PRODUCTION PLANNING AND SCHEDULING IN MULTI-STAGE BATCH PRODUCTION ENVIRONMENT

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ABSTRACT

We address the problem of jointly determining production planning and scheduling decisions in a complex multi-stage, multi-product, multi-machine, and batch-production environment. Large numbers of process and discrete parts manufacturing industries are characterized by increasing product variety, low product volumes, demand variability and reduced strategic planning cycle. Multi-stage batch-processing industries like chemicals, food, glass, pharmaceuticals, tire, etc. are some examples that face this environment. Lack of efficient production planning and scheduling decisions in this environment often results in high inventory costs and low capacity utilization.

In this research, we consider the production environment that produces intermediate products, by-products and finished goods at a production stage. By-products are recycled to recover reusable raw materials. Inputs to a production stage are raw materials, intermediate products and reusable raw materials. Complexities in the production process arise due to the desired coordination of various production stages and the recycling process. We consider flexible production resources where equipments are shared amongst products. This often leads to conflict in the capacity requirements at an aggregate level and at the detailed scheduling level. The environment is characterized by dynamic and deterministic demands of finished goods over a finite planning horizon, high set-up times, transfer lot sizes and perishability of products. The decisions in the problem are to determine the production quantities and inventory levels of products, aggregate capacity of the resources required and to derive detailed schedules at minimum cost.

We determine production planning and scheduling decisions through a sequence of mathematical models. First, we develop a mixed-integer programming (MIP) model to determine production quantities of products in each time period of the planning horizon. The objective of the model is to minimize inventory and set-up costs of intermediate products and finished goods, inventory costs of by-products and reusable raw materials, and cost of fresh raw materials. This model also determines the aggregate capacity of the resources required to implement the production plan. We develop a variant of the planning model for jointly planning sales and production. This model has additional market constraints of lower and upper bounds on the demand. Next, we develop an MIP scheduling model to execute the aggregate sales and productions plans obtained from the planning model. The scheduling model derives detailed equipment wise schedules of products. The objective of the scheduling model is to minimize earliness and tardiness (E/T) penalties.

We use branch and bound procedure to solve the production-planning problem. Demand of finished goods for each period over the planning horizon is an input to the model. The planning model is implemented on a rolling horizon basis.

We consider flowshop setting for the finished goods in the production environment. The due dates of finished goods are based on the customer orders. We report some new results for scheduling decisions in a permutation flowshop with E/T penalties about a common due date. This class of problems can be sub-divided into three groups- one, where the common due date is such that all jobs are necessarily tardy; the second, where the due date is such that the problem is unrestricted; and third is a group of problems where the due date is between the above two. We develop analytical results and heuristics for flow shop E/T problems arising in each of these three classes. We also report computational performance on these heuristics. The intermediate products follow a general job shop production process with re-entrant flows. We develop heuristics to determine equipment wise schedule of intermediate products at each level of the product structure. The due date of an intermediate product is based on the schedule of its higher-level product.

The models developed are tested on data for a chemical company in India. The results of cost minimization model in a particular instance indicated savings of 61.20 percent in inventory costs of intermediate products, 38.46 percent in set-up costs, 8.58 percent in inventory costs of by-products and reusable raw materials, and 20.50 percent in fresh raw material costs over the actual production plan followed by the company. The results of the contribution maximization model indicate 42.54 percent increase in contribution. We also perform sensitivity analysis on results of the production planning and scheduling problem.

The contribution of this research is the new complexities addressed in the production planning and scheduling problem. Traditional models on multi-stage production planning and scheduling are primarily based on assembly and fabrication types of product structures and do not consider the issues involved in recycling process. Scheduling theory with E/T penalties is largely limited to single machine environment. We expect that models developed in this research would form basis for production planning and scheduling decisions in multi-stage, multi-machine batch processing systems. The sensitivity analysis of the models would provide an opportunity to the managers to evaluate the alternate production plans and to respond to the problem complexities in a better way.

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Table of Contents

List of Figures

List of Tables

1 Introduction

1.1 Introduction

Today's business environment has become highly competitive. Manufacturing firms have started recognizing the importance of manufacturing strategy in their businesses. Firms are increasingly facing external pressures to improve customer response time, increase product offerings, manage demand variability and be price competitive. In order to meet these challenges, firms often find themselves in situations with critical shortages of some products and excess inventories of other products. This raises the issue of finding the right balance between cutting costs and maintaining customer responsiveness. Firms are facing internal pressures to increase profitability through improvements in manufacturing efficiency and reductions in operational costs.

There are several instances in industry where the above-mentioned changes in business environment have affected the profitability of firms. Harris Corporation, an electronics company based in U.S.A., increased its product range considerably and invested in flexible manufacturing resources in the early 1990s. They had to provide competitive on-time delivery performance over a much greater product mix. Their inefficient handling of a large product variety resulted in late deliveries, lost sales and average losses of $75 million annually (Leachman et al., 1996). IBM faced record losses in 1993 in the manufacturing and distribution operations of its computer business due to high operational costs. They could not handle the high demand variability of their products and reported high inventory costs and stock outs (Feigin et al., 1996). Fuel inventory costs have risen considerably in electric utility industries in U.S.A. in the last two decades as a result of electricity demand fluctuations

(Chao et al., 1989). H&R Johnson, the largest tile manufacturer in India, had to increase its product variety in terms of size and design, in order to meet the demand of expanding construction market. This resulted in high inventory costs. Their customer response time increased considerably, resulting in loss of sales (Gupta, 1993). Synpack, an Indian chemical manufacturing firm, increased its product portfolio in the mid 1990s. However, it could not handle the delivery commitments. The company's market share reduced considerably and they incurred high inventory costs (Akthar, 2004).

Indian manufacturing firms are facing stiff global competition, especially from China. Today, China has become the world's largest manufacturing base. China's capability to offer a large variety of products at low prices, and its fast responsiveness to the market has severely affected the sales of many Indian manufacturing firms. Indian companies are now forced to be competitive on prices, increase product offerings, and have shorter lead times in production.

The implications of the above–mentioned challenges in the business environment are that manufacturing firms are now forced to focus on cost-leadership issues, optimize the use of available resources, and reduce their operational costs. They have to constantly explore manufacturing strategies to meet these objectives.

Since the mid 1980s, the business press has highlighted the success of many Japanese, European, and North American firms in achieving a high degree of efficiency in manufacturing (Silver et al., 1998). In recent years, many of these firms have started to coordinate with other firms in their supply chain. For example, instead of responding to demand variability, firms share information with their partners to analyze demand pattern. It is observed that this notion, although useful, is not a sufficient way of facing some of the challenges discussed earlier. Most managers assume that new levels of efficiency can be obtained simply by sharing information and forming alliances with their partners. They do not realize that information and data have to be used with very clear objectives. Here, the role of inventory management and production planning and scheduling is introduced. Developing sound production planning and scheduling strategies may seem mundane in comparison to strategy formulation, but it is observed that these strategies are critical to long-term survival and competitive advantage.

Production planning and scheduling help considerably in reducing operational costs, improving customer service and utilizing the resources optimally. In the examples discussed above of high operational costs incurred by firms, significant savings have been realized using production planning and scheduling. By applying optimization based production-planning system, Harris Corporation raised its on-time deliveries from 75 to 95 percent without increasing inventories and converted its huge losses to an annual profit $40 million. Over the past two decades, IBM's operations research team developed production-planning systems and helped save hundreds of millions of dollars, while improving operations and competitive strategies. H&R Johnson implemented production-planning tools and reduced its production lead times and inventory costs.

Production planning and scheduling find their applicability in both discrete parts manufacturing and process industries. APICS[1] dictionary provides the key elements to classify industries as process or discrete parts (Blomer and Gunther, 1998; Crama et al., 2001). More and more process industries are shifting to specialties market with customized

[1] American Production and Inventory Control Society

products and are no longer operating on make-to-stock policy alone. This is especially true of batch process industries such as pharmaceuticals, food, and glass, etc. These industries do not restrict themselves to commodity products only. The first significant applications of production planning and scheduling methods in process industries were in oil refineries, food processing and steel manufacturing. Through the years, production planning and scheduling methods have been developed and applied to process manufacturing of other products such as chemicals, paper, soap and industrial gases.

The main motivation for this research is to observe the potential benefits of production planning and scheduling in manufacturing industries. The aim is to investigate the benefits of production planning and scheduling in complex production environments.

The remainder of this chapter is organized as follows. In the next section, we discuss the production planning and scheduling problem addressed in this research. We begin by describing the production environment in sub section 1.2.1. In sub-section 1.2.2, we discuss the complexities in the production environment. We describe the decisions to be addressed in the production planning and scheduling problem in sub-section 1.2.3. The summary of this chapter is provided in section 1.3.

1.2 Production Planning and Scheduling Problem

In this section, we describe the production planning and scheduling problem addressed in this research. First we describe the production environment. The motivation for the production environment considered in this research is largely from our observations on characteristics of chemical plants. Then we describe the complexities in the production environment. Subsequently we focus on the decisions to be addressed in the production planning and scheduling problem.

1.2.1 Production Environment

We consider multi-stage production environment that produces both intermediate products and finished goods. A stage in the production environment corresponds to the production of an intermediate product or a finished good. The concept of multi-stage in the environment considered is equivalent to the multi-level product structure, as shown below for illustration in figure 1.1. In figure 1.1, level 0 products are finished goods (E1, E2, E3), level 1 and level 2 products are intermediate products (I1,I2,…I6). The levels in the product structure diagram are various stages of the production process. For instance, level 1 and level 2 in figure 1.1 are the intermediate products stages. The intermediate products at level 2 are inputs to the intermediate products at level 1. Level 1 intermediate products are inputs to level 0 products, which are finished goods, and at the finished goods production stage.

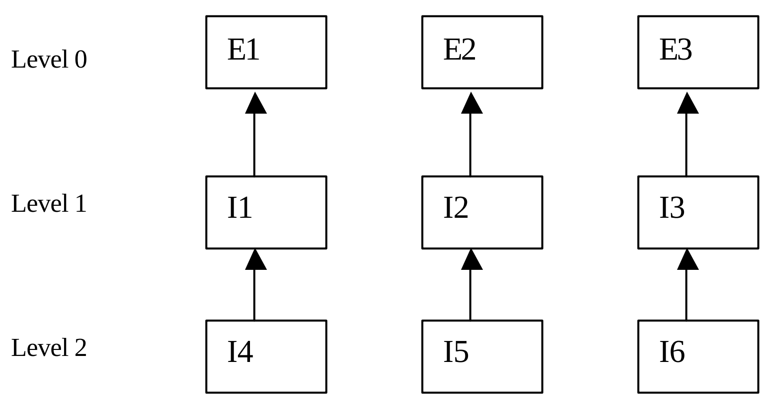

Figure 1.1: Multi-level Product Structure and Concept of Stage

The production environment has multiple production plants to produce intermediate products and finished goods. A production plant consists of number of equipment, called as ‘machines’. Intermediate products and finished goods are processed on machines in a production plant in a specific order. The processing of a product on a machine is called an ‘operation’. A ‘route’ is defined as the sequence of machines used for processing a product. To illustrate these concepts, we use figure 1.2 below. Consider a product ‘P’, it requires four operations in a production plant. There are five machines in the plant in this example (M1,M2,..,M5). As indicated in figure 1.2, there is choice of machines between M3 and M4 for third operation. That is, based on the machine used for third operation of product P, there are two different routes, Route 1 and Route 2 to produce product P. Route 1 comprises machines M1, M2, M3, M5 and Route 2 comprises machines M1, M2, M4 and M5.

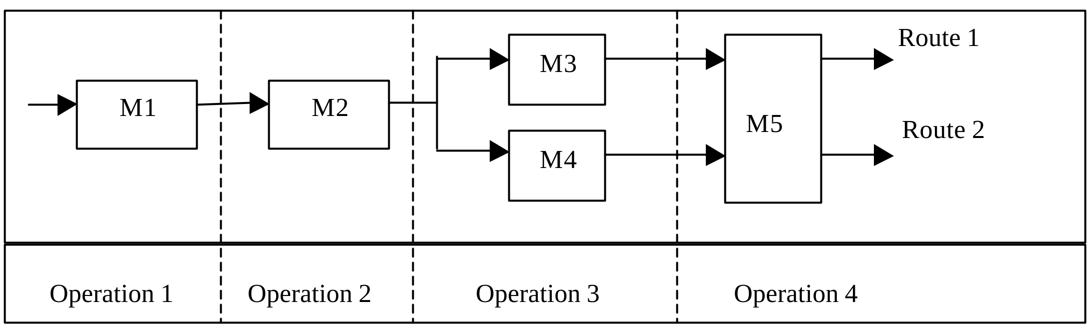

Figure 1.2: Machines, Operations and Routes of a Product

There are two types of production plants in the production process. One is the dedicated production plant. In the dedicated production plant, only one type of product is produced. The second type is the flexible plant. In the flexible production plant, intermediate products and finished goods share machines.

A by-product is generated, when an intermediate product or a finished good is produced in a production plant. A by-product consists of reusable raw materials. By-products are processed in a separate recycling plant, and some reusable raw materials are recovered from the recycling process. Part of the raw materials that is not recovered for reuse becomes waste. Figure 1.3 shows the inputs and outputs of the production process and linkages between the production plants and the recycling plants.

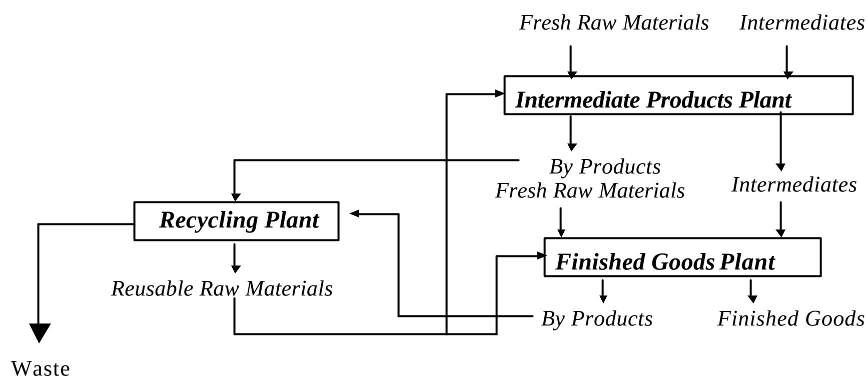

Figure 1.3: Inputs and Outputs of a Production Process

It can be seen in figure 1.3 that inputs to production process in a plant are the fresh raw materials, reusable raw materials and intermediate products. The outputs of a production process from a plant are intermediate products, finished goods and by-products. By-products are processed in recycling plants to recover reusable raw materials. Reusable raw materials are used again as inputs in the production process.

We consider flowshop setting for the finished goods in the production environment. In a flowshop, all products follow a similar route in a production plant. Intermediate products follow a general job shop setting with re-entrant flows. In a general job shop, the routes of products are distinct. The characteristic of a re-entrant job shop is that jobs are processed on a particular machine for more than one operation.

1.2.2 Complexities in the Production Environment

In this sub-section, we describe some of the complexities that exist in the production environment. The production environment discussed in previous sub-section, and the complexities in the production environment, form the basis for production planning and scheduling decisions.

