

Long-Term and Penalty Contracts in a Two-Stage Supply Chain with Stochastic Demand *

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December 12, 2003

* Under review at *Management Science*; not for quotation without the permission of the authors.

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Abstract

Recent applications of game-theoretic analysis to supply chain efficiency have focused on constructs between a buyer (the retailer or *manufacturer*) and a seller (the *supplier*) in successive stages of a supply chain. If demand for the final product is stochastic then the supplier has an incentive to keep its capacity relatively low to avoid creating unneeded capacity. The manufacturer, on the other hand, prefers the supplier's capacity to be high to ensure that the final demand is satisfied. The manufacturer therefore constructs a contract to induce the supplier to increase its production capacity. Most research examines contracting when final demand is realized *after* the manufacturer places its order to the supplier. However, if final demand is realized *before* the manufacturer places its order to the supplier, these types of contracts can be ineffective. This paper examines two contracts under the latter timing scenario: long-term contracts in which the business relationship is repeated, and penalty contracts in which the supplier is penalized for too little capacity. Results indicate long term contracts increase the profit potential of the supply chain. Furthermore, the penalty contracts can ensure that the supplier chooses a capacity level such that the full profit potential is achieved.

1. Introduction

As supply chains have become more extended in recent years, coordination and information sharing among supply chain members to improve system efficiency have attracted much interest. Many studies have considered the order fulfillment dilemma faced by manufacturing firms selling customized, make-to-order products. The dilemma consists of having to deliver products with tight deadlines while reducing inventories to minimize risk. To resolve their dilemma, the manufacturers commonly begin procurement by requesting resource commitments from their suppliers based on forecasted orders (i.e., soft orders), as opposed to waiting for firm purchase orders from their customers. Benefiting from large supplier capacity commitments while not directly bearing the costs, the manufacturers have incentive to initially overforecast before eventually purchasing a lower quantity from their suppliers. On the other hand, the suppliers have an incentive to keep their capacity relatively low so as to avoid creating unneeded capacity.

Previous research has applied game-theoretic analysis to show that such opportunistic behavior results in supply chain inefficiencies (e.g., Tsay 1999, Cachon and Lariviere 2001, Wang 2002). A recent empirical study of the semiconductor equipment industry has confirmed these results by demonstrating that if forecasts are not credible, they will be ignored and supply chain performance suffers (Cohen et al. 2003). This study also concludes that information sharing among supply chain members by itself is not sufficient to build superior supply chain performance.

Supply chain contracts have been suggested as a coordination mechanism that provides incentives to all supply chain members so that the decentralized and uncoordinated supply chain behaves nearly or exactly the same as an integrated one (Wang 2002, Cachon 2002). Usually supply contracts are designed to shift some risk to the manufacturer, thereby inducing the supplier to build a higher capacity. Supply chain contracts such as buyback, quantity flexibility, backup, price protection, revenue sharing, and sales rebate have been studied extensively in the literature

(e.g., Eppen and Iyer 1997; Tsay and Lovejoy 1999; Lee *et al.* 2000; Cachon and Lariviere 2003). These contracts have been shown to be effective if demand is realized *after* the manufacturer places its order. Whang (1995), Cachon (1998), Lariviere (1998), Tsay et al. (1998), and Cachon (2002) provide excellent reviews of the literature.

While such contracts remain common and effective in a variety of industries, the scenario in which demand is realized *before* the manufacturer places its order to the supplier, or equivalently, the manufacturer does not need to place an order because the supplier has taken over that responsibility, is becoming more relevant. This timing scenario can result from collaborative relationships among supply chain members, and these relationships have been suggested as an effective alternative to the typical, self-serving and opportunistic behavior among the firms (Davis and Spekman, 2004). Although such collaborative relationships among supply chain members are not commonplace, a number of exemplary companies have begun to practice them with their best suppliers. The increased popularity of vendor managed inventory (VMI) systems, continuous replenishment programs (CRP), and collaborative planning, forecasting, and replenishment (CPFR) programs, are examples of the growing acceptance of such collaborative supply chain relationships (Davis and Spekman, 2004).

