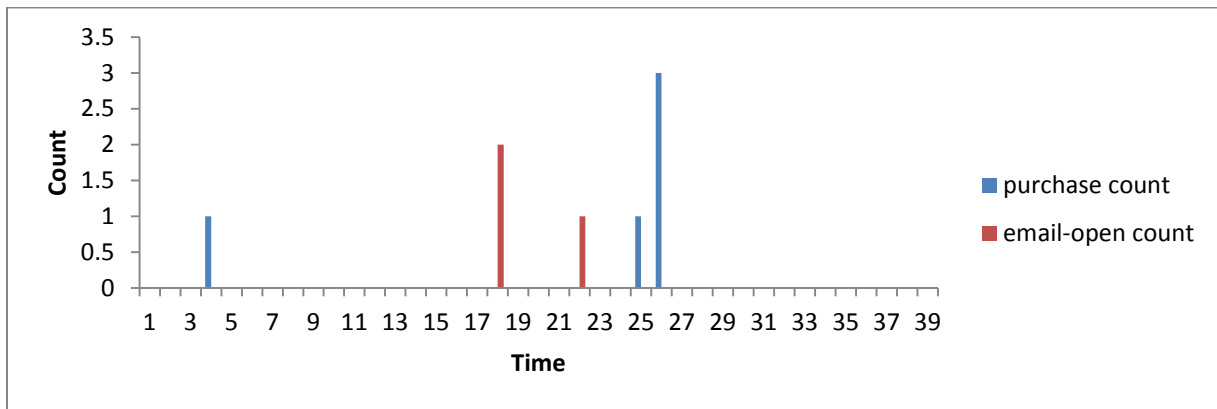
One of the limitations of this study is that we do not observe the information from the competitors. Customers' lack of response to emails could simply be due to the fact that they subscribe to too many email programs. Each email that is delivered to the customers' inbox is a load of information. Customers who are not capable of processing the information will be overwhelmed and stop responding. If the firm is aware of its customers' inbox activity, it is imperative to incorporate this information into the study. However, due to the sensitivity of such information, it is unlikely that the retailer can obtain this knowledge. Future studies could consider conduct field experiments to understand how the competing emails affect customers' reaction to the firm's emails.

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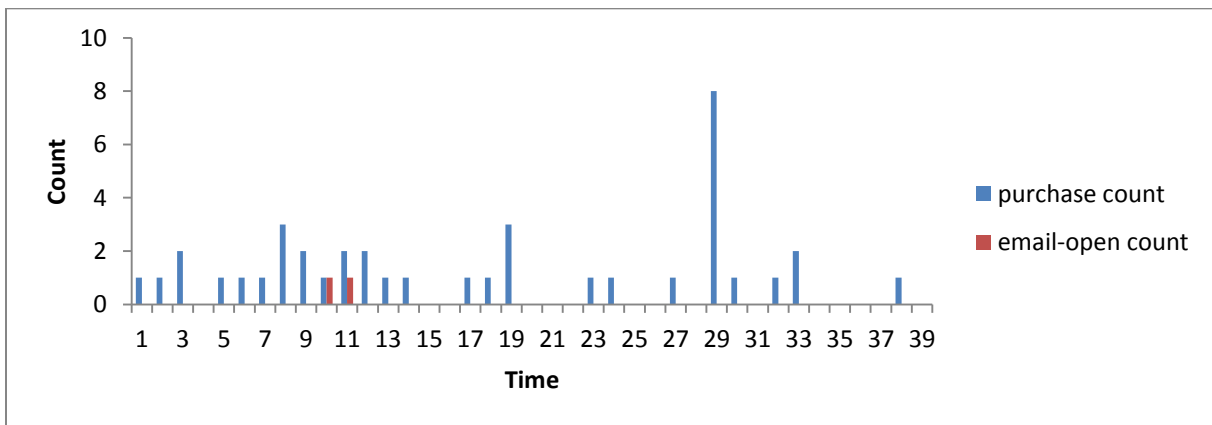
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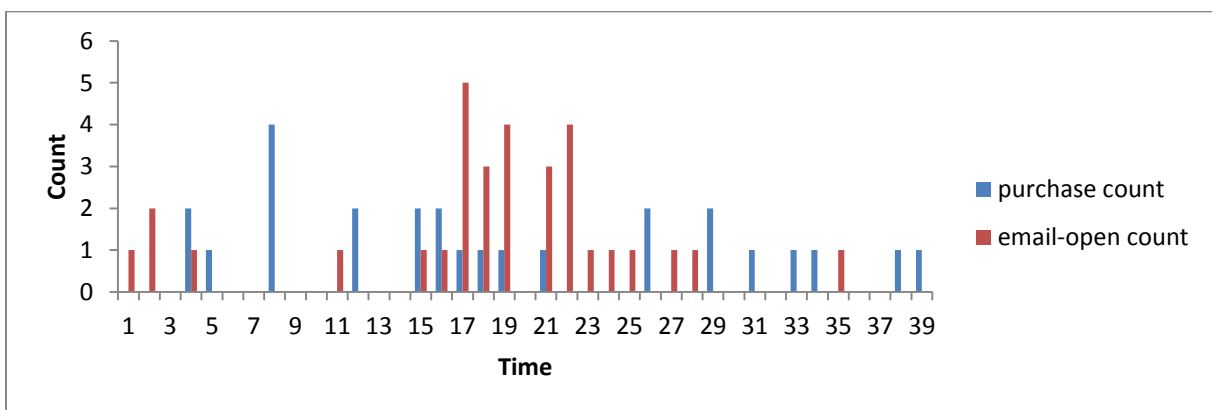
**Figure 1 Purchase and Email Open Count of Three Select Customers**