As seen in figure 1.3, raw materials are recovered from by-products through a recycling process and reused in the production process. The recycling process is an important tool in reducing the operational costs, as the cost of raw materials is very high. Maximum recovery of the raw materials would translate to less use of fresh raw materials in the production process. It is desirable to run the recycling plants when the production plants are in operation. The reason for this argument is that by-products and reusable raw materials have limited storage capacity. Simultaneous generation and recycling of by-products would minimize the storage of by-products and recovered raw materials. This also translates into maintaining lesser inventory of fresh raw materials, because more reusable raw materials are being used in the production process. The above discussion leads to requirement of coordinating the production process and the recycling process. The production plans of the plants should be synchronized with the recycling plants to reduce the operational costs.

In a multi-stage environment, inventory is in the form of intermediate products and finished goods. To minimize production costs, inventory of the products needs to be minimized. This objective results in complexity of coordinating the schedules of products across the production plants. If production plants were decoupled with each other while scheduling, considerably high amount of inventory would be required to avoid production delays. When an intermediate product or an end product is scheduled, intermediate products that are inputs to the product should be available. Inventory of products will be reduced if the production plants are synchronized, i.e., when an intermediate product is produced, its higher-level product (where it is an input) is ready for processing. Similarly, the availability of raw materials with their minimum inventory is to be ensured before scheduling products.

There are high setup times in the production process. During product changeover at a flexible machine, idle time is incurred. In chemical plants, because of the chemical properties of products, residues have to be removed thoroughly at each changeover, and this results in considerable amount of idle time. There are trade-offs between setup costs and inventory costs. Higher production run of a product in a setup would result in high inventory cost, whereas more number of setups would consume significant amount of capacity in setups.

Intermediate products and finished goods are perishable. They have to be consumed within a specific time period, else they become waste. To minimize wastage and to avoid any production delays resulting from wastage, production plans at the plants need to be synchronized based on the shelf life of products.

Intermediate products are transferred to another production plant or within the same production plant, for next stage production, through transfer lot size of products. Only after certain quantity specified by the transfer lot size is produced, the product is transferred for its consumption. This again leads to the requirement of coordinating the production plants on the basis of transfer lot sizes.

There is also a trade-off between purchasing the intermediate products and their in-house production. The implications of purchasing the intermediate products are twofold.

Purchasing would obviously result in higher production costs, but this also can help in minimizing production delays.

Demand variability adds to the complexity in the system. The production planning is done on the basis of combination of firm orders and demand forecast over a finite planning horizon. The implication of demand variability is that if the demand forecast is not correct, there would be high inventory levels of some products and stock outs of other products. Another implication of demand fluctuation is that within the planning period, frequent revision in production plan and schedule is required to absorb the variation in demand.

Based on the production environment and its complexities discussed above, we describe in the next sub-section, the production planning and scheduling problem. We also formalize the decisions to be addressed in the production planning and scheduling problem.

1.2.3 Production Planning and Scheduling Decisions

In this sub-section, we characterize the production planning and scheduling problem based on the decisions to be addressed in the problem. There are two sets of decisions in the problem. One set of decisions is the production planning decisions. The other set is the scheduling decisions.

Production planning decisions are aggregate decisions and tactical in nature. One of the production planning decisions is to determine the production quantity of intermediate products and finished goods in each time period of the planning horizon. Production planning also determines the aggregate capacity of resources required to meet the production plan in each time period is to be determined. The production planning costs are the inventory costs of products and setup costs incurred over the planning horizon. The production-planning problem is to determine the decisions discussed above at minimum cost.

Scheduling decisions are more detailed and operational in nature. The time horizon of scheduling decisions is relatively short. For each product, the start time and the completion time on each machine is to be determined. The scheduling costs consists of inventory costs and costs incurred due to delay in satisfying customer orders. The formal definition of scheduling costs is provided later in chapter 3. The scheduling problem in our research is to determine the scheduling decisions at minimum cost. We are dealing with deterministic scheduling, i.e., at the time of scheduling, all the information that defines a problem instance is known with certainty. The information lending the scheduling problem to be deterministic, for example, is the known processing time of products, and machine availability.

1.3 Summary

In this chapter, we have discussed some of the changes occurring in the business environment as a result of increasing global competitiveness of firms. We highlighted the increasing importance of reducing operational costs of firms in the changing environment. It was discussed that production planning and scheduling is one of the important tools in reducing the operational costs of firms. We provided a detailed description of production planning and scheduling problem addressed in this research. Then, we discussed the production environment in detail along with the complexities of the production environment. We also focused on the decisions to be addressed in the production planning and scheduling problem.

The rest of the thesis is organized as follows. In the next chapter, we provide the literature review of the production planning and scheduling problem considered in this research. Chapter 3 describes the mathematical models for addressing the production planning and scheduling decisions. In chapter 4, we discuss the solution algorithms for solving the production planning and scheduling problem. In chapter 5, we report the results of the solution algorithms used to solve production planning and scheduling problem. We also provide sensitivity analysis on results of the production planning and scheduling problem in this chapter. In chapter 6, we apply the production planning and scheduling models to a real life problem of pharmaceutical company in India. The results of this application and the sensitivity analysis on the results are provided in this chapter. In chapter 7, we provide the summary of this research, contribution from this research, and discuss some issues relating to future research.

2 Literature Review

In this chapter, we review the research on production planning and scheduling problems in discrete parts manufacturing and process industries. There has been a renewed interest in application of mathematical programming to address production planning and scheduling decisions (Graves et al., 1993). The interest is mainly due to recent advances in information technology as it allows production managers to acquire and process production data on a real-time basis. As a result, managers are actively seeking decisions support systems to improve their decision-making. We will review some of the mathematical programming models developed and applied to the industry problems.

Primarily, there exist two types of approaches to address the production planning and scheduling decisions. One is the integrated approach, where production planning and scheduling decisions are determined simultaneously in a single monolithic model. The other approach is the hierarchical approach, where production planning and scheduling decisions are determined sequentially through separate models at an increasing level of detail. Both the approaches have been applied to solve the production planning and scheduling problems. We will study the mathematical models in both the approaches in this chapter.

Most of the research in scheduling theory with consideration of due dates has focused on minimizing the delay in customer orders (tardiness). The formal definition of tardiness is provided later in the chapter. Recently, the scheduling researchers have started investigating issues related to earliness of a job. Just-in-Time (JIT) philosophy has been the main driving force for this interest. We will study several other reasons for considering earliness as one of scheduling objectives later in the chapter.

The plan of this chapter is as follows. In the next section, we discuss the integrated mathematical models developed in discrete parts manufacturing and process industries. In section 2.2, we review hierarchical production planning and scheduling models. Section 2.3 describes the work done in scheduling with earliness and tardiness penalties. In section 2.4, we identify certain research gaps from this literature review.

2.1 Integrated Production Planning and Scheduling Models

We begin by reviewing the integrated models applied to single-stage and multi-stage production environment in discrete parts manufacturing industries. Then we will consider the models in process industries. Manne (1958) was the first to propose a production-scheduling model for multi–product, single-stage, and batch processing environment. Manne developed a linear program that provided a good approximation when the number of products being manufactured is large in comparison to the number of time periods. The solution procedure developed by Manne does not provide optimal solution to the problem. Dzielinski and Gomory (1965) further developed the model suggested by Manne (1958) by applying Dantzig-Wolfe decomposition to the problem. Application of the decomposition principle yields an equivalent linear program, called the master program, with fewer constraints and variables. The decomposition methods in the solution procedure provided by Dzielinski and Gomory helped in reducing the computations, but the solution obtained is far from optimal. The linear program being decomposed is only an approximation to an integer program whose solution is actually desired. Lasdon and Terjung (1971) applied the column generation procedure to the multi-product, single-stage integrated production-scheduling problem. They do not consider the master problem as done by Dzielinski and Gomory. Instead, the large number of variables is handled by column generation via sub-problems. They derive a lower bound of the problem and use it as the termination criterion for computations. The solution procedure from Lasdon and Terjung requires half the number of iterations as compared to the work of Dzielinski and Gomory. However, the solution obtained by Larson and Terjung also is quite far from the optimal solution. In fact, the solutions suggested by Manne (1958), Dzielinski and Gomory (1965) and Lasdon and Terjung (1971) are not necessarily feasible and the reported costs are not necessarily correct. This is because setup times and costs are charged only once even when a batch is split between periods. The authors have approximated these costs with the reason that with many products produced in each period, the percentage of unaccounted setups is usually small. Thus, in all three papers, the costs are underestimated and the capacity is not sufficient to allow for setups in some periods. This will sometimes result in infeasible schedules. Eppen and Martin (1987) developed tighter linear programming and lagrangian relaxation for multi-product, single stage production scheduling problems. They show that the linear programming relaxation generates bounds equal to those generated using lagrangian relaxation or column generation. Eppen and Martin report on successful experiments with models consisting of upto 200 products and 10 time periods. Trigeiro, Thomas and McClain (1989) reported on computational experience using lagrangian relaxation on large multi-product, single-stage models with high setup times. They improved on the weakness of underestimating set-up times and set-up costs in above mentioned three papers. However, the solution procedure provided by Trigeiro, Thomas and McClain also does not guarantee feasibility of scheduling decisions.

Multi-stage production environment introduces dependent demand of products. Production quantities and schedules of products at a particular level depend on the decisions made for the products at higher levels (parents or successors). Earlier work in multi-stage batch processing system is by Zangwill (1969) and Veinott (1969). They presented efficient solution techniques with dynamic programming for un-capacitated serial product structure. The computational requirements increase considerably with problem size in the solution procedures of Zangwill and Veinott. Love (1972) shows that if production costs are non-decreasing from intermediate products stage to end products stage, then an optimal schedule has the property that if in a given period, stage *j* produces, then stage *j + 1* also produces. This nested structure is exploited by Love in an algorithm for finding an optimal schedule. Crowston, Wagner and Williams (1973) analyzed multi-machine lot sizing decisions by constructing dynamic programming algorithm in serial and assembly product structures, with constant demand in an infinite planning horizon. However, they consider only one component at a level, and the solution procedure is characterized by excessive computational requirements. Crowston and Wagner (1973) extended the results of Love (1972) to present dynamic programming algorithm for assembly structures with known but varying demand over the finite-planning horizon. The solution time of the algorithm increases exponentially with the number of time periods, but only linearly with the increase in number of stages. Crowston and Wagner also apply branch and bound algorithm for large number of time periods but with serial product structures only. Lambrecht and VanderEecken (1978) present a heuristic approach for serial product structure with only one capacity constraint. Blackburn and Millen (1982) consider serial and assembly product structures with un-capacitated production facility. Through series of simulation experiments, Blackburn and Miller report potential errors in single- pass, stage-by-stage heuristic approaches for lot-sizing decisions in multi-stage systems. One major weakness in all the research discussed so far on multi-stage environment is that they do not consider component commonality, i.e., a product with more than one successor or parent. This assumption is unrealistic for many plant environments. Steinberg and Napier (1980) were the first to consider product commonality by proposing a formulation that is a constrained generalized network framework. This work brings out the importance of commonality and serves as a benchmark for evaluating heuristic algorithms. However, the model is solved with a mixed integer programming code, which limits its application to small problems. Billington, McClain and Thomas (1983) formulate a mixed-integer program to model the capacity constrained multi-stage general product structure production-scheduling problem for determining lot–sizing decisions, production lead-times and capacity planning. They allow product commonality in the product structure, a feature largely ignored in the previous work. Billington, McClain and Thomas develop heuristic procedures to reduce the problem size on the basis on number of common products. Their solution procedure is not useful for large problems and the heuristic solution is found to be very far from the optimal solution. Afentakis, Gavish and Karmarkar (1984) developed algorithms to obtain optimal solutions for single-product assembly product structures for un-capacitated systems. They decompose the problem into set of single stage production planning problems linked by a set of dual prices. They solve these single stage problems using a fast shortest path algorithm. This natural decomposition has been used as an efficient way to develop lower bounds to the optimal solution. They incorporate the lower bounds in a branch and bound procedure and solve problems up to 50 products in 15 stages for 18 periods in the planning horizon. However, their solution is for assembly product structures only. Aftentakis and Gavish (1986) relax this restriction and examine the lot-sizing problem in the general product structure systems with un-capacitated production facilities. The solution procedure for the problem defined with general product structure is more complex than the one defined on assembly systems. Afentakis and Gavish transform the general product structure problem into an equivalent and larger assembly system. They apply lagrangian relaxation that yields easily solvable sub-problems. However, this approach significantly increases the number of variables. They report computational results with only 3 end products and 15 stages over a 12 period planning horizon. Franca, Armentano, Berretta and Clark (1996) consider the lot sizing decisions in multi-stage capacitated systems with assembly and general product structures. They develop heuristic algorithms that perform well only with large capacity, fewer setups, and assembly product structures. They report computational results upto 17 products and 10 time periods. Pongcharoen, Hicks and Braiden (2004) consider multi-stage, capacitated production planning and scheduling problem in assembly product structures. They use genetic algorithms based heuristics but report results for small problems only. Bahl, Ritzman and Gupta (1987) and Karimi, Ghomi and Wilson (2003) provide a review of the production planning models for discrete parts manufacturing applications. Table 2.1 summarizes the models developed in single and multi-stage batch processing systems for discrete parts manufacturing environment.

Mathematical programming applications for production-planning decisions have been used in process industries like oil, steel, petroleum, food etc. Eilon (1969) proposed a mixed integer program (MIP) for production scheduling in multi-product, single stage environment with capacity constraints in a chemical industry. He developed heuristic algorithms based on batch scheduling approach to schedule 5 products, subject to normal demand distribution with known parameters. In a two-stage production environment, Prabhakar (1974) studied lot sizing and sequence dependent setup time sequencing in the chemical industry using an MIP to obtain production schedules only for a single planning period. He considers the complexity of job splitting while determining the scheduling decisions. Prabhakar used branch and bound algorithm to solve the MIP and reported results for small problems. Zanakis and Smith (1980) present a goal programming approach for production planning decisions in chemical

Source	**Production Environment**
Manne (1958) Dzielinski and Gomory (1965) Lasdon and Terjung (1971) Trigeiro, Thomas and McClain (1989) Eppen and Martin (1987) Bahl, Ritzman and Gupta (1987)	Multi-product, single stage, capacitated production facilities.
Zangwill (1969) Veinott (1969) Love (1972) Lambrecht and VanderEecken (1978)	Multi-product, multi-stage series product structure, capacitated production facilities.
Billington, McClain and Thomas (1983) Franca, Armentano, Berretta and Clark (1996)	Multi-product, general product structure with single end product, capacitated multi-production facilities.
Blackburn and Millen (1982) Afentakis and Gavish (1986)	Multi-product, general product structure with single end product, un-capacitated multi-production facilities.
Steinberg and Napier (1980) Afentakis, Gavish and Karmarkar (1984) Bahl, Ritzman and Gupta (1987) Roundy (1993)	Multi-product, multi-stage assembly system, un-capacitated production facilities
Crowston, Wagner and Williams (1973) Pongcharoen, Hicks and Braiden (2004)	Multi-product, multi-stage series and assembly product structures, capacitated facilities, constant demand.
Crowston and Wagner (1973)	Multi-product, multi-stage series and assembly product structures, capacitated facilities, varying demand.