Dell spearheads a specific example of a collaborative supply chain relationship in which a "back-end" intranet links Dell's material planners directly to supplier inventories. Dell uses these supplier connections to share a wide variety of real-time information regarding its customers and its own assembly plants. Incoming order information is shared immediately with Dell's suppliers, who deliver to Dell plants from supplier-owned warehouses located near the plants in less than 90 minutes after receiving a replenishment order (Mayersohn, 2001). The suppliers also utilize this information to improve the accuracy of their forecasts, because of long delivery lead times of some of their parts from second- and third-tier suppliers. From the time a customer order is received at Dell's plant, a finished PC can be shipped in less than 4 hours, and the computer can be in customer's possession the next day.

If indeed the final demand is realized before the manufacturer places its order (but after the supplier creates its capacity) then a contract designed for the previous timing scenario may not be appropriate. For example, the buyback contract is designed to handle orders that exceed demand. Because this cannot happen if demand is realized first, this contract would be ineffective. The supplier, who bears the full cost of creating the capacity while not receiving the full benefit, will tend to underinvest. An effective contract in this scenario must therefore address differences between the supplier's capacity and final demand. To bring capacity closer to the ideal level, the contract should encourage the supplier to increase capacity and/or discourage it from choosing too little. To that end, we examine two types of contracts: long-term and penalty. The long-term contract increases the marginal benefit to the supplier from increasing capacity because the additional production periods increase the likelihood that the extra capacity is actually used. The penalty contract, in which the supplier must pay a fee if it cannot meet the manufacturer's order, increases the cost to the supplier of underinvesting.

The model, in which demand is stochastic and realized before the manufacturer places its order, is introduced in Section 2. Then, a general solution is characterized in a one-period game with no penalties in Section 2.1. Section 3 introduces long-term and penalty contracts, and it is shown that the profit potential increases and is fully realized under any well-behaved demand distribution. To highlight and visualize these results, a specific example is then given in which the demand follows an exponential distribution. Finally, in Section 4, the conclusions are presented.

2. The Model

In the model we examine two successive links in a supply chain, a manufacturer and its supplier, that are involved in a collaborative relationship which ensures that final demand is known before the supplier fills the manufacturer's order. Each period the risk-neutral manufacturer sells its output at a pre-determined, per-unit price r which creates an uncertain

demand. Demand is IID on the interval $[a, b)$, where $0 \leq a < b \leq \infty$. The density function for demand is denoted $f(x)$, where $f(x) > 0$ for $x > 0$. The cumulative density function is denoted $F(x)$, where $F(0) = 0$ and $F(b) = 1$. Demand for the manufacturer's product translates into a corresponding demand for the components purchased from the supplier.

The risk-neutral supplier can create capacity at a cost of c_k per unit at the beginning of each period. Included in c_k is the supplier's opportunity cost when devoting a unit of capacity to the manufacturer. The supplier therefore agrees to any transaction with the manufacturer that yields a non-negative expected economic profit, which is to say that the supplier's expected profit with the manufacturer is at least as great as its best alternative. A unit of capacity can be converted into one usable component per period at a production cost of c_p . The supplier incurs the production costs only when filling the manufacturer's order. The sum of the costs, $c_k + c_p$, is less than the final product price r . All other costs are normalized to zero. The depreciation rate of capacity per period is equal to $\delta \in [0, 1]$. The salvage value for capacity is assumed to be zero.