Customer 1

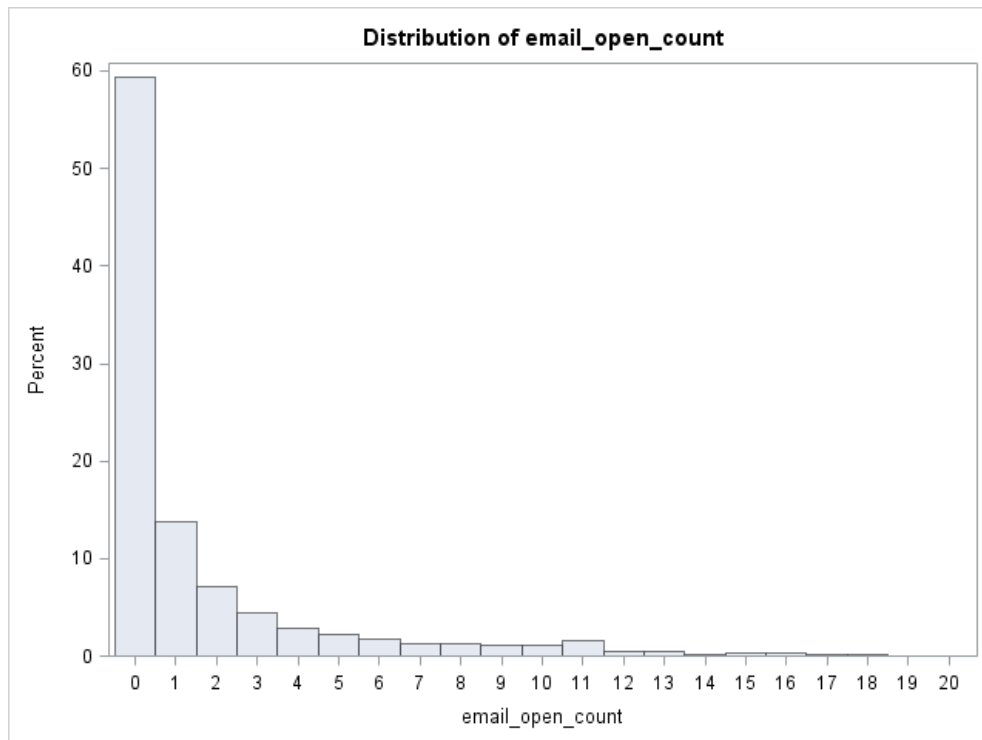
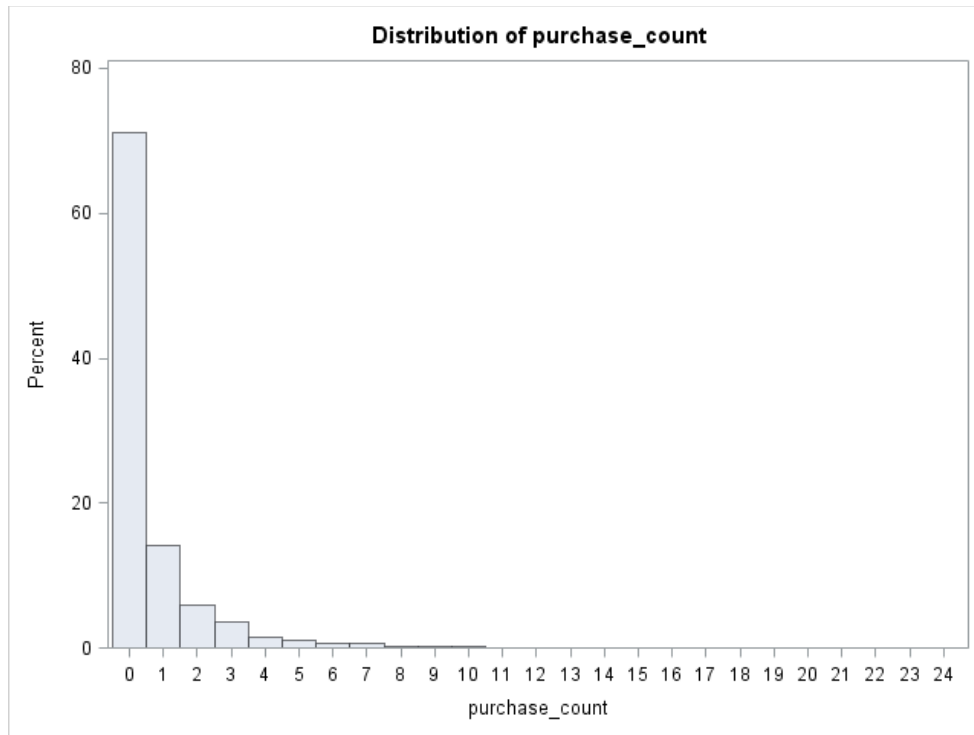