Table 2.1: Integrated Models in Discrete Parts Manufacturing Industries

industries. There exist some non-linearities in the cost structures and production process in the chemical plants. These non-linearities arise when there is a pooling of products. Non-linearities may also arise in blending final products if the qualities of the component streams affect the qualities of the blended product in a non-linear manner. There are non-linearities in process yields also in chemical plants. Baker and Lasdon (1985) provide treatment of non-linearities through use of Successive Linear Programming (SLP) in their work. Vickery and Markland (1985) develop an integer goal programming approach in capacitated multi-product production environment in serial production system for a pharmaceutical company. They develop heuristic algorithms for solving large-scale problems. Smith-Daniels and Smith-Daniels (1986) present an MIP for lot sizing in packaging lines with joint family costs and sequence dependent setup times. They use branch and bound algorithm for solving the problem and report results for small problem sizes only. Smith-Daniels and Ritzman (1988) present an MIP for lot sizing and sequencing in process industries. They report successful implementation of models in food industry with problem size of 160 integer variables and 1760 continuous variables. They also compare their solution with the approach that considers lot sizing and sequencing as independent decisions. They argue that decomposing the problem into sub-problems can result in infeasible production schedules. However, integrated solution of Smith-Daniels and Ritzman is tested only for small problems. Shapiro (1993) developed a LP production-planning model for an oil refinery. He applies Dantzig-Wolfe decomposition method to solve the problem. Shapiro also developed an MIP to capture the non-linear characteristics in chemical industries, and reports results with 15 products. Numao (1995) solves an integrated production planning and scheduling problem in petrochemical production process. They design a heuristic based decision support system to address the production planning and scheduling decisions, although the performance of the heuristics is not reported. Table 2.2 summarizes the large-scale monolithic mathematical models applied in process industries for production planning and scheduling. In the next section, we study some of the hierarchical production planning and scheduling models.

Source	**Production Environment**
Eilon (1969) Smith-Daniels and Smith-Daniels (1986)	Single stage, multi-plant, capacitated production facilities.
Prabhakar (1974) Zanakis and Smith (1980) Baker and Lasdon (1985) Vickery and Markland (1986) Smith-Daniels and Ritzman (1988) Shapiro (1993) Numao (1995)	Multi-stage, multi-plant, capacitated production facilities.

Table 2.2: Integrated Models in Process Industries.

2.2 Hierarchical Production Planning and Scheduling Models

Hax and Meal (1975) and Bitran and Hax (1977) did earlier work in formalizing the hierarchical production-planning framework in a multi-product, multi-plant, single-stage, and batch-processing environment. They present procedures to partition the overall production planning and scheduling problem into manageable and interlinked sub-problems. An important input in hierarchical modeling philosophy is the number of levels recognized in the product structure. Hax and Meal (1975) recognized three levels for the purpose of aggregating the product data. They state that aggregation is often achieved by grouping end products into product families and product families into product types. Product families are groups of products that share a common manufacturing set-up cost. Product types are groups of families whose production quantities are to be determined by an aggregate production plan. Families belonging to a type normally have similar costs per unit of production time and similar seasonal demand patterns. In practical applications, more or fewer levels might be needed. The hierarchical approach can be extended to different numbers of aggregation levels by defining adequate sub-problems. Hax and Meal (1975) provide heuristics to perform four levels of computations. First, products are assigned to plants using MIP, which makes long-term capacity provision and utilization decisions. Second, a seasonal stock accumulation plan is prepared using LP, making allocation of capacity in each plant among product types. At the third level, detailed schedules are prepared for each product family using standard inventory control methods, allocating the product type capacity among the product families and at the fourth level, individual run quantities are calculated for each product in each family, again using standard inventory control methods.

A significant aspect of the hierarchical approach is the ability of disaggregation procedures to obtain feasible solutions of aggregate decisions at the detailed level. Bitran and Hax (1977) conducted a series of experiments to examine the performance of the single-stage hierarchical system to determine the size of the forecast errors, capacity availability, magnitude of setup costs and nature of planning horizon. Bitran, Haas and Hax (1981) compare various disaggregation procedures and analyze the impact of different aggregation schemes on production planning costs. They also modify the procedures of Bitran and Hax (1977) to incorporate high setup cost. Liberatore and Miller (1985) developed hierarchical models for production planning and scheduling in single stage, multi-product capacitated production facilities. They develop a LP model for production planning decisions and an MIP for daily scheduling decisions. Their solution procedure is useful for single stage problems only. Resource allocation in single stage, parallel machine scheduling application has been described in Bitran and Tirupati (1988a,b). They develop mixed integer, quadratic program aggregate planning model to homogenize the product group. This resulted in reduction in complexity for the scheduling problem. Bowers and Jarvis (1992) applied hierarchical framework for multi-product, single-stage production and scheduling problem. The three phase models developed by Bowers and Jarvis implements inventory planning, short-term production planning and daily sequencing tasks.

Meal (1978) describes an integrated distribution planning and control system citing the complexities in extending the hierarchical approach to multistage systems. The two stages are the parts production and assembly operations and the third stage is the distribution system. This work lacks the consistency between aggregation and disaggregation procedures, i.e., the link between the production and a distribution module is relatively weak. Gabbay (1979) addressed multi-product, capacitated multi-stage production environment in hierarchical planning framework. He does not provide a proposal to address the infeasibility in production schedules. Bitran, Haas and Hax (1982) apply the extension of single stage hierarchical stage production planning to two-stage production process. The two stages are the parts production and the assembly process. Maxwell et al. (1983) propose a hierarchical set of models for production planning in discrete parts manufacturing and assembly systems. Their solution procedure works well with large capacity only. They apply the models in stamping plants in US automotive industry. Bitran and Tirupati (1993) comprehensively review the work done in single stage and multi stage hierarchical models in production planning and scheduling. Ozdamar, Bozyel and Birbil (1998) develop hierarchical decision support system for production planning in parts production and assembly process. They develop models for planning at product type level, product family level and planning at end product level. However, the disaggregation procedures suggested in this work do not guarantee feasibility. Ozdamar and Yazgac (1999) propose hierarchical models for production distribution system. In the planning model, Ozdamar and Yazgac consider aggregation of time periods and products while omitting detailed capacity consumption by setup. In table 2.3, we summarize the application of hierarchical models in discrete parts manufacturing environment.

Bradley, Hax and Magnanti (1977) described an application of hierarchical production systems to a continuous manufacturing process. Leong, Oliff and Markland (1982) developed hierarchical models for production planning in process industries. They apply the models in a fiberglass company with multi-product and parallel processor production environment and report substantial cost savings. Oliff and Burch (1985) develop three phase hierarchical models for production scheduling in process industries.

Source	**Production Environment**
Hax and Meal (1975) Bitran and Hax (1977) Bitran, Haas and Hax (1981) Liberatore and Miller (1985) Bowers and Jarvis (1992) Bitran and Tirupati (1993)	Single stage, batch manufacturing systems
Bitran and Tirupati (1988a,b)	Single Stage, Parallel Machine
Gabby (1979) Bitran, Haas and Hax (1982) Maxwell et al. (1983) Bitran and Tirupati (1993) Ozdamar, Bozyel and Birbil (1998)	Multi-stage fabrication and assembly System
Meal (1978) Ozdamar and Yazgac (1999)	Multi-stage, Distribution and Planning System

Table 2.3: Hierarchical Models in Discrete Parts Manufacturing Industries

Lot sizes, line assignments and inventory levels are determined for individual products through LP. Final job sequencing is accomplished by scheduling heuristics. Kleutgchen and McGee (1985) developed mathematical models for Pfizer Pharmaceuticals. Implementation of the models reduced inventories significantly. The main weakness in this work is that it is restricted to inventory management and does not addresses other production planning and scheduling decisions. Lin and Moodies (1989) develop two mathematical programming models and sequencing heuristic for production planning and scheduling in steel industry. Katayama (1996) propose a two stage hierarchical production planning system for process industries. Katayama applies the hierarchical models in petrochemical plants with use of MIP and neural network approach. Qiu and Burch (1997) develop hierarchical planning model for production planning in process industries. MIP is developed for aggregate planning and sets of heuristics are developed for daily scheduling. A brief summary of the work in hierarchical production planning in process industries is given below in table 2.4. In the next section, we review the research on scheduling with earliness and tardiness penalties.

Source	**Production Environment**
Bradley, Hax and Magnanti (1977)	Continuous manufacturing, job shop environment
Oliff and Burch (1985) Kleutghen and McGee (1985) Lin and Moodies (1989) Katayama (1996)	Multi-product, capacitated production facility, continuous production
Leong, Oliff and Markland (1982) Qiu and Burch (1997)	Multi-product, parallel machine

Table 2.4: Hierarchical Models in Process Industries

2.3 Earliness and Tardiness Scheduling

The study of earliness and tardiness penalties in scheduling models is a relatively recent area of research. Most of the existing literature on scheduling focuses on problems that have objective functions such as minimizing makespan (completion time of schedule) and tardiness. Conway et al. (1967) refer to these objectives as regular performance measures, and these measures are non-decreasing in completion times. Minimizing tardiness has been the usual performance measure that considers the due dates of jobs. Recent interest in Just-In-Time (JIT) production has created the notion that earliness, as well as tardiness should be discouraged. The concept of penalizing both earliness and tardiness has resulted in new and rapidly developing line of research in the scheduling field. As the use of both earliness and tardiness penalties gives rise to a non-regular performance measures (non-increasing in completion times), it has led to new methodological issues in the design of solution procedures. The majority of research on earliness and tardiness scheduling is focused on single machine scheduling, although some single machine models have been extended to multi-machine setting. We begin by reviewing the research on single machine scheduling.

Single Machine Scheduling

Baker and Scudder (1990) review the research on single machine scheduling with earliness and tardiness (E/T) penalties. Primarily, the literature has grown from the generality of assumptions made about due dates and penalty costs. A generic E/T model is defined in the following way. There are n jobs to schedule. Each job i is described by processing time p_i and a due date d_i. Scheduling decision would provide completion time of job C_i. Earliness E_i and tardiness T_i of a job i is defined by $E_i = max\ (0,\ d_i - C_i)$ *and* $T_i = max$

$(0, C_i - d_i)$ respectively. Associated with each job i are earliness penalty, $a_i > 0$ and tardiness penalty, $b_i > 0$. Assuming the penalty functions are linear, the basic objective function for minimizing E/T costs, for any schedule S can be written as, $f(S) = \sum_{i=1}^{n} a_i.E_i + b_i.T_i$. In some formulations of the E/T problem, the due date is given, while in others the problem is to find the optimal due date and the job sequence simultaneously. Allocating different penalties for earliness and tardiness suggests that the associated cost components of both are different from each other in many practical settings. However, all penalty functions are primarily to guide the solution towards meeting the due date exactly. This implies that an ideal schedule is the one in which all due dates are met exactly. An important special case in the family of E/T scheduling problems is when $a_i = b_i = 1$, i.e., un-weighted E/T penalties. Common due date of jobs is another notion in E/T scheduling. This represents situations where several jobs belong to a single customer's order or the assembly environment where components should be ready at the same time to avoid production delays. The objective function in these special cases becomes, minimizing the absolute deviation of job completion times from a common due date, $f(S) = \sum_{i=1}^{n} E_i + T_i = \sum_{i=1}^{n} \left| C_i - d \right|$

One of the preliminary works on single machine E/T scheduling is by Sidney (1977), who provides an efficient algorithm to minimize the maximum earliness or tardiness penalty. This algorithm is improved by Lakshminarayan et al. (1978). The origins of a different research direction can be traced to the work of Kanet (1981a). He considers the problem of minimizing the total un-weighted earliness and tardiness around an unrestricted common due date, i.e., due date that is not tight enough to act as a constraint on scheduling decision. E/T problem with tighter due date is called restricted version. Unrestricted due date is defined as follows. If p_i the processing time of job i and jobs are arranged such that p_1 £ p_2 £ p_3…£ p_n, the E/T single machine problem is unrestricted, if due date d is such that:

d ³ $D = p_2 + p_4 + p_6 + \ldots\ldots + p_{n-4} + p_{n-2}, + p_n,$ *if n is even.*

$d \geq \Delta = p_1 + p_3 + p_5 + \ldots\ldots + p_{n-5} + p_{n-3} + p_n,$ *if n is odd.*

Under this condition, Kanet provides an algorithm for finding an optimal solution in polynomial time. Baker and Scudder (1990) have shown the optimal solution to the unrestricted due date problem has following properties:

1. There is no idle time in the schedule. This means that if job j immediately follows job i in the schedule with completion time, $C_j = C_i + p_j$
2. The optimal schedule is V Shaped. Jobs for which C_i £ d are sequenced in non-increasing order of processing time, while jobs for which $C_i > d$ are sequenced in non-decreasing order of processing times. Raghavachari (1986) establish the V-shape of an optimal schedule for any common due date.
3. One job completes precisely at the due date, i.e., $C_i = d$ for some i.

Sundararaghavan and Ahmed (1984) generalize Kanet's problem to a scheduling environment with several identical parallel machines. The optimality conditions discussed above for Kanet's problem and the availability of large number of optimal solutions are discussed by Hall (1986). Another generalization of Kanet's problem is studied by Bagchi, Chang and Sullivan (1987), where all jobs have equal earliness and tardiness weights. The authors describe optimality conditions that, in the case where the due date is unrestricted, characterize an efficient algorithm.

The restricted version of the problem occurs when common due date $d < \Delta$. Hall, Kubiak and Sethi (1991) have shown that the restricted version of single machine E/T problem is NP-complete. Bagchi, Chang and Sullivan (1986) present an algorithm for solving the restricted problem. However, their procedure implicitly assumes that the start time of the schedule is zero. Szwarc (1989) proposes that the optimal start time may be nonzero, so that the Bagchi, Chang and Sullivan algorithm does not guarantee optimality. The solution procedures due to Szwarc (1989) and Bagchi, Chang and Sullivan (1986) are both enumerative in nature. Sundararaghavan and Ahmed (1989) present a heuristic algorithm that work effectively when the start time is zero. The worst case of enumerative approaches of the solution procedures of restricted problem requires analysis of 2^n schedules, where n is the number of jobs.

Variants of E/T problems in single machine have been researched on the basis of distinct due dates and weighted E/T penalties. Garey, Tarjan and Wilfong (1988) study the problem of minimizing total un-weighted earliness and tardiness on a single machine with distinct due dates of jobs. The single machine weighted earliness and tardiness scheduling problem with distinct due dates is studied by Abdul-Razaq and Potts (1988). They provided a branch and bound algorithm for the problem. For the same problem, Ow and Morton (1989) provide a computational study of several heuristic algorithms. Li (1997) proposes lagrangian relaxation based branch and bounds algorithms that guarantee the optimality of the solution, the algorithms are useful for small problems only. Wan and Yen (2002) investigate single machine E/T problem with distinct due dates and weighted E/T penalties. They develop heuristic algorithms that have tabu search procedure and report computational performance of heuristics. Ventura and Radhakrishnan (2003) focus on single machine E/T scheduling with varying processing times and distinct due dates. They decompose the constraints in two sets. One set of constraints, they solve as assignment problem, and relax the other set of constraints to form the lagrangian dual problem. They solve the lagrangian problem using the sub-gradient algorithm.