The supplier chooses capacity each period before the final demand is realized. It is committed to supply components on short notice to the manufacturer, or to manage the inventory at the manufacturer based on regular information exchange. The manufacturer places an order for a number of components equal to demand. If the supplier has created k units of capacity, it can fill the manufacturer's order only if demand is less than or equal to k . Otherwise the supplier cannot fill the order, and some of the manufacturer's demand is left unsatisfied. The manufacturer's expected sales in a period then equal

$$\int_0^k x f(x) dx + k(1 - F(k)), \quad (1)$$

which simplifies to

$$k - \int_0^k F(x) dx. \quad (2)$$

An efficient supply chain is one that chooses k to maximize total profits, which equal the combined profits of the supplier and manufacturer:

$$\Pi_T = \left(k - \int_0^k F(x) dx \right) (r - c_p) - c_k k. \quad (3)$$

The first order condition for the optimal capacity k^* is $\frac{\partial \Pi_T}{\partial k} = 0$, and simplifies to the condition

$$F(k^*) = 1 - \frac{c_k}{r - c_p}. \quad (4)$$

Because $F(x)$ is strictly increasing there is a unique solution to this optimal capacity problem.¹

The optimal capacity for the supply chain, k^* , is termed the *efficient capacity*.

2.1. The One-Period Market Solution

The efficient capacity would be chosen if the manufacturer and supplier act as a single firm, without concern for the division of profits between them. If, however, the manufacturer and supplier act independently to maximize their individual profits, then the pursuit of their own self-interests can cause a deviation from the interests of the supply chain as a whole. To examine this deviation we derive the Nash Equilibrium outcome in which the manufacturer and supplier unilaterally choose their own strategic variables – the component price and capacity, respectively – to serve their own interests.

The final product price r is fixed, and so in the first stage the manufacturer offers an individually rational, incentive compatible contract to the supplier specifying the component price w . The supplier uses this in determining its capacity k , and accepts the contract if its own expected profits are non-negative.

¹ The second derivative of the profit function equals $\frac{\partial^2 \Pi_T}{\partial^2 k^2} = -n(w - c_p)f(x)$. This is negative, indicating that k^* represents an optimum.

The problem is solved by backward induction, solving first the supplier's optimal capacity in terms of w . The supplier's profit function equals

$$\Pi_S = \left(k - \int_0^k F(x) dx \right) (w - c_p) - c_k k. \quad (5)$$

The first order condition for the supplier's optimal capacity k_s is $\frac{\partial \Pi_S}{\partial k} = 0$, which simplifies to the condition

$$(1 - F(k_s))(w - c_p) = c_k. \quad (6)$$

The right-hand-side of (6) represents the marginal cost to the supplier of increasing capacity, and the left-hand-side represents the marginal net revenue from increasing capacity, which results from an increase in the expected value of sales. The supplier maximizes its own profit by choosing capacity such that the two sides are equal. For analytical convenience condition (6) is restated as follows:

$$F(k_s) = 1 - \frac{c_k}{w - c_p}. \quad (7)$$

Condition (7) is identical to condition (4) except that the final product price r is replaced with the component price w . Note that the supplier's optimal capacity is positively related to w .²

Given the supplier's optimal response to w , the manufacturer's profit function becomes

$$\Pi_M = \left(k_s - \int_0^{k_s} F(x) dx \right) (r - w). \quad (8)$$

The first order condition, $\frac{\partial \Pi_M}{\partial w} = 0$, simplifies to

$$\frac{\partial k_s}{\partial w} (r - w^*) (1 - F(k_s)) = k_s - \int_0^{k_s} F(x) dx. \quad (9)$$

² It is easy to see that $\frac{\partial k_s}{\partial w}$ is positive. If w increases, the right-hand-side of (7) increases. Since $F(x)$ is strictly increasing in x , the value k_s must also increase to restore equality.

The right-hand-side of (9) represents the marginal loss to the manufacturer's profits resulting directly from an increase in the component price w . The left-hand-side represents the marginal gain in profits resulting from the positive effect that an increase in w has on capacity. The manufacturer's profits are maximized when w is chosen such that the two sides are equal.³ The Nash Equilibrium capacity and component price are the values w and k that simultaneously satisfy conditions (7) and (9). The supplier's capacity choice is characterized by Theorem 1:

Theorem 1: *The supplier's capacity choice, k_s , is less than the efficient capacity, k^* .*

Proof: Since the right-hand-side of (9) is strictly positive, it follows that w^* is less than r . The right-hand-side of condition (4) is therefore greater than that of condition (7). Since $F(x)$ is strictly increasing in x , it must be the case that $k^* > k_s$. The supplier therefore chooses a capacity less than the efficient level. \square