Customer 2

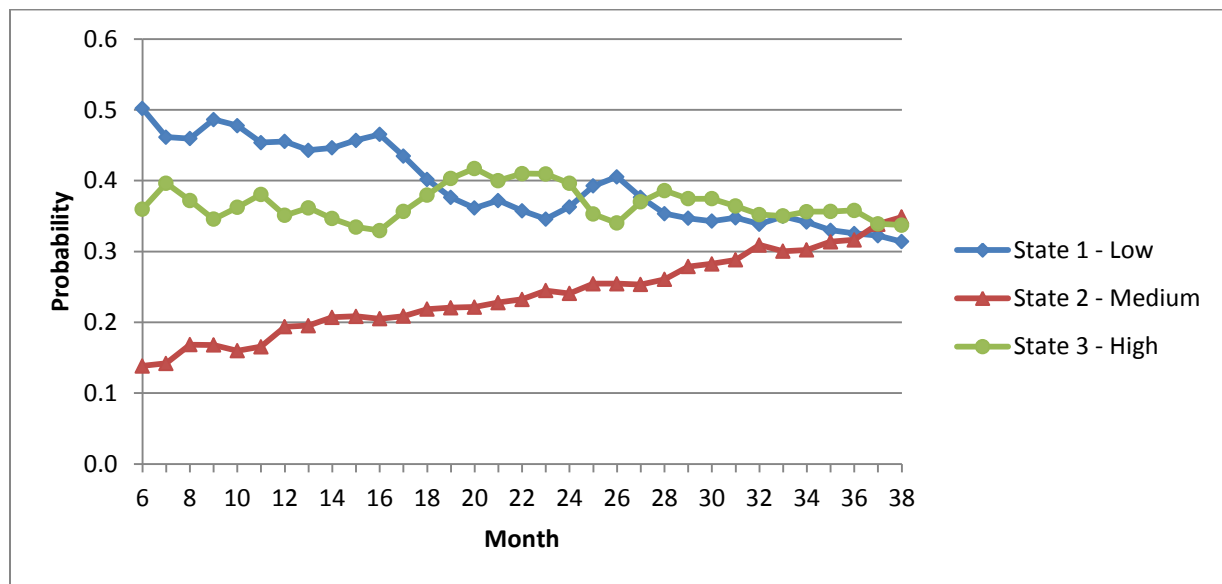


Customer 3

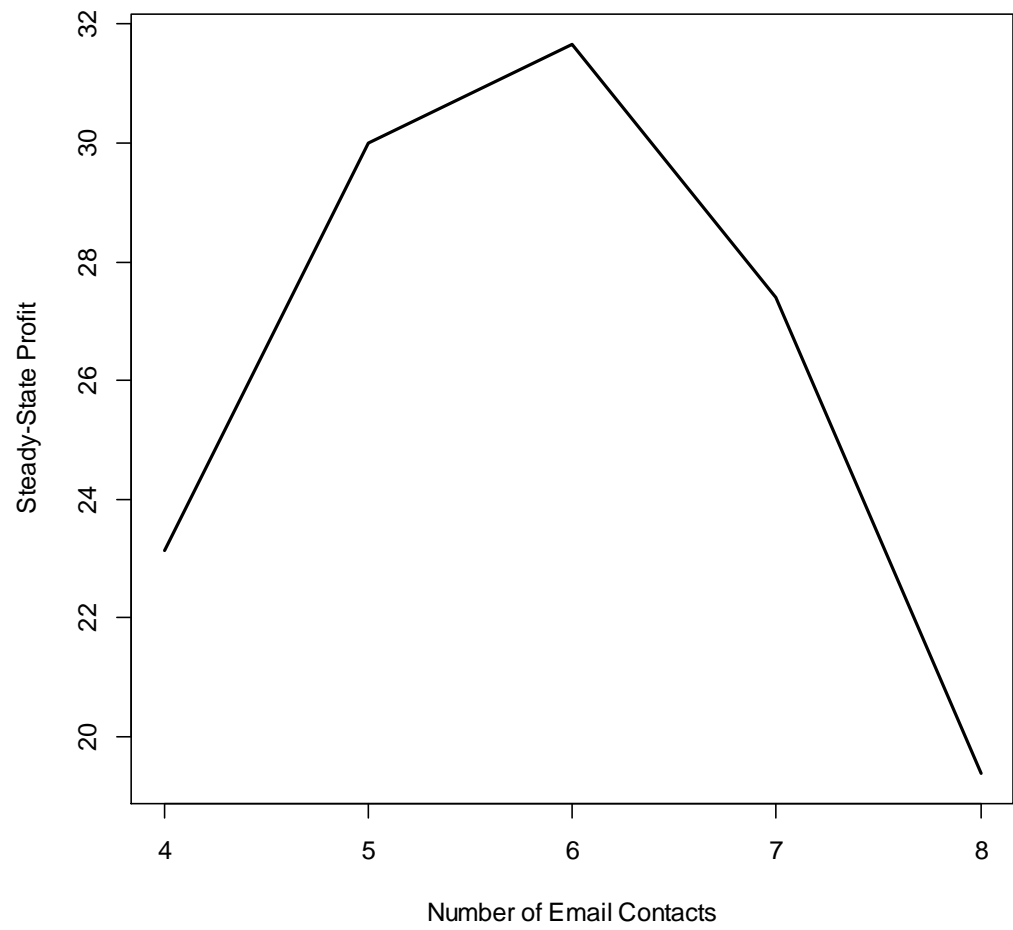
**Figure 2 Distributions of Purchase and Email Open Count**



**Figure 3 Evolution of Relationship States over Time**



**Figure 4 Distribution of Steady-State Profit against the Number of Email Contacts**



**Table 1 Descriptive Statistics**

	Mean	Std. Dev.	Lower 5%	Upper 95%
Number of Purchases	0.69	1.63	0	4
Number of Emails Sent	6.90	4.91	0	15
Number of Emails Opened	1.64	3.16	0	10

**Table 2 Selecting the Number of States**

HMM States	Log-Likelihood	BIC
1	-8949.95	18007.54
2	-8712.93	17659.08
3	-8582.33	17541.41
4	-8550.44	17639.10