Some work is done on non-linear penalties in single machine E/T scheduling. Merten and Muller (1972) introduced the completion time variance problem (CTVP) as a model for file organization decisions in which it is important to provide uniform response times to users. They also demonstrated the equivalence of the CTVP and the waiting-time variance problem (WTVP). Schrage (1975) proposed the first exact algorithm for scheduling CTVP up to 5 jobs. Eilon and Chowdhury (1977) provided an enumerative algorithm for determining an optimal schedule when the number of jobs n is relatively small (n = 20). Their algorithm minimizes WTVP. For large n, they proposed five heuristic procedures for approximately solving the problem. Three of the heuristic procedures utilize pair wise interchanges of adjacent jobs to improve the solution. Kanet (1981b) proved that CTVP is equivalent to minimizing the sum of squared differences of job completion times. He adapted an algorithm for the absolute deviation problem as a heuristic for the CTVP and showed that the performance of his heuristic is superior to those proposed by Eilon and Chowdhury. Vani and Raghavachari (1987) proposed heuristic algorithms for CTVP and claimed that their heuristic procedure compares favorably with the heuristics of Eilon and Chowdhury and Kanet. Bagchi et al. (1987) showed that the CTVP is equivalent to the mean squared deviation problem (MSDP) of job completion times about some common due-date. They noticed that for any given schedule, the optimal due date is equal to the mean completion time. They proposed a branching procedure to find the optimal solution. Although they utilized several dominance properties in order to accelerate their enumerative procedure, the procedure is clearly inadequate for solving large problems (i.e., n = 20). Gupta et al. (1990) proposed another heuristic, which is based on the complementary pair-exchange principle, for finding a good approximate solution to the CTVP. Their heuristic procedure has been shown through computational experiments to generate better solutions than other heuristics. De et al. (1992) have presented a pseudo-polynomial dynamic programming algorithm for optimally solving instances of the CTVP where processing times are small integers. They also proposed a fully polynomial approximation scheme. Kubiak (1993) showed that the CTVP is NP-complete. Kubiak (1995) proposed a quadratic integer programming formulation and two new pseudo-polynomial dynamic programming algorithms for the CTVP. In table 2.5, we provide taxonomy of the research done in single machine E/T scheduling.

We now address the other class of problems in E/T scheduling which has the property of inserted idle time.

Issue of Inserted Idle Time

Most of the E/T work in scheduling does not consider the issue of inserted idle time (IIT) either by restricting the solution to be a non-delay schedule or by assuming a common due date for all jobs. Inserted idle occurs when a resource is deliberately kept idle in the face of waiting jobs. Kanet and Sridharan (2000) provide a comprehensive review of IIT scheduling. However, they do not consider the review of Baker and Scudder (1990), as these papers are restricted to non-IIT and non-delay schedules. For the $n|1|d_i=d|\Sigma E_i+T_i$ problem (common due date problem), Cheng and Kahlbacher (1991) proved that it is unnecessary to consider schedules with inserted idle time except prior to the first job in the schedule. Both the review papers, Kanet and Sridharan, Baker and Scudder, observe that the essence of E/T problem lies in its non-regular performance measure. Imposing the restriction of no inserted idle time diminishes the objective. In the light of this observation IIT-E/T literature is scanty. We now study some of the work done in multi-machine scheduling, specifically in flowshop environment with earliness and tardiness penalties.

Source	**Objective Function**
Kanet (1981a); Sundararaghvan and Ahmed (1984); Bagchi, et al.(1986), Sullivan and Chang (1986); Szwarc (1989); Hall, Kubiak and Sethi (1989)	$f(S)=\sum_j (d-C_j)^+ + \sum_j (C_j-d)^+ = \sum_j \lvert C_j-d \rvert$ Common due date, un-weighted E/T penalties
Panwalker, Smith and Seidmann (1982); Emmons (1987); Bagchi, Chang and Sullivan (1987); Hall, Kubiak and Sethi (1991)	$f(S)=a\sum_j (d-C_j)^+ + b\sum_j (C_j-d)^+$ Common due date, weighted E/T penalties
Bagchi, Chang and Sullivan (1987); De, Ghosh and Wells (1989a, b)	$f(S)=\sum_j (C_j-d)^2$ Common due date, unweighted E/T penalties
Eilon and Chowdhury (1977); Kanet (1981b); Vani and Raghavachari (1987)	$f(S)=\sum_j (C_j-C)^2$ Common due date, unweighted E/T penalties, completion time variance
Bagchi, Chang and Sullivan (1987);	$f(S)=a\sum_j [(d-C_j)^+]^2 + b\sum_j [(C_j-d)^+]^2$ Common due date, weighted E/T penalties
Cheng (1987); Emmons (1987); Quaddus (1987); Bector, Gupta and Gupta (1988); Hall and Posner (1989)	$f(S)=\sum_j a_j (d-C_j)^+ + \sum_j b_j (C_j-d)^+$ Common due date, unequal weighted E/T penalties
Fry, et al.(1987); Abdul-Razaq and Potts (1988); Ow and Morton (1988, 1989); Li (1997); Wan and Yen (2002); Ventura, et al. (2003)	$f(S)=\sum_j a_j (d_j-C_j)^+ + \sum_j b_j (C_j-d_j)^+$ Distinct due date, unequal weighted E/T penalties
Gupta and Sen (1983); Cheng (1984); De et al. (1992); Kubiak (1993); Kubiak (1995)	$f(S)=\sum_j a_j [(d_j-C_j)^+]^2 + \sum_j b_j [(C_j-d_j)^+]^2$ Distinct due date, unequal weighted E/T penalties

Table 2.5: Single Machine Schedule with Earliness and Tardiness Penalties

Multi-Machine Scheduling

Flowshop scheduling problems have attracted many researchers since the work of Johnson (1954) for 2 machine flowshops. In flowshop problems, n jobs are processed on m machines in the same order. We are going to review the research on flowshop scheduling that has following assumptions. A machine processes only one job at a time; a job can be processed on only one machine at a time; the operations are non-preemptable and setup times of jobs on machines are independent of sequences. Since the early seventies, scheduling researchers have been analyzing the computational complexity of various flowshop models. NP-completeness of the flowshop problems minimizing makespan (completion time of schedule) for $m \geq 3$ has been shown by Garey et al. (1976), where m is the number of machines. Koulamas (1994) has shown NP-hardness of $F| | \Sigma T_i$ problem for $m \geq 3$. The above complexity result coupled with the nature of flowshops has limited the possibility of developing efficient solution algorithms for F2| | T. Sen, Dileepan and Gupta (1989) proposed a branch-and-bound algorithm for F2 | | T. They first derived a local optimality condition for sorting two adjacent jobs in a sequence that is sufficient, but not necessary. As a result, this condition has a limited effect on reducing the size of the branch-and-bound solution tree. Also Sen, Dileepan and Gupta lower bounds are rather weak because they are based only on the tardiness of the already scheduled jobs and they do not include a lower bound on the tardiness of the still unscheduled jobs. Kim (1993) proposed an improved branch-and-bound algorithm for F2 | | T. Kim derived a condition for identifying jobs that could be placed last in a optimal sequence which is analogous to Elmaghraby's lemma for 1 | | T. Kim developed stronger lower bound on the tardiness of the still unscheduled jobs. However. Kim's lower bounds are also weak because they are based on conservative estimates on the completion time of the unscheduled jobs. The main drawback of the branch-and-bound algorithms is that they do not utilize any dominance conditions for reducing the size of the branch-and-bound solution tree. As a result, they can be applied only to small problems with $n < 15$ jobs. Kim's experiments also showed that his branch-and-bound algorithm performs better than the algorithm of Sen, Dileepan and Gupta.

Since $F|\,|\,\Sigma T_i$ is NP- Hard, $F|\,|\,\Sigma E_i+T_i$ is also NP-Hard. The research on E/T penalties in flowshop settings is very scanty. Gowrishankar et al. (2001) looked at minimizing the completion time variance and the sum of squares of completion time deviations from a common due date. They develop lower bound for both the problems. Using lower bound, they propose branch and bound algorithms for the two problems. For larger problems, they propose heuristics for both the problems. Other objective functions have not been looked in flowshop E/T scheduling.

2.4 Research Gaps

In the previous sections, we have reviewed various production planning and scheduling models applied to discrete parts manufacturing industries and process industries. It is seen that models have been developed in single stage and multi-stage production environment. Most of the models in multi-stage production environment have focused on fabrication and assembly types of product structures. The production environment with recycling process and its associated complexities has not been addressed in the literature. We discussed in chapter 1, that recycling is an important issue in bringing down production costs. We have studied the impact of recycling process on production planning and scheduling decisions. The existing models on production planning and scheduling do not address the complexities of the production environment we discussed in chapter 1. In integrated and hierarchical models addressing production planning and scheduling decisions, inconsistency often occurs in capacity requirements of production planning decisions and scheduling decisions. Aggregate capacity of resources is considered in production planning decisions. We discussed that while determining the scheduling decisions, infeasibilities may occur due to excess capacity requirements. The complexities of the planning problem make scheduling decisions even more difficult. The issue of alternate machines availability (resulting in multiple routes of a product) is not addressed in the literature on multi-stage and multi-machine environment. Also not addressed in the literature is the issue of backlogging of demand over the planning horizon in multi-stage environment. This becomes an important issue in situations when the schedule has tardiness.

In literature review of scheduling theory with earliness and tardiness (E/T) penalties, we discussed reasons for considering earliness as a recent area of research. Most of the work in E/T scheduling is limited to single machine scheduling with certain assumptions about the due dates. Multi-stage environment like flowshop and jobshop production environment is largely unattended in scheduling with E/T penalties. In the next chapter, we describe the mathematical models to address the production planning and scheduling decisions.

3 Production Planning and Scheduling Models

3.1 Introduction

In this chapter, we describe the mathematical models that address the production planning and scheduling decisions described in chapter 1. The production planning decisions determine production quantity of products, inventory level of products and aggregate capacity of production resources. The scheduling decisions determine the schedule of products at each machine where they are processed. The schedule of a product comprises start time and completion time of product at each machine.

We have seen in chapter 2 that modeling at different levels is called hierarchical modeling in literature (Bitran and Hax, 1977; Bitran, Haas and Hax, 1981; Bitran and Tirupati, 1993). We discussed that determining production planning and scheduling decisions in one integrated model is computationally not efficient (Qiu et al., 1997). The motivation to develop hierarchical models is also driven by the planning process observed in the production environment. Production planning and scheduling decisions are required to be made sequentially at increasing level of detail. Capacity requirements, timing and sizing of production runs in the planning horizon are determined in production planning. Machine-wise allocation of products to be produced is done in detailed scheduling. Hierarchical modeling postpones the detailed scheduling decisions till they are actually required. The detailed scheduling decisions are therefore, based on more accurate information. However, there can be situations when the scheduling decisions are not feasible. The reason for this is that aggregate capacity is considered at the time of determining production-planning decisions. In scheduling, issues like job precedence constraints, and operation precedence constraints may lead to capacity requirement, which is more than the available aggregate capacity. This will result in an infeasible schedule. We will address the issue of infeasibility in detail when we solve the production planning and scheduling problem.

We model the production planning and scheduling decisions in two steps. In the first step, we model production-planning decisions. This is a mixed integer programme. The decisions of the production-planning model over a finite planning horizon are:

- Quantity of each product to be produced on each production plant in each time period
- Inventory levels of finished goods, intermediate products, by-products and raw materials in each time period
- Quantity of fresh raw material required in each time period.

The production-planning model also determines the aggregate capacity of the resources required, in order to derive the production planning decisions.

In the second step, we model scheduling decisions. There are two scheduling models to address scheduling decisions; one for finished goods scheduling and the other for intermediate products scheduling. Detailed machine wise scheduling decisions, i.e., start times and completion times of each product on each machine is derived from the scheduling model. The rest of the chapter describes the formulation of production planning and scheduling models. In the next section, we describe the formulation of production-planning model. Mathematical formulations of scheduling models are discussed in section 3.3. We summarize this chapter in section 3.4.

3.2 Production Planning Model

The production-planning model is developed for addressing medium range time horizon decisions. The objective of the production-planning model is to minimize the production costs. Production costs are the inventory costs and set up costs of end products, intermediate products, inventory costs of by-products and recovered raw materials and cost of fresh raw materials. We now provide the formulation of production planning model.

3.2.1 Formulation of Production Planning Model

The production-planning model is formulated as follows:

Indices

i = index of end products and intermediate products
t = index of time period in the planning horizon
j = index of the production plants
m = index of by-products
s = index of reusable raw materials that are recovered from by-products
p = index of recycling plants
u = index of reusable raw material storage tanks
v = index of by-products storage tanks
e = index of machines in the production lines
r = index of routes of a product

Parameters

E = Set of end products, $\{i \mid i = 1,2,\ldots,b\}$
I = Set of intermediate products, $\{i \mid i = b+1,\ldots\ldots,n\}$
T = Set of time periods, $\{t \mid t = 1,2,\ldots\ldots T\}$
J = Set of production plants, $\{j \mid j = 1,2,\ldots..J\}$
A_i = Set of products in bill of material of i, $i \in E, I$
N = Set of machines used in the production plants, $\{e \mid e = 1,2,\ldots.N\}$
R_i = Set of products (E and I) for which i is an input, $\{k \mid a_{ki} > 0$, k Î $E \; U \; I\}$.
AR = Set of products that share machines but do not have alternate routes
BR = Set of products that share machines but have alternate routes
RT_i = Set of routes of product i, i Î BR
RE_e = Set of routes on machine e, e Î N

M = Set of by-products from which raw materials are recovered, $\{m \mid m = 1,2,\ldots,M\}$
S = Set of raw materials which are recovered from by-products, $\{s \mid s = 1,2,\ldots,S\}$
P = Set of plants where by-products are processed to recover raw materials, $\{p \mid p = 1,2,\ldots P\}$
TS = Set of tanks used for storing raw materials
TM= Set of tanks used for storing by-products.
A_s = Set of tanks used for storing raw material s, s Î S
B_u = Set of raw materials stored in tank u, u Î TS
A_m = Set of tanks used for storing by-product m, m Î M
B_v = Set of by-products stored in tank v, v Î TM
a_{ik} = Amount of k required per unit of i, i Î E, I, k Î A_i.
r_{jt} = Capacity (in hours) of production plant j in period t, j Î J, t Î T
r_{ejt} = Capacity (in hours) of machine e in production plant j in time period t, e Î N, j Î J, t Î T
d_{it} = Demand of product i in period t, i Î E
C_i = Cost (in Rs per unit) of input materials to i, i Î E, I
S_{ij} = Setup cost for product i on phase j, i Î E, I, j Î J
h_i = Inventory cost (in Rs per unit) for product i, i Î E, I
= (Inventory carrying rate) * C_i
t_{ij} = Time (in hours) to produce one unit of product i on production plant j, i Î E, I, j Î J
t_{iej} = Time (in hours) to produce one unit of product i on machine e in production plant j, i Î E U I, e Î N, j Î J
t_{ij} = Setup time (in hours) for product i on production plant j, i Î E, I, j Î J
N_i = Number of batches of product i that can be produced between two setups, i Î E, I
B_i = Output batch size of product i, i Î E, I
ss_i = Safety stock of product i, i Î E, I
y_{smp} = Ratio of raw material s recovered from by-product m at plant p, s Î S, m Î M, p Î P
c_{is} = Amount of raw material s required per unit of i, i Î E, I, s Î S
c_{pis} = Minimum percentage of fresh raw material s required in product i, i Î E, I, s Î S
M_{mi} = Amount of by-product m generated per unit of i, m Î M, i Î E, I
K_{mp} = Processing capacity of plant p to process by-product m, m Î M, p Î P
f_{pt} = Available time (in hours) of plant p in time period t, p Î P, t Î T
f_s = Cost (in Rs) of fresh raw material s, s Î S
h_m = Inventory carrying cost (in Rs per unit per month) of by-product m, m Î M
h_s = Inventory carrying cost (in Rs per unit per month) of reusable raw material; s, s Î S
A_m = Set of products generating by-product M = {i½$M_{mi}>0$} i Î E U I, m Î M
B_s = Set of products using reusable raw material s = {i½$c_{is}>0$}, i Î E U I, s Î S
C_u = Capacity of tank u, u Î TS
C_v = Capacity of tank v, v Î TM