The intuition for Theorem 1 has to do with the marginal costs and net revenues of investment. Profits are maximized, whether for the supply chain or for the supplier, when capacity is chosen so that the marginal cost of capacity equals the marginal net revenue. From the supply chain perspective, the marginal cost of capacity equals c_k , and the marginal net revenue equals the final product price r , less the production cost c_p , times the additional sales. From the supplier's perspective, the marginal cost of capacity is the same, but the marginal net revenue equals the component price w , less the production cost, times the additional sales. The

³ We have not modeled explicitly the constraint that the supplier's profits should be non-negative. If indeed $\Pi_S < 0$ at w^* , the manufacturer must raise w to $c_k + c_p$: the value at which $\Pi_S = 0$. The supplier would still choose a capacity less than k^* because $r > c_k + c_p$.

marginal net revenue to the supplier is lower, and so it chooses a level of capacity that is lower than what is optimal for the supply chain.

This section demonstrates that a one-period arrangement between the supplier and the manufacturer, in which the manufacturer chooses only the component price, results in an underinvestment in capacity. Supply chain profits could be increased if the supplier increases its capacity. The following section examines contracts the manufacturer can use to induce the supplier to do so.

3. Incentive Contracts

The goal of the manufacturer is to influence the supplier's costs and benefits to induce it to increase capacity. An effective contract therefore must increase the benefits of raising capacity and/or increase the costs of underinvesting.

Increasing the price per component does increase the benefits of a higher capacity, but this method falls short of yielding the efficient capacity because it comes at a cost to the manufacturer. Another method stems from the risk of overinvesting. If the supplier chooses a capacity that turns out to be too high, it receives no return on the unused capacity.⁴ If some of this unused capacity could potentially generate a positive return, the supplier would be more willing to create it. Repeating the transaction over multiple periods, in addition to supporting the collaborative spirit, creates this potential. If the supplier happens to overinvest one period, it still has a chance to utilize some of the excess capacity the following period. This raises the marginal benefit of capacity. The first contract we examine is therefore a long-term contract in which the business relationship between the manufacturer and supplier is repeated.

⁴ Having a positive salvage value for capacity would surely increase the supplier's capacity choice, but it would also increase the optimal capacity. The supplier would still underinvest.

The second contract we examine addresses the cost to the supplier when underinvesting. Underinvesting costs the supplier foregone sales, and so there is an opportunity cost of w per unit that capacity falls short of demand. The manufacturer influences this cost by choosing w . But again, raising w increases its own costs as well. It is more effective to add an explicit cost to the supplier per unit that capacity falls short of demand, especially if the cost is paid to the manufacturer. To this end the manufacturer can specify a penalty, paid by the supplier to the manufacturer, for each unit of excess demand. As long as its expected economic profit from the contract is non-negative, the supplier accepts it. The second contract we examine is therefore one that specifies both a component price and a penalty per unit of excess demand.

3.1 The Long-Term Contract

Suppose the manufacturer offers a contract to the supplier that specifies a component price w as well as a number of periods n in which the transaction is conducted. Demand is IID each period, and so the expected value of demand never changes. Because of depreciation, for the supplier to maintain a capacity of k it must create k units in the first period and δk units each period thereafter. By the end of n periods, the total units of capacity the supplier has created equals

$$k + (n - 1)\delta k. \quad (10)$$

From (2) the expected value of the total sales over n periods is found to be

$$S_T = n \left(k - \int_0^k F(x) dx \right), \quad (11)$$

and the total supply chain profits become

$$\Pi_T = n \left(k - \int_0^k F(x) dx \right) (r - c_p) - c_k (k + (n - 1)\delta k). \quad (12)$$

The efficient capacity satisfies the first order condition

$$F(k^*) = 1 - \frac{c_k(1 + (n-1)\delta)}{n(r - c_p)}. \quad (13)$$

Theorem 2: *As n increases, the efficient capacity increases, thereby increasing the profit potential of the supply chain.*

Proof: If n increases, the right-hand-side of (13) increases. Since $F(x)$ is strictly increasing in x , the value k^* must increase to restore equality.