**Table 3 Estimation Results for the 3-States Hidden Markov Model**

	Estimates	Standard Error	T-Value
<b>Transition Matrix</b>			
Intercept for transition (State 1 to 2)	-4.917	0.326	-15.087
Intercept for transition (State 1 to 3)	-1.213	0.219	-5.549
Intercept for transition (State 2 to 2)	-0.191	0.447	-0.427
Intercept for transition (State 2 to 3)	-1.242	1.587	-0.783
Intercept for transition (State 3 to 2)	-2.425	0.318	-7.626
Intercept for transition (State 3 to 3)	2.041	0.210	9.740
Time since last purchase on transition to State 2 (log)	0.384	0.093	4.118
Time since last purchase on transition to State 3 (log)	-0.312	0.058	-5.381
Time since last open on transition to State 2 (log)	0.901	0.124	7.266
Time since last open on transition to State 3 (log)	0.042	0.060	0.697
Lag Email Sent on transition to State 2	-0.173	0.031	-5.487
Lag Email Sent on transition to State 3	-0.040	0.018	-2.292
Lag Email Sent Square on transition to State 2	0.013	0.003	4.825
Lag Email Sent Square on transition to State 3	0.008	0.002	4.833
<b>Conditional Purchase Frequency (ZNBD)</b>			
<i>Negative Binomial Equation</i>			
Intercept State 1	-3.349	0.106	-31.550
Intercept (additional State 2, exp)	0.993	0.055	18.196
Intercept (additional State 3, exp)	-1.427	0.633	-2.252
Time Since Last Purchase State 1 (log)	0.249	0.073	3.397
Time Since Last Purchase State 2 (log)	-1.229	0.245	-5.011
Time Since Last Purchase State 3 (log)	-0.970	0.149	-6.517
Email Sent State 1	1.129	0.022	51.350
Email Sent State 2	-1.173	0.102	-11.500
Email Sent State 3	-0.102	0.047	-2.159
Email Sent Square State 1	-0.095	0.003	-36.556
Email Sent Square State 2	-0.752	0.156	-4.823
Email Sent Square State 3	-0.006	0.005	-1.114
Theta, exp	0.727	0.391	1.860
Variance for the intercept, exp	-0.633	0.212	-2.989
<i>Excess of Zeros Equation</i>			
Intercept State 1	0.955	0.242	3.952
Intercept State 2	-1.561	0.516	-3.025
Intercept State 3	2.692	0.864	3.117
Time since last purchase State1 (log)	-0.693	0.110	-6.284
Time since last purchase State2 (log)	2.486	0.913	2.723
Time since last purchase State3 (log)	-0.898	0.456	-1.971
<b>Email Open Frequency (Binomial)</b>			
Intercept State 1	-2.255	0.185	-12.193
Intercept State 2	0.428	0.192	2.228
Intercept State 3	-6.024	0.598	-10.073
Time since last open state 1 (log)	-0.275	0.121	-2.278
Time since last open state 2 (log)	-0.274	0.133	-2.060
Time since last open state 3 (log)	-1.056	0.565	-1.870
Variance for the intercept, exp	0.806	0.076	10.550
<b>Correlation (Email Open and Purchase)</b>			
Frank copula correlation coefficient	1.593*	0.753	2.115

\*The corresponding Spearman's rho in the parenthesis (see Trivedi and Zimmer 2005 for the transformation of the dependence measures).

**Table 4 Transition Probability Matrix of the HMM**

	To State 1	To State 2	To State 3
<i>Transition Probabilities</i>			
<i>Without Email Contacts</i>			
From State 1	91.8%	2.1%	6.1%
From State 2	27.4%	70.8%	1.8%
From State 3	33.2%	9.2%	57.6%
<i>Transition Probabilities</i>			
<i>With One Email Contacts</i>			
From State 1	81.4%	4.3%	14.3%
From State 2	14.0%	83.6%	2.4%
From State 3	16.1%	10.3%	73.6%
<i>Transition Probabilities</i>			
<i>With Five Email Contacts</i>			
From State 1	82.1%	3.0%	14.9%
From State 2	19.0%	77.6%	3.4%
From State 3	16.3%	7.1%	76.6%
<i>Transition Probabilities</i>			
<i>With Ten Email Contacts</i>			
From State 1	76.2%	3.1%	20.7%
From State 2	17.2%	78.3%	4.5%
From State 3	11.7%	5.7%	82.5%