Variables

X_{ijt} =	Quantity of product i produced on production plant j in time period t, $i \in E, I, j \in J, t \in T$
XR_{irjt} =	Quantity of product i on route r on production plant j in time period t, $i \in E \cup I, r \in RT_i, j \in J, t \in T$
I_{it} =	Inventory of product i at the end of period t, $i \in E, I, t \in T$
O_{ijt} =	Number of setups of product i on production plant j in time period t, $i \in E, I, j \in J, t \in T$
Y_{st} =	Reusable raw material s used at all production plants in period t, $s \in S, t \in T$
F_{st} =	Quantity of fresh raw material s used at all production plants in period t, $s \in S, t \in T$
F_{sit} =	Quantity of fresh raw material s used in product i at all production plants in period t, $s \in S, i \in E, I, t \in T$
Q_{mpt} =	Quantity of by-product m processed at plant p in period t, $m \in M, p \in P, t \in T$
IS_{st} =	Inventory of reusable raw material s at the end of period t, $s \in S, t \in T$
IST_{sut} =	Inventory of reusable raw material s in tank u at the end of period t, $s \in S, u \in TS, t \in T$
IM_{mt} =	Inventory of by-product m at the end of period t, $m \in M, t \in T$
IMT_{mvt} =	Inventory of by-product m in tank v at the end of period t, $m \in M, v \in TM, t \in T$

$$\min\ z = \sum_i \sum_t h_i.I_{it} + \sum_i \sum_t S_{ij}.O_{ijt} + \sum_s \sum_t hs.IS_{st} + \sum_m \sum_t h_m.IM_{mt} + \sum_s \sum_t f_s.F_{st}$$

subject to:

$$I_{it} = I_{it-1} + \sum_j X_{ijt} - d_{it} \qquad \forall\ i,t.\ i \in E \qquad (1)$$

$$I_{it} = I_{it-1} + \sum_j X_{ijt} - \sum_{k \in R_i} \sum_j X_{ijt}.a_{ki} \qquad \forall\ i,t.\ i \in I \qquad (2)$$

$$\sum_i (X_{ijt}.t_{ij} + O_{ijt}.t_{ij}) \le r_{jt} \qquad \forall\ j,t.\ i \in E,I \qquad (3)$$

$$\sum_{i \in AR} (X_{ijt}t_{iej} + O_{ijt}t_{ij}) + \sum_{i \in BR} \sum_{r \in RE_e} (XR_{irjt}t_{iej} + O_{ijt}t_{ij}) \le r_{ejt} \qquad \forall\ e,j,t \qquad (4)$$

$$X_{ijt} = \sum_{i \in RT_i} XR_{irjt} \qquad \forall\ i,j,t.\ i \in E,I \qquad (5)$$

$$X_{ijt} \le O_{ijt}.N_i.B_i \qquad \forall\ i,j,t.\ i \in E,I \qquad (6)$$

$$I_{it} \ge ss_i \qquad \forall\ i,t.\ i \in E,I \qquad (7)$$

$$IS_{st} = IS_{st-1} + \sum_{m \in M} \sum_P Q_{mpt}.y_{smp} - Y_{st} \qquad \forall\ s,t.. \qquad (8)$$

$$IM_{mt} = IM_{mt-1} + \sum_{i \in A_M} \sum_j X_{ijt}.M_{mi} - \sum_P Q_{mpt} \qquad \forall\ m,t.\ i \in E,I \qquad (9)$$

$$Y_{st} + F_{st} = \sum_{j} \sum_{i \in Bs} X_{ijt}.C_{is} \qquad \forall\ s,t.\ i \in E,I \qquad (10)$$

$$F_{sit} \geq c_{is}.c_{pis}.\sum_{j} X_{ijt} \qquad \forall\ s,i,t.\ i \in E,I \qquad (11)$$

$$\sum_{i} F_{sit} = F_{st} \qquad \forall\ s,t. \qquad (12)$$

$$\sum_{M} Q_{mpt}.k_{mp} \leq f_{pt} \qquad \forall\ p,t. \qquad (13)$$

$$\sum_{s \in B_u} IST_{sut} \leq C_u \qquad \forall\ u,t\ .u \in TS \qquad (14)$$

$$\sum_{u \in A_s} IST_{sut} = I_{st} \qquad \forall\ s,t. \qquad (15)$$

$$\sum_{m \in B_v} IMT_{mvt} \leq C_v \qquad \forall\ v,t.\ v \in TM \qquad (16)$$

$$\sum_{v \in A_M} IMT_{mvt} = I_{mt} \qquad \forall\ m,t. \qquad (17)$$

$$X_{ijt}, I_{it}, IS_{st}, IM_{mt}, IST_{sut}, IMT_{mvt}, Y_{st}, F_{st}, F_{sit,} Q_{mpt} \geq 0$$
$$O_{ijt} \ \ integer \qquad (18)$$

Constraint 1 indicates that demand for each end product has to be met in each time period. Constraint 2 is for derived demand of intermediate products. It indicates that demand of each intermediate product in each time period is based on the production of intermediate and end products where the product is an input. Constraint 3 is the capacity constraint of dedicated production plants. It restricts the production quantity of intermediate and end products produced on the basis of available capacity of plants in each time period. Constraint 4 is the capacity constraint of flexible production plants. The first summation in the constraint is capacity required in each time period for processing and setups of products that share machines but do not have alternate routes. The second summation is for the capacity requirement in each time period of products that share machines and have alternate production routes. Constraint 5 sums the total production of a product across all its routes in each time period. Constraint 6 ensures that the required numbers of setups are done in one production run of a product in each production plant in each time period. Constraint 7 provides lower bounds on the inventory levels of end products and intermediate products in each time period. Constraint 8 is the inventory balance for recovered raw materials. It states that in each time period recovered raw materials are generated by processing of by-products and are consumed in production of intermediate and end products. Constraint 9 is the inventory balance constraint for by-product. It indicates that in each time period the by-products are generated by intermediate and end products produced and are consumed in the recycling plants to recover raw materials. Constraint 10 is the total raw material requirement in each time period, i.e., the sum of fresh raw material and recovered raw material would be the total requirement of raw material across all products. Constraint 11 is for minimum quantity of fresh raw materials required in each time period. It provides a lower bound on the use of fresh raw material for each product. Constraint 12 equates that the total fresh raw material consumption in each time period to the fresh raw material consumed across all products. Constraint 13 restricts the processing of by-products in each time period on the basis of available capacity of recycling plants. Constraint 14 limits the inventory of recovered raw material in each time period with the storage tank capacity. Constraint 15 equates the sum of inventory of recovered raw material in each tank to its total inventory in each time period. Constraint 16 restricts the inventory of by-products in each time period with the available storage tank capacity. Constraint 15 states that inventory of by-products in each tank in each time period is equal to its total inventory. In the next section, we describe the formulations of finished goods scheduling model and intermediate products scheduling model.

3.3 Scheduling Models

In this section, we describe the formulations of scheduling models in order to derive scheduling decisions of the production planning and scheduling problem. The scheduling decisions determine start time and completion time of a job at each machine. The aggregate production plan derived from the production-planning model is input to the scheduling model. The production plan of the planning model imposes constraints on the scheduling model.

Scheduling problem consists of two parts, one is the finished goods scheduling and the other is the intermediate products scheduling. They are different problems because the production environment is different in finished goods and the intermediate products. As discussed in chapter 1, finished goods in our problem have flowshop pattern. In a flowshop, each product has same sequence of operations. For determining the optimal schedule of any performance measure, jobs may or may not be processed in the same sequence at each machine. If jobs are processed in the same sequence at all machines, the flowshop is known as permutation flowshop. Finding an optimal schedule in a flowshop for any objective when sequence of jobs may vary at machines is significantly harder than for determining the sequence for permutation flowshop (Baker, 1974; Pinedo, 1998). As a result, we have considered the permutation flowshop production environment in the scheduling problem.

The finished goods have a due date that is specified by the customer orders and demand forecast. One of the objectives of the scheduling model is to meet the customer orders with minimum tardiness. Tardiness of a job T_i is defined as: $T_i = max\ (C_i - d_i, 0)$, where C_i is the completion time of job i on the last machine and d_i is the due date of job i. Tardiness is a regular performance measure, i.e., non-decreasing in C_i for all i. Garey, Johnson and Sethi (1976) provide NP-hardness proof of the m machine permutation flowshop tardiness problem. In a multi-stage environment, minimizing inventory costs also becomes important as inventory costs are incurred at various stages of producing finished goods in the form of intermediate products. Also some intermediate products have limited shelf life. Thus minimizing earliness is also one of the objectives of the scheduling model. Earliness of a job E_i is defined as: $E_i = \max(d_i - C_i, 0)$. Earliness is a non-regular performance measure, i.e., non-increasing in C_i for all i. The overall objective of the scheduling model is to minimize earliness and tardiness (E/T) penalties, i.e., to minimize absolute deviation of job completion times about their due date. The flowshop E/T problem is harder than the flowshop tardiness problem, hence we focus on analyzing special case of flowshop E/T problem which has common due date of jobs.

We now provide the formulation of permutation flowshop problem of minimizing earliness and tardiness penalties with common due date d. This is the MIP model for finished goods scheduling decisions.

3.3.1 Finished Goods Scheduling Problem Formulation

Indices

i = index of jobs
j = index of machines

Sets

N = set of jobs, $\{i \mid i=1,2,\ldots,n\}$
S = set of machines, $\{j \mid j=1,2,\ldots,m\}$

Parameters

d =	common due date of jobs
p_{ij} =	processing time of job i on machine j

Variables

S_{ij} =	start time of job i on machine j
C_{ij} =	completion time of job i on machine j
T_i =	tardiness of job i, $T_i = \max(C_{im} - d, 0)$
E_i =	earliness of job i, $E_i = \max(d - C_{im}, 0)$

$$y_{ik} = \begin{cases} 1, & \text{if job } i \text{ is before job } k \text{ in a sequence, } i, k \in N \\ 0, & \text{otherwise} \end{cases}$$

$$b_i = \begin{cases} 1, & \text{if } T_i \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\min \quad Z = \sum_i E_i + T_i = \sum_i \left| C_{im} - d \right|$$

subject to:

$$C_{ij} \geq C_{ij-1} + p_{ij} \qquad \forall\, i \in N, j \in S \qquad (1)$$

$$C_{kj} - C_{ij} + M(1 - y_{ik}) \geq p_{kj} \qquad \forall\, i, k \in N, j \in S \qquad (2)$$

$$C_{ij} - C_{kj} + M\, y_{ik} \geq p_{ij} \qquad \forall\, i, k \in N, j \in S \qquad (3)$$

$$C_{im} - d = T_i - E_i \qquad \forall\, i \in N \qquad (4)$$

$$T_i \leq M\, b_i \qquad \forall\, i \in N \qquad (5)$$

$$E_i \leq M(1 - b_i) \qquad \forall\, i \in N \qquad (6)$$

$$C_{ij} = S_{ij} + p_{ij} \qquad \forall\, i \in N, j \in S \qquad (7)$$

$$C_{ij}, S_{ij}, E_i, T_i \geq 0$$
$$y_{ik}, b_i \in \{0,1\} \qquad (8)$$

Constraint 1 is operation precedence constraint for a job. It ensures that an operation cannot start until the previous operation is complete. Constraint 2 and constraint 3 indicate job precedence at a machine. They ensure that if a job i is scheduled before job k, then at each machine job k is started only after job i is completed. Constraint 4 determines E_i or T_i of a job, as the case may be. Constraint 5 and constraint 6 ensure that only one of E_i or T_i is incurred as by definition $E_i = -T_i$ and both E_i and T_i are non-negative. Constraint 7 indicates that preemption is not allowed for a job and determines the start times of each job at each machine.

Finished goods have external demand in the form of customer orders and forecast. Intermediate products have derived demand based on the production of products, where intermediate products are inputs. Finished goods derive their due dates from customer orders. Intermediate products derive their due dates from the production schedule of products where they are required. In our problem the production process of finished goods and intermediate products differ on the basis of production routes. As we have seen, finished goods follow flowshop pattern whereas intermediate products have general route, similar to jobshop environment. In a jobshop, each product has a distinct route that may or may not be similar to the route of other products. An intermediate product in our problem has an additional complexity that it may require a particular machine several times in its route, i.e., job shop with re-entrant flows. We now provide the formulation of scheduling model for intermediate products scheduling.

3.3.2 Intermediate Products Scheduling Problem Formulation

Indices

i, k = index of jobs
j = index of machines
l, s = index of operations

Sets

$N =$ set of jobs, $\{i \mid i=1,2,\ldots,n\}$

$S =$ set of machines, $\{j \mid j=1,2,\ldots,m\}$

Parameters

$d =$ common due date of jobs

$e_i^s =$ machine used by job i for s^{th} operation

$p_{ij} =$ processing time of job i on machine j

$L_i =$ last operation of job i

Variables

$S_{ij} =$ start time of job i on machine j

$C_{ij} =$ completion time of job i on machine j

$T_i =$ tardiness of job i, $T_i = \max(C_{im}-d, 0)$

$E_i =$ earliness of job i, $E_i = \max(d-C_{im}, 0)$

$$y_{ilkse} = \begin{cases} 1, & \text{if } l^{th} \text{ operation of job } i \text{ is before } s^{th} \text{ operation of job } k \text{ at machine } e \quad i,k \in N,\ e \in S \\ 0, & \text{otherwise} \end{cases}$$

$$b_i = \begin{cases} 1, & \text{if } T_i \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\min \quad z = \sum_i E_i + T_i = \sum_i \left| C_{ie_i^{L_i}} - d \right|$$

subject to:

$$C_{ie_i^l} - C_{ie_i^{l-1}} \geq p_{ie_i^l} \qquad \forall\, i \in N, j \in S \qquad (1)$$

$$C_{kj} - C_{ij} + M(1 - y_{ilske}) \geq p_{kj} \qquad \forall\, i,k \in N, j \in S \qquad (2)$$

$$C_{ij} - C_{kj} + M\, y_{ilkse} \geq p_{ij} \qquad \forall\, i,k \in N, j \in S \qquad (3)$$

$$C_{ie_i^{L_i}} - d = T_i - E_i \qquad \forall\, i \in N \qquad (4)$$

$$T_i \leq M\, b_i \qquad \forall\, i \in N \qquad (5)$$

$$E_i \leq M(1 - b_i) \qquad \forall\, i \in N \qquad (6)$$

$$C_{ij} = S_{ij} + p_{ij} \qquad \forall\, i \in N, j \in S \qquad (7)$$

$$C_{ij}, S_{ij}, E_i, T_i \geq 0 \qquad (8)$$

$$y_{ilkse}, b_i \in \{0,1\}$$

Constraint 1 indicates that an operation of a job can be started only after its previous operation is completed. Constraint 2 and 3 ensure that there is no overlapping of jobs at a machine. They indicate that if at machine j, l^{th} operation of job i is scheduled, then s^{th} operation of another job k can be started only when job i has finished processing on machine j. Constraint 4 determines the earliness and tardiness of each job. Constraint 5 and constraint 6 ensure that only one of E_i or T_i is incurred as by definition $E_i = -T_i$ and both E_i and T_i are non-negative. Constraint 7 indicates that preemption of job is not allowed and determines the completion time of each job. We now summarize this chapter.