The total profits per period equal $\frac{\Pi_T}{n}$, and from (12) we can see that

$$\frac{\partial\left(\frac{\Pi_T}{n}\right)}{\partial n} = \frac{c_k k(1 - \delta)}{n^2}. \quad (14)$$

This is positive, meaning that a higher n increases total profit per period if k remains constant. But if k then increases to its efficient level, profits increase even more. The profit potential per period thus unambiguously increases as n increases. \square

Let us now determine the effect of long-term contracts on the supplier's capacity choice if the manufacturer and supplier act independently to maximize profits. The supplier's profit function is

$$\Pi_S = n \left(k - \int_0^k F(x) dx \right) (w - c_p) - c_k (k + (n-1)\delta k), \quad (15)$$

and its first order condition simplifies to

$$F(k_n) = 1 - \frac{c_k(1 + (n-1)\delta)}{n(w - c_p)}. \quad (16)$$

The manufacturer's profit function is

$$\Pi_M = n \left(k_n - \int_0^{k_n} F(x) dx \right) (r - w), \quad (17)$$

and its first order condition for the optimal component price equals

$$\frac{\partial k_n}{\partial w}(r - w^*)(1 - F(k_n)) = k_n - \int_0^{k_n} F(x)dx. \quad (18)$$

Theorem 3: *When n increases, the supplier's equilibrium capacity increases, but remains less than the efficient level for any value of n .*

Proof: Denote k_1 and w_1 as the equilibrium capacity and price when the number of periods is n_1 . This implies that the first order conditions for the supplier and manufacturer, (16) and (18), simultaneously hold. Recall that the right-hand-side of (18) is the marginal loss to the manufacturer resulting from a small increase in w , and the left hand side is the marginal gain resulting from the positive effect that a higher w has on capacity. Profits are maximized when the two are equal.

By differentiating both sides of (16) with respect to w we find that

$$\frac{\partial k_1}{\partial w} = \frac{c_k(1 + (n_1 - 1)\delta)}{n_1(w - c_p)^2 f(k_1)}. \quad (19)$$

This value is positive, meaning that the equilibrium capacity increases as w increases.

Suppose that n increases to n_2 and w remains unchanged for the moment. The right-hand-side of (16) increases, and so the supplier increases capacity to restore equality. Suppose now that the manufacturer lowers w to the value, denoted \underline{w} , at which the supplier reduces capacity back to k_1 . From (16) it can be found that

$$\underline{w} = c_p + \frac{n_1(w_1 - c_p)(1 + (n_2 - 1)\delta)}{n_2(1 + (n_1 - 1)\delta)}. \quad (20)$$

From (19) and (20) it can be seen that $\frac{\partial k_1}{\partial w}$ is higher when $w = \underline{w}$ than when $w = w_1$.

Because of this and the fact that $\underline{w} < w_1$, it is evident from (20) that at capacity k_1 and price \underline{w} ,

$$\frac{\partial k_1}{\partial w}(r - \underline{w})(1 - F(k_1)) > k_1 - \int_0^{k_1} F(x)dx. \quad (21)$$

The inequality in (21) implies that the marginal benefit from increasing w exceeds the marginal loss, and so the manufacturer improves profits by raising w . The supplier responds by increasing capacity to a value higher than k_1 . A higher n therefore results in a higher equilibrium capacity.

Though the supplier increases its capacity as n increases, it does not choose the efficient level. Theorem 2 demonstrates that the efficient level of capacity increases as well, and conditions (13) and (16) indicate that the efficient capacity is again only chosen if $w = r$. The manufacturer always chooses $w < r$, and so the supplier underinvests relative to the efficient capacity for any value of n . \square

The significance of Theorem 3 is that the manufacturer can induce the supplier to increase its capacity by offering a repeating contract. The higher capacity induced by this collaborative measure changes the profit structure as outlined in Theorem 4:

Theorem 4: *In equilibrium, as n increases the manufacturer's profits and total supply chain profits per period both increase.*

Proof: The total profits per period equal $\frac{\Pi_r}{n}$, and from (14) we know that this value increases as n increases and k remains constant. Theorem 3 proves that the supplier then raises k toward (but not surpassing) the efficient level, increasing profits even more. Total supply chain profits per period thus unambiguously increase as n increases.

The manufacturer's equilibrium profits unambiguously increase with n . The reason is that if n increases, capacity increases even if the manufacturer does not change w . This in itself would raise the manufacturer's periodic profits. But the manufacturer then adjusts w to its

benefit, thereby increasing its periodic profits even further. The manufacturer's profits per period thus unambiguously increase as n increases. \square

The fact that the manufacturer's profits increase when n increases is significant because it means the manufacturer would choose to offer a repeating contract. The supplier accepts the contract so long as its economic profits are non-negative. If the supplier's economic profits are negative in equilibrium, this implies it could earn a greater return on its capital elsewhere. The manufacturer must then increase w until its contract is at least as attractive as the supplier's best alternative.

Offering a long-term contract is beneficial in that it induces a higher capacity, but it does not induce the supplier to choose the efficient capacity. In the next section we show that the use of penalties in conjunction with a long-term contract can increase supply chain profit potential *and* ensure that the supplier chooses the efficient capacity.

3.2. The Penalty Contract

In a penalty contract the manufacturer specifies the component price w and the penalty p , which the supplier pays to the manufacturer per unit that demand exceeds capacity. When the supplier chooses capacity k , the expected value of the total penalty paid per period equals p times the expected value of the amount demand exceeds k . This equals

$$p \left(\int_k^b f(x)(x-k)dx \right), \quad (22)$$

which simplifies to

$$p \left(b - k - \int_k^b F(x)dx \right). \quad (23)$$

The expected value of the total penalty paid over n periods equals n times this amount, and the expected value of the supplier's total profits becomes

$$\Pi_S = n \left(k - \int_0^k F(x) dx \right) (w - c_p) - c_k (k + (n-1)\delta k) - np \left(b - k - \int_k^b F(x) dx \right). \quad (24)$$

The first order condition $\frac{\partial \Pi_S}{\partial k} = 0$ simplifies to

$$F(k^*) = 1 - \frac{c_k (1 + (n-1)\delta)}{n(w + p - c_p)}. \quad (25)$$

Note from (13) and (25) that the supplier chooses the efficient capacity if $w + p = r$.

To maximize its profits the manufacturer should choose w and p such that the supplier chooses the efficient capacity and so that the contract is just as attractive to the supplier as its best alternative. The following theorem shows that such a contract is possible.

Theorem 5: *There exist values w^* and p^* such that $w^* + p^* = r$ and $w^*, p^* \in (0, r)$, and such that $\Pi_S(w^*, p^*) = 0$. A contract specifying w^* and p^* maximizes supply chain profits.*

Proof: Since w is a revenue to the supplier and p is cost, it follows that $\frac{\partial \Pi_S}{\partial w} > 0$ and $\frac{\partial \Pi_S}{\partial p} < 0$.

Lowering w and raising p such that $w + p = r$ unambiguously reduces Π_S . If $w = r$ and $p = 0$, then $\Pi_S > 0$ because $w > c_k + c_p$. If $w = 0$ and $p = r$, then $\Pi_S < 0$ if the supplier accepts the contract and builds any positive capacity. Because Π_S is continuous in w and p , there must exist $w^*, p^* \in (0, r)$ such that $w^* + p^* = r$ and $\Pi_S(w^*, p^*) = 0$. By choosing $w = w^*$ and $p = p^*$ the manufacturer induces the supplier to choose the efficient capacity, and since $\Pi_S(w^*, p^*) = 0$ the supplier agrees to the contract. \square

The penalty contract is clearly ideal for the manufacturer. It maximizes its profits and ensures the efficient capacity for any value of n . The manufacturer can also increase its profits further by raising n .