3.4 Summary

In this chapter we have described the mathematical models to address the decisions of production planning and scheduling problem considered in this research. We have discussed the reason for modeling the decisions sequentially through hierarchical models. The production planning is a mixed integer linear programming model. We have developed two scheduling models, one for finished goods scheduling and the other for intermediate products scheduling. Both scheduling models are mixed integer linear programming models. In the next chapter we describe the solution algorithms for solving the production planning and scheduling problem.

4 Solution Procedure for Production Planning and Scheduling Problem

4.1 Introduction

We have modeled the production planning and scheduling decisions in chapter 3 in two steps. In the first step, we have developed the production-planning model as a mixed integer programme (MIP). The decisions of the production-planning model are production quantities of products, inventory levels of products and aggregate capacity of resources required to meet the production plan. In the second step, we have modeled scheduling decisions, which are start times and completion times of each product on each machine. Scheduling problem consists of two parts, finished goods scheduling and intermediate products scheduling. Finished goods follow flowshop pattern of production process. In chapter 3, we presented an MIP formulation for finished goods scheduling problem. Intermediate products follow a general job shop pattern of production process with re-entrant flows. In chapter 3, we also presented an MIP formulation of intermediate products scheduling problem. We discussed in chapter 3, the rationale for modeling the production planning and scheduling decision in a hierarchical manner. The decisions of production-planning model are constraints, within which, the detailed scheduling decisions are made. The decisions of the production planning model, production quantity and inventory levels of products, are input parameters to the detailed scheduling model.

In this chapter, we discuss the solution procedure for solving the production planning and scheduling problems. In section 4.2, we define the framework that we have used to solve the production planning and scheduling problem. In section 4.3, we provide the solution procedure for production-planning problem. Next, we develop solution procedures for solving the scheduling problems. In section 4.4, we develop the solution procedure for solving the finished goods scheduling problem. In section 4.5, we solve the intermediate products scheduling problem. We discussed in chapter 1, that the production environment has dedicated production plants. In section 4.6, we describe the solution procedure solving the dedicated plant-scheduling problem. We summarize this chapter in section 4.7.

4.2 Solution Procedure for Production Planning and Scheduling Problem

As we discussed in section 4.1, we are solving production planning and scheduling problem in two steps. In the first step, we solve the production-planning problem as shown in figure 4.1. Production quantities of products and inventory levels of products are the decisions of production planning model. Production planning decisions are input to the scheduling model. The scheduling model has to determine the schedule of production plan proposed by the production-planning model. In the second step, we develop solution procedure for solving finished goods scheduling problem. We develop analytical results and heuristics for solving the finished good scheduling problem. Then, we develop solution procedure for intermediate products scheduling problem. We report results of the solution procedure for the production planning and scheduling problem in chapter 5. In the next section, we describe the solution procedure for production planning problem

4.3 Solution Procedure for Production Planning Problem

The production-planning model is solved using the branch and bound algorithm. Demand for finished goods in each period of the planning horizon is an input to the model. Aggregate capacity is considered in the production-planning model. For dedicated plants, capacity of the bottleneck machine is considered as the plant capacity. In case of flexible plants, capacity of each machine processing multiple products is considered. The language complier used to solve the mathematical model is General Algebraic Modeling System (GAMS), version 19.8 with solvers integrated in the compiler. We use the branch and bound algorithm of CPLEX solver to solve the production-planning model. In the next section, we describe the solution procedures for finished goods scheduling problem.

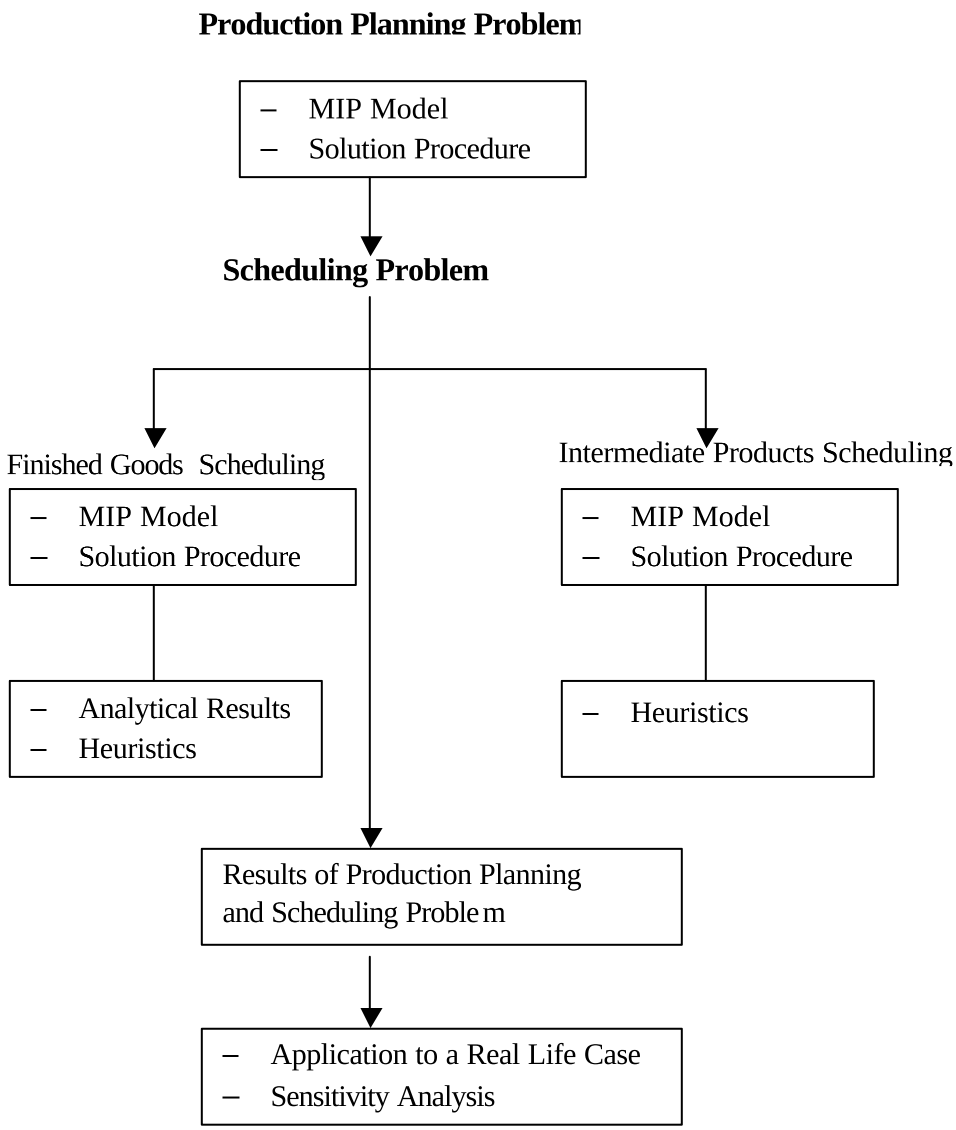

Figure 4.1: Schematic of Solution Procedure for Production Planning and Scheduling Problem

4.4 Solution Procedure for Finished Goods Scheduling Problem

As we discussed in chapter 3, that to solve finished goods scheduling problem, we are considering permutation flowshop with common due date. The objective of the scheduling model is to minimize earliness and tardiness (E/T) penalties. We discussed in chapter 2, that the existing results on earliness and tardiness penalties have focused on single machine scheduling. We also discussed the unrestricted due date in single machine scheduling (Baker and Scudder, 1990). Let us call the unrestricted due date for single machine as d_0. Let d be the common due date for all the jobs. When jobs are arranged such that $p_1 \le p_2 \le \ldots\ldots\ldots\ldots \le p_n$, a problem is called unrestricted if:

$d \ge d_0 = (p_n + p_{n-2} + p_{n-4} + \ldots\ldots + p_4 + p_2)$, *if n is even.*

$d \ge d_0 = (p_n + p_{n-2} + p_{n-4} + \ldots\ldots + p_3 + p_1)$, *if n is odd.* For any value of $d < d_0$, the problem is restricted Let *SUD(d)* be the single machine E/T problem for common due date $d \ge d_0$. Baker and Scudder (1990) have derived the optimal sequence, properties of optimal sequence and the schedule of *SUD(d)*. The optimal sequence of *SUD(d)* for common due date $d \ge d_0$ is:

$(n, n\text{-}2, n\text{-}4, \ldots, 1, \ldots, 2, \ldots 4, \ldots\ldots n\text{-}3, n\text{-}1)$, if n is odd

$(n, n\text{-}2, n\text{-}4, \ldots, 2, \ldots, 1, \ldots 3, \ldots\ldots n\text{-}3, n\text{-}1)$, if n is even.

The properties of optimal sequence of *SUD(d)* are:

a) No idle time between jobs in the schedule.

b) V-shaped optimal sequence. In a V shaped sequence, jobs scheduled before the due date d are in longest processing time first *(LPT)* sequence and jobs scheduled after d are in shortest processing time first *(SPT)* sequence.

c) One job finishes at due date d.

To determine the schedule of the optimal sequence obtained above for common due date, $d \geq d_0$, let S_i and C_i be the start time and completion time of job i respectively. If the optimal sequence is $1,2,\ldots e\text{-}1,\ e,\ e+1,\ldots.n$, where e is the job that finishes at common due date d, i.e., $C_e = d$ and $S_e = C_e - p_e$. There is no idle time in this schedule, $C_{e-1} = S_e$ and $S_{e-1} = C_{e-1} - p_{e-1}$. This way the schedule of the optimal sequence is determined.

We use these results of single machine E/T problem for unrestricted common due date and exploit some of its properties in solving special cases of multi-machine problems. In multi-machine problems, we are studying permutation flowshop with common due date, which is a more tractable case of flowshop environment. Next, we develop the unrestricted and restricted due dates for permutation flowshops with common due date.

We would like to begin by stating that there could be alternate optimal sequences of *SUD(d)* for any $d > d_0$. The optimal sequence shown above is at $d = d_0$. This optimal sequence is:

(n, n-2, n-4,…..,1,..,2,.... 4,…….n-3, n-1), *if n is odd*

(n, n-2, n-4,…..,2,..,1,….3,…….n-3, n-1), if *n* is even.

No matter what the optimal sequence is, the cost of the schedule of any of the alternate optimal sequences is obviously same. It is difficult to obtain all alternate optimal sequences for $d > d_0$. However, all the alternate optimal sequences can be obtained for *SUD(d)* at $d = d_0$. The set of all alternate optimal sequences at $d = d_0$ would be used later to solve flowshop E/T problem, hence we now describe a procedure to generate all alternate optimal sequences at $d = d_0$. It is to be noted that there will be alternate optimal sequences at $d = d_0$, only if, processing times of any two jobs are same. The alternate optimal sequences at $d = d_0$ are generated as follows. If the optimal sequence obtained above is index from 1 *to* n,

<u>Procedure for Generating Alternate Optimal Sequences at $d = d_0$ (GAOS)</u>

Step 1: $j = 1$

Step 2: $x = j + 1$

Step 3.1: Is $p_{xm} = p_{jm}$

Yes→ Create new sequence by interchanging j and x
- $x = x + 1$
- is $x = n + 1$
  - yes→ $j = j + 1$ and goto step 3.2
  - no→ repeat step 3.1

No→ $x = x + 1$ and repeat step 3.1

Step 3.2 if $j = n$
STOP else goto step 2

We now define some terms before deriving unrestricted and restricted due dates for flowshop E/T problem.

Notation

i =	index of jobs,	$i = 1,2,\ldots n.$
j =	index of ordered machines in a flowshop,	$j=1,2\ldots..m.$
s =	index of sequences of jobs,	$s = 1,2,\ldots l$
d =	common due date of jobs	
p_{ij} =	processing time of job i on machine j	
d_0 =	unrestricted common due date for single machine, $p_{nm} + p_{n-2m} + p_{n-4m} + \ldots\ldots + p_{4m} + p_{2m}$, *if n is even* $p_{nm} + p_{n-2m} + p_{n-4m} + \ldots\ldots + p_{3m} + p_{1m}$, *if n is odd.*	
$SUD(d_0)$ =	single machine E/T problem for unrestricted common due date d_0	
$S(m, d_0)$ =	set of optimal sequences of $SUD(d_0)$ at last machine m with common due date d_0. $S(m, d_0)$ is generated by procedure described above of generating optimal sequences.	
$E(s, d_0)$ =	set of early and on-time jobs in sequence s with common due date d_0, $s \in S(m, d_0)$.	
$T(s, d_0)$ =	set of tardy jobs in sequence s with common due date d_0, $s \in S(m, d_0)$.	

$r(s, d_0) =$ schedule of optimal sequence s, consisting of S_i and C_i $\forall i$, $s \in S\ (m,\ d_0)$. Schedule is generated as described in the procedure above in this section, when we discussed the single machine results from Baker and Scudder (1990).

$Z_1\{r\ (s,\ d_0)\} =$ earliness and tardiness costs of schedule $r\ (s,\ d_0)$.

$S_{ij} =$ start time of job i on machine j

$C_{ij} =$ completion time of job i on machine j

$F(s) =$ Flowshop schedule of sequence s, $s \in S(m,d_0)$. $F(s)$ is determined as follows. Let the sequence be $1,2,\ldots\ldots n$.
$S_{11} = 0$,
for $i = 1$ to n
for $j = 1$ to m,

$$S_{ij} = max\ \{C_{ij-1},\ C_{i-1j}\}$$
$$C_{ij} = S_{ij} + p_{ij}$$

$M_{F(s)} =$ Makespan of schedule $F(s)$, $M_{F(s)} = C_{nm}$, $s \in S\ (m,\ d_0)$. This is the completion time of last job in the sequence.