In equilibrium the manufacturer should offer the penalty contract and set $n = \infty$. There are of course practical limitations that would preclude an infinite-term contract. For example, fluctuations in demand and costs over time, which are not permitted in this model, can make reforming the contract from time to time both desirable and necessary. Nevertheless, these findings illustrate the potential impact of long-term contracts, together with a penalty stipulation, on the capacity and profits.

The results illustrate that combining long-term and penalty contracts can improve supply chain profits for any well-behaved, continuous demand distribution. To visualize these results, however, it is helpful to solve the model with a specific demand distribution. Like the example provided in Cachon and Lariviere (2001), the next section assumes demand is distributed exponentially.

Example: Demand Follows an Exponential Distribution

Suppose demand is distributed according to the exponential distribution function $f(x) = \alpha e^{-\alpha x}$ and $F(x) = 1 - e^{-\alpha x}$, where the mean of the distribution is $\frac{1}{\alpha}$. Assume also that the parameterization of the model is as follows: $\alpha = \frac{1}{3}$, $c_k = 2$, $c_p = 1$, $\delta = 0.2$, and $r = 6$. From (13) the efficient capacity is found to be

$$k^* = \ln\left(\frac{5n}{1.6 + 0.4n}\right)^3. \quad (26)$$

If the supplier and manufacturer act independently and there is no penalty, then the Nash Equilibrium capacity and component price are found from conditions (9) and (16). If there is a

penalty, then equilibrium capacity, component price, and penalty are found from condition (26) and from applying Theorem 5. The plots as n goes from 1 to 10 are as follows:

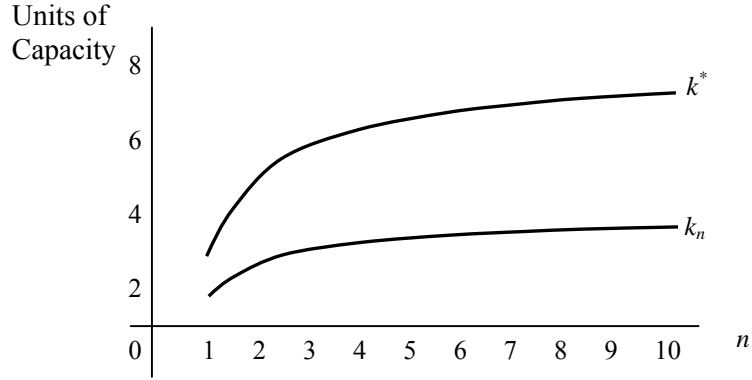


Figure 1
Capacities Under Long-Term Contracts

Figure 1 shows, as Theorems 2 and 3 predict, that the supplier underinvests, and both the supplier's and the efficient capacity increase with n .

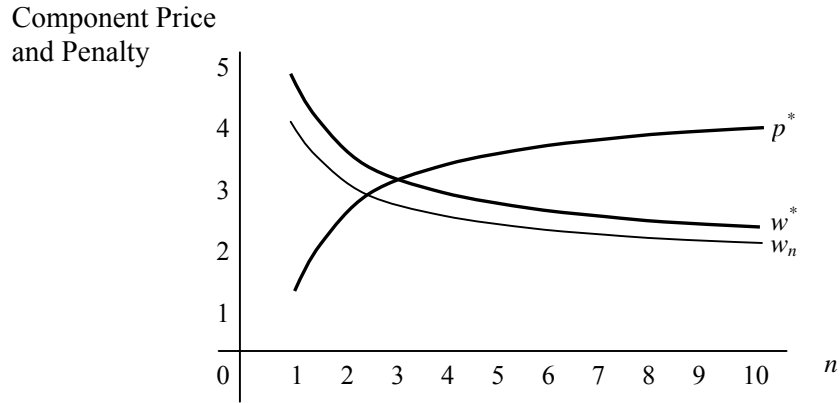


Figure 2
Component Prices and Penalties Under Long-Term Contracts

Figure 2 shows that the component price with and without the penalty drops. Furthermore, the component price is higher with the penalty contract. The penalty increases with n .