Makespan of the schedule is defined as the completion time of last job in the sequence. $M_{F(s)}$ is the makespan of schedule $F(s)$ of permutation flowshop sequence s. We define k as the sequence with minimum makespan, i.e., $k = \arg \min_{s \in S(m,d_0)} M_{F(s)}$. The unrestricted due date d_1 in permutation flowshop environment is defined as $d_1 = M_{F(k)} - \sum_{j \in T(k,d_0)} p_{jm}$. The first term at right hand side is the makespan of sequence k. The second term is the sum of tardy jobs in sequence k. Now we develop the restricted due date d_2 in permutation flowshop setting. Let us define: $a = \arg\min_i \sum_{j=1}^{m} p_{ij} \quad \forall\, i = 1,2,\ldots.n$. a is the minimum of sum of processing times of job at all machines amongst all jobs. We call this sum as the restricted due date, i.e.,

$$d_2 = \sum_{j=1}^{m} p_{oj}$$

We have defined in the above paragraphs, the unrestricted due date d_1 and restricted due date d_2 in a permutation flowshop environment. We now define another range of due date, that is in between the restricted and unrestricted due date, and we call it as intermediate due date. Thus, for flowshop E/T problem for common due date, we have problems for $d \geq d_1$(unrestricted due date); $d_2 < d < d_1$ (intermediate due date) and $d \leq d_2$ (restricted due date). On the basis of the classification of due dates, we have decomposed the flowshop E/T problem into three sub problems as shown in figure 4.2.

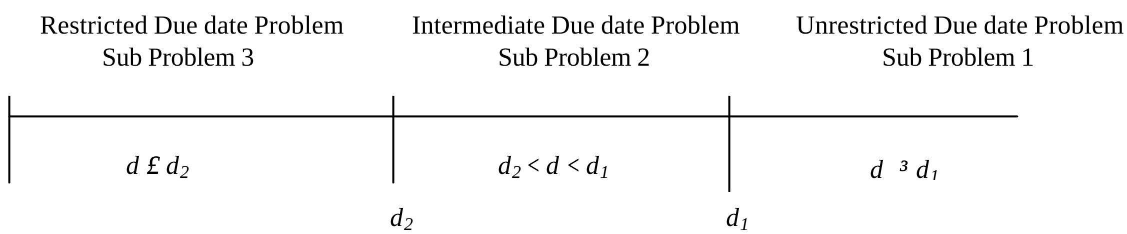

Figure 4.2: Flowshop E/T Problem Decomposition Based on Due Dates

Sub-problem 1 is the flowshop E/T problem defined over the unrestricted common due date $d \geq d_1$, sub-problem 2 is flowshop E/T problem defined over the intermediate due date $d_2 < d < d_1$ and sub-problem 3 is the flowshop E/T problem defined over the for restricted due date $d \leq d_2$. Sub-problem 3 has a special structure by definition of d_2, that all jobs will be necessarily tardy. We will discuss in detail about the special properties of sub-problem 3 when we will describe the solution procedure for solving sub-problem 3 later in this chapter. In the following sub-sections, we describe each of the sub-problems and solution algorithms to solve them. In sub-section 4.4.1, which follows next, we solve sub-problem 1.

4.4.1 Sub-Problem 1: Flowshop E/T Problem for Unrestricted Common Due Date

In this sub-section, we develop the solution procedure for solving the permutation flowshop E/T problem for unrestricted common due date $d \geq d_1$. The objective of sub-problem 1 is to minimize E/T penalties, i.e., $Minimize\ Z = \sum_i E_i + T_i = \sum_i \left| C_{im} - d \right|$, where C_{im} is the completion time of job i on the last machine m.

One of the optimal properties of $SUD(d)$ is that there is no idle time in the schedule. If there is an idle time, it should be removed while maintaining the feasibility of the schedule. We now develop a procedure to remove idle time in the schedule $F(s)$ at the last machine. This procedure will be used later in the solution procedure for solving sub-problem 1.

<u>Procedure for Removing Idle Time at Last Machine (RIT)</u>

Let the sequence *s* be 1,2,….n

.

Step 1: $i = n$
Step 2: $t = S_{im} - C_{i-1m}$
Step 3: If $t > 0$
Yes→ for $x = 1$ to $i-1$
$S_{xm} = S_{xm} + t$
$C_{xm} = S_{xm} + p_{xm}$
If $i = 1$, STOP else
$i = i-1$ and goto Step 2
No→ If $i = 1$, STOP else
$i = i-1$ and goto Step 2

In step 1, the last job in the sequence is selected. Step 2 checks if there is an idle time between the jobs. Step 3 removes the idle time between the jobs while maintaining the feasibility of the schedule. This procedure would result in following schedule at machine *m*.

$C_{nm} = M_{F(s)}$
$S_{nm} = C_{nm} - p_{nm}$
For $i = n\text{-}1$ to 1
$C_{im} = S_{i+1.m}$
$S_{im} = C_{im} - p_{im}$

We now state a theorem to determine optimal solution for sub-problem 1.

Theorem 1: *For a flowshop E/T problem with common due date $d \geq d_1$, there is an optimal sequence k with $Z\{F(k)\} = Z_1\{r(k,d_0)\}$.*

Proof: By definition of $SUD(d_0)$, sequence k is optimal for $d \geq d_0$. It follows that for $d \geq d_0$, $Z_1\{r(k, d)\} = Z_1\{r(k,d_0)\}$. By definition, $d_1 \geq d_0$. Thus for $d \geq d_1$, sequence k is optimal for $SUD(d)$ and $Z_1\{r(k,d_1)\} = Z_1\{r(k,d_0)\}$. $Z\{F(k)\}$ is function of completion time of jobs at machine m, i.e., $Z\{F(k)\} = \sum_{j=1}^{n} |C_{jm} - d_1|$ for $d = d_1$. It follows that $Z\{F(k)\} \geq Z_1\{r(k,d_1)\}$ as $Z_1\{r(k,d_1)\}$ is optimal for $d = d_1$.

In schedule $F(k)$ at machine m, if $S_{im} = C_{i-1.m}$ $\forall$ $i = n, n\text{-}1, n\text{-}2, \ldots, 2$, sequence k has all optimal properties of $SUD(d)$ at $d = d_1$. If $S_{i.m} \geq C_{i-1.m}$ $\forall$ $i = n, n\text{-}1, n\text{-}2, \ldots, 2$, this idle time can be removed by the procedure RIT defined above.

It follows that sequence k has now all properties of $SUD(d)$ at $d = d_1$. Thus, $Z\{F(k)\} = Z_1\{r(k, d)\}$ at $d = d_1$. If d_1 is increased to $d_1 + D$, the optimal schedule at stage m would be $C_{im} = C_{im} + D$ for $i = n\text{-}1$ to 1 and $C_{nm} = M_{F(k)} + D$. For $d > d_1$, all properties of $SUD(d)$ hold. Hence for $d \geq d_1$, $Z\{F(k)\} = Z_1\{r(k, d_0)\}$ and sequence k is optimal.

Q.E.D.

We have derived above optimal solution for sub problem 1. We would like to state that the value of unrestricted due date d_1 in sub problem 1 is determined on the basis of set of all optimal sequences of single machine E/T problem at $d = d_0$. As mentioned earlier in section 4.4, it is difficult to obtain optimal sequences for single machine E/T problem for $d > d_0$. In that sense the value of d_1 could be made tighter. This is because some of the optimal sequences for $d > d_0$ could have lesser makespan than $M_{F(k)}$, and d_1 is a function of $M_{F(k)}$ as defined above. In the next sub section we describe sub problem 2 and develop its solution procedure.

4.4.2 Sub-Problem 2:Flowshop E/T Problem for Intermediate Common Due Date

The objective of sub problem 2 is same as that of sub-problem 1, i.e., $Minimize\ Z = \sum_i E_i + T_i = \sum_i |C_{im} - d|$. The difference between sub problems 1 and 2 is in the value of the common due date d. The common due date value for sub problem 2 is between d_2 and d_1, i.e., $d_2 < d < d_1$. Garey et al. (1976) provide proof of NP-completeness of this problem. We were able to use some of the optimal properties of single machine E/T problem, and construct optimal results for flowshop E/T problem for d ³ d_1. For common due date $d < d_1$, we find that it is difficult to obtain analytically optimal solution for flowshop E/T problem. We have developed a heuristic algorithm to solve sub-problem 2. We now describe the proposed heuristic algorithm to solve sub problem 2.

4.4.2.1 Heuristic Algorithm (H1) for Sub Problem 2

The proposed heuristic for solving sub-problem 2 is based on permutation sequence of jobs at the bottleneck machine. Bottleneck machine is identified in this problem as the machine that requires maximum sum of processing time of all jobs amongst all machines.

The solution of multi-machine problems is often useful by decomposing the problem into single machine problems. As a result, we solve the single machine E/T problem at the bottleneck machine. The pre-bottleneck processing times of a job is captured by considering release dates of job at the bottleneck machine. The release date of a job in this problem is defined as the earliest time at which the job is available for processing at the bottleneck machine. The post-bottleneck processing times of a job is captured by determining the due date of a job at the bottleneck. The resulting problem is single machine E/T problem with release dates and distinct due dates, $n/1/r_i/S(E_i+T_i)$. We solve this single machine problem at the bottleneck machine. To solve this, we refer some results on $n/1/r_i/S(E_i+T_i$ by Chu (1992) and Chu and Portmann (1992). They derive a sequence of jobs on single machine. In our heuristic, using a priority function (defined below in the detailed heuristic steps), a job is selected and appended to a partial sequence. Schedule of the partial flowshop sequence is developed (explained below). Based on this schedule, release dates and due dates of a job are updated at each iteration of appending the job. The schedule of the complete permutation sequence is then modified to improve earliness and tardiness costs. In the end, local neighborhood search procedure (tabu search) is applied to improve the solution. We now explain the detailed steps of the heuristic.

Notation

d =	common due date for all jobs
i =	index of jobs, $i = 1,2,\ldots n$
j =	index of machines, $j = 1,2,\ldots m$
p_{ij} =	processing time of job i on machine j
k =	bottleneck machine
S_{ij} =	start time of job i on machine j
C_{ij} =	completion time of job i on machine j
r_{ik} =	earliest time at which job i is available for processing at machine k
d_{ik} =	due date of job i at bottleneck machine k
s =	a permutation flow shop sequence of n jobs

p = set of partial sequence of jobs

$s(s, i)$ = schedule of sequence s consisting of S_{ij} and C_{ij} for " *i Î s*, $j = 1,2,\ldots,m$

$Z\{s(s, i)\}$ = cost of permutation flowshop schedule

$$Z\{s(s, i)\} = \sum_{i=1}^{n} \left| C_{im} - d \right|$$

The problem is to determine s and $s(s, i)$ so as to minimize $Z\{s(s, i)\}$.

<u>Heuristic (H1) for Solving Sub-Problem 2</u>

Step 1 Determining bottleneck machine k

$$k = \arg\max_{j} \sum_{i=1}^{n} p_{ij}$$

Step 2 Determining permutation flowshop sequence (s) and schedule $s(s, i)$ for s

Step 2.1 Determining release date of job i at bottleneck machine k

$$r_{ik} = \sum_{x=1}^{k-1} p_{ix} \quad \forall\, \mathrm{i} = 1,2,\ldots \mathrm{n}$$

Determining due date of job i at bottleneck machine k

$$d_{ik} = d - \sum_{x=k+1}^{m} p_{ix} \quad \forall\, \mathrm{i} = 1,2,\ldots \mathrm{n}$$

Step 2.2 Determining priority u_i of jobs

$u_i = r_{ik}$ if $r_{ik} + p_{ik}$ ³ d_{ik}

$u_i = d_{ik} - p_{ik}$ if $r_{ik} + p_{ik} < d_{ik}$

Step 2.3 Appending a job to p (partial sequence)

Select job with minimum u_i and add to p

Step 2.4 Schedule $s(p, i)$ as follows:

for i to $|p|$, *i Î p*,

for $j = 1$ to m

$S_{11} = 0$

$S_{ij} = \max \{C_{ij-1}, C_{i-1j}\}$

$C_{ij} = S_{ij} + p_{ij}$

Step 2.5 Updating r_{ik} $\forall\ i \notin \pi$

Add i to π and call it π_i

Determine $s(\pi_i, i)$ according to step 2.3 $\forall\ i \in \pi_i, j = 1$ to m

$r_{ik} = C_{\pi i\, k-1}$(completion time of i at $(k-1)$ after being appended to π)

This is based on the logic that we schedule the partial sequence π_i according to step 2.4 and determine the time when job i is available for processing at bottleneck machine.

Step 2.6 Updating d_{ik} $\forall\ i \notin \pi$

$$d_{ik} = \max \{d_{ik}, C_{\pi k+1}, \max_{k+2 \le x \le m} \{C_{\pi x} - \sum_{y=k+1}^{x-1} p_{iy}\}\}$$

This is based on the logic that a job is not required till the time the partial sequence π is already scheduled on post- bottleneck stages.

Step 2.7 Repeat steps 2.1 to 2.6 for $i \notin \pi$ till $|\Pi| = n$, i.e. a complete sequence σ is obtained.

Step 3 Adjusting the schedule at $j = m$ (last machine)

Shifting all early jobs towards right (increasing C_{im}) before 'd'

Define e: set of early jobs, $e = \{i \mid C_{im} < d\}$

o: set of ontime job: $o = \{i \mid C_{im} = d\}$

t: set of tardy jobs: $t = \{i \mid C_{im} > d\}$

$l = \{i \mid S_{im} < d$ and $C_{im} > d\}$

for $i = 1$ to n,

if($C_{im} < S_{i+1m}$ and $C_{im} < d$),

get $z = \min\{S_{i+1m} - C_{im}, d - C_{im}\}$

for $x = 1$ to i

$S_{xm} = S_{xm} + z$

$C_{xm} = C_{xm} + z$

With this all jobs that complete before due date d are shifted towards d so that earliness costs are reduced. This procedure maintains the feasibility of schedule.

Step 4 Improving E/T costs further

if $|e| \geq |o| + |t|$

check if $|o| = 1$

Yes $\rightarrow$ for $i = 1$ to n,

$S_{im} = S_{im} + p_{xm}, x \in o$

$C_{im} = C_{im} + p_{xm}, x \in o$

No $\rightarrow z = d - S_{xm}, x \in l$

for $i = 1$ to n

$S_{im} = S_{im} + z$

$C_{im} = C_{im} + z$

Step 4.1 Bring back (reduce C_{im}) tardy jobs (if they can be) that got shifted towards right after 2.7.2

for $i = 1$ to $|t|$, $i \in t$;

if $C_{im} < S_{i+1m}$ and $S_{i+1m} > C_{im-1}$

Yes → $S_{i+1m} = S_{i+1m}$ – min $\{S_{i+1m} - C_{im},\, S_{i+1m} - C_{im-1}\}$

$C_{i+1m} = S_{i+1m} + p_{i+1m}$

No → $S_{i+1m} = S_{i+1m}$

$C_{i+1m} = C_{i+1m}$

Step 5 Determine $Z\{s(s, I)\} = \sum_{i=1}^{n} | C_{im} - d|$

Step 6 Improving the objective value by performing neighbor hood search scheme (tabu search) to get a better sequence and schedule. The tabu search procedure is described below.

<u>Tabu Search Procedure (TS)</u>

Z_c = objective function of the current best solution
s_c = current best sequence
Z_e = objective function of the best ever solution
s_e = best ever sequence
p = number of pairs, $p = n(n-1)/2$
t = number of tabu iterations
Z_{xj} = objective function of the candidate sequence x formed by interchanging j^{th} pair, $j = 1,2,\ldots p$
s_{xj} = sequence of candidate sequence x formed by interchanging j^{th} pair, $j = 1,2,\ldots p$.
a_j = $Z_c - Z_{xj}$,
ts_j = tabu structure of the j^{th} pair, $0 £ ts_j £$ tabu tenure

Step 6.1 for $i = 1$ to t

Step 6.1.1 for $j = 1$ to p

Generate p candidate sequences s_{xj} by interchanging j^{th} pair from the current best sequence s_c, $x = 1,2,\ldots p$

Schedule the sequence x from step 2.4, step 3 and step 4.