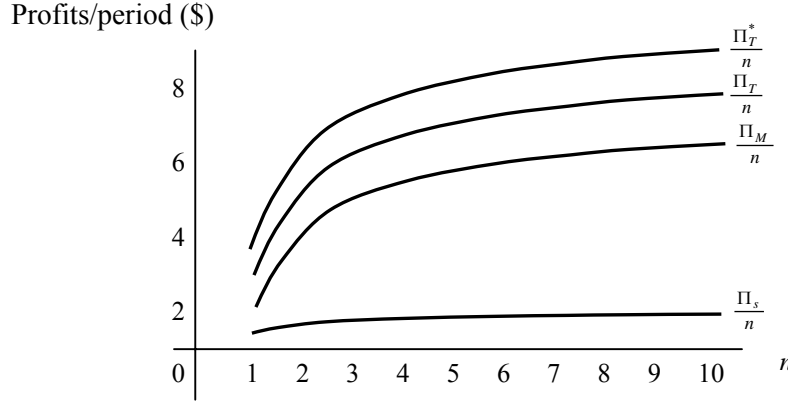


Figure 3
Profits Per Period Under Long-Term Contracts

Figure 3 depicts the profits per period. The value $\frac{\Pi_T^*}{n}$ represents total supply chain profits per period when the efficient capacity is created. This value equals the manufacturer's profits under the optimal penalty contract because the supplier earns a return equal to that of its best alternative. As Theorem 2 predicts, this value increases with n . The values $\frac{\Pi_T}{n}$, $\frac{\Pi_M}{n}$, and $\frac{\Pi_s}{n}$ represent, respectively, the supply chain's, manufacturer's, and supplier's profits per period with no penalty contract. As Theorem 4 predicts, $\frac{\Pi_T}{n}$ and $\frac{\Pi_M}{n}$ both increase with n .

4. Conclusion

Recent research primarily considers contracting when final demand is realized after the manufacturer places its order to the supplier. However, if the final demand is realized before the manufacturer places its order to the supplier or if the manufacturer does not place an order since the supplier has taken over that responsibility (due to collaboration among the supply chain members), the proposed contracts can be ineffective. If demand is realized first, effective contracts must either provide the incentive to the supplier to increase capacity, or decrease the

incentive to underinvest. To that end, this paper examined two types of contracts: a long-term contract in which the business relationship is repeated, and a penalty contract in which the supplier is penalized for too little capacity.

The results indicate that suppliers tend to create a capacity below the supply chain optimal level (i.e., they underinvest in capacity). However, as was illustrated, the manufacturer can induce the supplier to create higher capacity by offering a repeated (i.e., long-term) contract. It was also shown that the higher capacity induced by this collaborative measure increases the per-period manufacturer's and supply chain profit potential. The addition of the penalty contract can encourage the supplier to further increase its capacity to the efficient level, ensuring that the supply chain profit potential is realized. The proposed contracts are increasingly relevant as the traditional self-serving and opportunistic behavior among supply chain members are gradually changing to more collaborative ones. The increased application of VMI, CRP and CPFR programs in a variety of industries indicate the growing acceptance of collaborative supply chain relationships in recent years.

There are a number of directions in which this research can be extended. One possible extension stems from the reason the supplier might underinvest – it bears the full cost of creating the capacity while receiving less than the full benefit. In principle, efficiency can be achieved if the manufacturer and the supplier share the capacity costs in a proportion equal to that at which the financial benefits are shared. At issue, however, is how the supply chain members could arrive at such an agreement. It could perhaps result from a better balance of negotiating power between the supply chain parties and/or an asymmetry of information with regard to the costs of creating capacity. Further study could determine how the supply chain members might agree to share the costs in this manner.

Another potential direction for future research lies in the application of the contracts. Long-term and penalty contracts are effective because they increase the incentive to create capacity. While we have applied these contracts to a scenario in which final demand is realized

before the manufacturer places its order, the incentives they create suggest they could also be useful when demand is realized after the order is placed. The effectiveness of these contracts could be compared to or used in conjunction with the contracts proposed in the previous literature.

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