Determine Z_{xj} from step 5

Determine $a_j = Z_c - Z_{xj}$

Sort d_j*'s* in non-increasing order and re-index d_j from 1 to n

Step 6.2 $j = 1$

Step 6.3

Case 1: Candidate solution is worse than current solution and the pair is tabu as well

a_j £ 0 and $ts_j > 0$

$j = j+1$ and repeat step 6.3

Case 2: Candidate solution is better than current solution and the pair is not tabu

if $a_j > 0$ and $ts_j = 0$

step 6.3.1 $Z_c = Z_{xj}$

$s_c = s_{xj}$

ts_j = tabu tenure

for $j = 1$ to p

if $ts_j > 0$

$ts_j = ts_j - 1$

if $Z_c < Z_e$

$Z_e = Z_c$

$s_e = s_c$

Case 3: Candidate solution is worse than the current solution and the pair is tabu

if a_j £ 0 and $ts_j = 0$

goto step 6.3.1

Case 4: Candidate solution is better than the current solution, better than best ever solution but the pair is tabu (Aspiration) if $a_j > 0$ and $ts_j > 0$ and $Z_e > d_j$

goto step 6.3.1

Step 6.4 If $i = t$, STOP, else $i = i + 1$ and goto Step 6.1.1.

Step 2.1 determines the release dates and due dates at the bottleneck machine for all jobs. Step 2.2 determines the priority of a job that is yet to be selected in a partial sequence. The job with the highest priority is selected and appended to the partial sequence in step 2.3. Schedule of the partial sequence is developed in step 2.4. In step 2.5, based on the completion time of the last job of the partial sequence at the bottleneck machine, release dates of the jobs not in the partial sequence are updated. Similarly, in step 2.6, due dates of the jobs not in the partial sequence at the bottleneck stage are determined. In step 2.7, a complete permutation flowshop sequence is determined. In step 3, we shift jobs that complete before the due date and have idle times at the last machine towards the due date. This reduces the earliness costs while maintaining the feasibility of the schedule. In step 4 we reduce the earliness and tardiness costs by increasing the completion time of jobs at the last machine as long as the number of early jobs are more than the number of on-time jobs and tardy jobs. Step 5 determines the objective value of the schedule.

In step 6 we apply tabu search, a local neighbor hood search procedure to improve the value of objective function. In tabu search procedure, parameters are the number of tabu iterations and the tabu tenure. Tabu iterations are the number of iterations over which the tabu procedure is applied. In this procedure, typical tabu iteration would have following steps. From the n jobs, $p = n(n-1)/2$ pairs are created. From the current sequence derived after step 5, p candidate sequences are obtained by applying pair wise interchange at the current sequence. All p sequences are scheduled based on the steps described in heuristic and the objective value of each sequence is determined. Tabu move is performed based on the objective values of the *p* sequences. The tabu moves are described in the heuristic. Tabu tenure is the number of iterations for which the pair that just performed the tabu move would not be considered. Next, we describe the solution procedure for solving sub-problem 3.

4.4.3 Sub-Problem 3:Flowshop Tardiness Problem for Common Due Date

We now discuss the sub problem 3 of minimizing earliness and tardiness penalties in a flowshop for common due date $d < d_2$ (d_2 is obtained in sub section 3.6.2). This sub problem has a special structure by definition of d_2, that no job is early. Thus problem reduces to that of minimizing tardiness. Since the due date in our problem is common for all jobs, minimizing tardiness is same minimizing flowtime, if all jobs are necessarily tardy. Further since all jobs are simultaneously available, the minimizing flowtime problem is same as minimizing completion time. Thus our problem is to minimize tardiness or flowtime or completion time of all jobs. We now derive analytical solution of sub-problem 3. We begin that by defining few terms.

Notation

i =	index of products,	$i = 1,2,\ldots n$
j =	index of machines,	$j = 1,2\ldots..m$
q =	index of sequences of jobs	
S =	set of permutation flowshop sequences	
d, d' =	common due date of jobs	
p_{ij} =	processing time of job i on machine j	
S_{ij} =	start time of job i on machine j	
C_{ij} =	completion time of job i on machine j	

E_i = earliness of job i, $E_i = max\{d-C_{im},0)$

T_i = tardiness of job i, $T_i = max(C_{im}-d, 0)$

$s(q, d)$ = permutation flow shop schedule of sequence q and due date d, q Î S.

$Z\{s(q, d)\}$ = Early/Tardy cost of schedule $s(q, d)$,

$$Z\{s\ (q,d)\} = \sum_{j=1}^{n} \left| C_{im} - d \right|$$

$$k = \arg\min_{i} \sum_{j=1}^{m} p_{ij}$$

$$d_2 = \sum_{j=1}^{m} p_{kj}$$

Proposition 1: *In a flowshop E/T problem with common due date d, an optimal sequence s for* $d = d_2$ *is optimal for* $d < d_2$.

Proof: Suppose the optimal sequence s for $d = d_2$ is not optimal for $d < d_2$. From definition of d_2, in any flowshop sequence q, no job is early ($E_i = 0$, " $i = 1,2,\ldots n$) for $d = d_2$. Hence schedule $s(q, d)$ has regular performance measure (non-decreasing in C_{ij}) for $d = d_2$. For regular performance measures, the cost of any schedule with inserted idle time $t = D$ can be improved by removing D as C_{ij} " i, j are reduced by $t = D$. Hence we consider $s(q, d_2)$ without inserted idle time and all jobs are scheduled as early as possible. $s(q, d_2)$ is derived as follows:

for $i = 1$ *to* n

for $j = 1$ *to* m

$S_{11} = 0$

$S_{ij} = max\ \{C_{ij-1}, C_{i-1j}\}$

$$C_{ij} = S_{ij} + p_{ij}$$

$$Z\{s\ (q, d_2)\} = \sum_{j=1}^{n} \left| C_{im} - d_2 \right|$$

From definition of $Z\{s(q, d_2)\}$, it can be seen that:

for $d = d_2 - 1$, $Z\{s(q, d)\}$ increases by n,

for $d = d_2 - 2$, $Z\{s(q, d)\}$ increases by $2n$,

for $d = d_2 - x$, $Z\{s(q, d)\}$ increases by xn.

Thus for any $d < d_2$, $Z\{s(q, d)\}$ increases by $(d2-d)n$,

Hence for $d < d_2$, $Z\{s(q, d)\} = Z\{s(q, d_2)\} + (d_2-d)n$

Now consider an optimal sequence s for $d = d_2$. Suppose s is not optimal for a due date d' where $d' < d_2$. Consider another sequence $s1$, which is optimal for $d' < d_2$. Then we have,

$Z\{s(s, d')\} = Z\{s(s, d_2)\} + (d_2-d')\ n$ (1)

$Z\{s(s1, d')\} = Z\{s(s1, d_2)\} + (d_2-d')\ n$ (2)

If s is not optimal for d',

$Z\{s(s, d')\} > Z\{s(s1, d')\}$ (3)

From (1), (2) and (3) ,

$Z\{s(s, d_2)\} + (d_2-d')n > Z\{s(s1, d_2)\} + (d_2-d')n$

Thus, $Z\{s(s, d_2)\} > Z\{s(s1, d_2)\}$. This is a contradiction as s is an optimal sequence for $d = d_2$. Hence s is an optimal sequence for $d < d_2$.

Q.E.D.

This result has implications that the optimal solution of flowshop tardiness problem for common due date d £ d_2 (sub-problem 3) remains same for range of d. It is, however, difficult to analytically obtain the optimal solution of sub-problem 3. We develop heuristic algorithm for the problem. Several researchers have investigated the problem of minimizing tardiness, flowtime, and completion time in permutation flowshops. The equivalence of these three objectives was shown above. We have compared the performance of our heuristic with the existing results and found our proposed heuristic to perform better.

The concept used in the heuristic is same used in heuristic algorithm of sub-problem 2. We derive permutation flowshop sequence at the bottleneck machine. The one minor difference between the heuristics of sub problems 2 and 3 is that the priority function of a job is determined differently. This is because in sub-problem 3 we are solving n/1/ri/ΣTi, whereas in sub-problem 2 we are solving n/1/ri/ΣEi +Ti. Secondly, the steps of improving earliness and tardiness costs of heuristic of sub-problem 2 are not required. The steps of the heuristic solution of sub-problem 3 are explained below.

4.4.3.1 Heuristic Algorithm (H2) for Sub-Problem 3

Heuristic H2 for Solving Sub-Problem 3

Steps 1 to steps 2.1 are same as in heuristic for solving sub-problem 2.

Step 2.2 Determining priority u_i of jobs

$$u_i = max(r_{ik}, t) + max\{max(r_{ik}, t) + p_{ik}, d_{ik}\}$$

where t = *current time* = C_{sk}

Step 2.3 to step 2.7 are same as in heuristic for sub-problem 2.

Steps 3 and steps 4 are not required as no job is early.

Steps 5 and steps 6 are same as in heuristic for sub-problem 2.

Next, we describe the solution procedure for solving the intermediate products scheduling problem.

4.5 Solution Procedure for Intermediate Products Scheduling Model

In this section, we develop the solution procedure for the intermediate products scheduling problem. The main difference between finished goods and intermediate products is in the production process. While the production process of finished goods resemble flowshop pattern, intermediate products are processed in a general jobshop pattern with re-entrant flows. This means that intermediate products do not have similar routes in the production process. This increases the complexity of scheduling in the flexible plant. One important consideration in the intermediate products scheduling is that there cannot be any tardiness in the schedule. This is because, intermediate products derive their due date from the schedule of higher-level products as seen in chapter 1. Based on the product structure, higher-level products are scheduled first, their schedule is translated in the requirements (due dates) of their lower level intermediate products. To maintain feasibility of the schedule of product structure, products at any level (except level 0 products, which are finished goods) cannot be tardy. Hence only earliness costs need to be minimized in the intermediate products scheduling problem. We have developed the solution algorithm for intermediate products on these lines. We describe the heuristic now to determine intermediate products schedule.

As discussed in section 4.4, we are minimizing earliness in the intermediate product scheduling, tardiness has to be zero to maintain feasibility of the schedule. At a particular level of product structure, we sort all jobs of the level on the basis of their due dates. Jobs derive their due dates from the schedule of their higher-level products. Starting from the job which has farthest due date, all operations of a job are scheduled. This way all jobs are scheduled at a particular level. Then the schedule of next lower level is considered till the last level is reached. In doing this, overlapping of jobs at a machine is avoided in following way. Figure 4.2 below shows that status of a machine that has jobs 1, 2 and 3 are already scheduled. $x1 + x2$ is the idle time between jobs 1 and 2, $x3$ is the idle time between jobs 2 and 3. d_4 is the due date of job 4 which is yet to be scheduled on this machine. If processing time of job 4 is less than $x2$, it will be scheduled as shown by dotted lines. Else it will be checked if job 4 can be scheduled between 2 and 3, i.e., if the processing time of job 4 is less than $x3$. If it is not possible to schedule job 4 in any of the two places, it would be placed before job 3 as shown in the figure below.

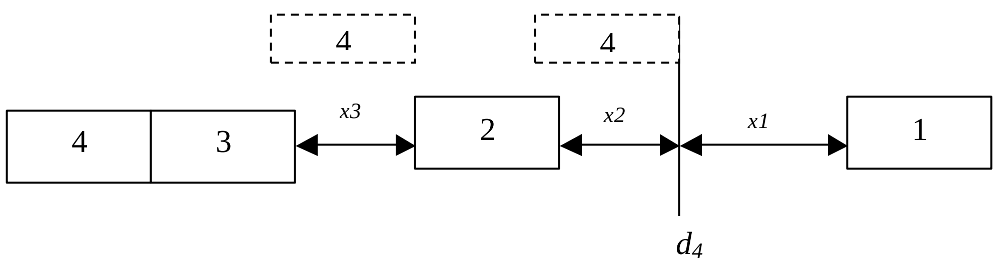

Figure 4.2: Conflict Removal at a Machine

We now provide detailed steps of the heuristic beginning with defining the parameters.

Indices
i = index of products, $i = 1,2,\ldots n$
j = index of machines, $j = 1,2,\ldots m$
E = index of operations

Sets
N = set of products, $\{i \mid i = 1,2,\ldots\ldots n\}$
EQ = set of machines, $\{e \mid e = 1,2,\ldots eqp\}$

Parameters
q_i = number of operations of product *i*, *i Î N*

p_{ij} = processing time of product i on machine j, $i \in N$

e_{ij} = machine used by product i at machine j, $i \in N$

X_i = quantity (in units) of product i to be produced as proposed by planning model, $i \in N$

I_i = inventory (in units) of product i at the beginning of the scheduling period, $i \in N$

B_i = standard batch size (in units) of product i, $i \in N$

m_i = number of batches between two setups for product i, $i \in N$

M_i = setup time (in hours) of product i, $i \in N$

r_{ik} = amount of i (in units) in one batch of product k, $i, k \in N$

P_i = set of products for which i is an input, $\{k \mid r_{ik}>0\}$, $i, k \in N$

D_i = number of due dates of product i, $i \in N$

d_{ix} = x^{th} due date of product i, $i \in N, x =1,2,\ldots D_i$

R_{ix} = Requirement of product i in x^{th} due date, $i \in N, x =1,2,\ldots D_i$

t_{ix} = time of x^{th} due date of product i at level 0, $i=1,2,3\ldots..n_0$, $x =1,2,\ldots D_i$

A_{ix} = Production quantity of product i in x^{th} due date, $i \in N, x =1,2,\ldots D_i$

n_{ix} = number of batches of product i in x^{th} due date, $i \in N, x =1,2,\ldots D_i$

int_i = number of intermediates of product i, $i \in N$

L = number of levels in the product structure i, $i \in N$

S_{abxi} = start time of a^{th} batch of b^{th} stage in x^{th} due date of product i, $i \in N, x =1,2\ldots D_i, b=1,2\ldots q_i, a=1,2,\ldots n_{ix}$.

C_{abxi} = completion time of a^{th} batch of b^{th} stage in x^{th} due date of product i, $i \in N, x=1,2\ldots D_i, b=1,2\ldots q_i, a=1,2,\ldots n_{ix}$.

c_e = number of machines scheduled on machine e, $e \in EQ$

$e1_{ec}$ = time (in hours) from which machine e available at y^{th} count, $y =1,2,..c_e+1$, $e \in EQ$

$e2_{ec}$ = time (in hours) for which machine e is available from $ef1_{ec}$ at y^{th} count, $y =1,2\ldots, c_e+1, e \in EQ$

nl_l = number of products at level l, $l = 1,2,\ldots L$.

SD_l = sorted values of products and their due dates in non-increasing order of due dates at level l, $l =1,2,\ldots L$.

Step 1 Determining production quantities after netting out inventory at all levels

for $l = 1 \text{ to } L$

for $i = 1 \text{ to } nl_l$

for $x =1 \text{ to } D_i$

Step 1.1

$$A_{ix} = \max\left\{\sum_{k=1}^{x} R_{ik} - I_i - \sum_{k=1}^{x-1} A_{ik},\ 0\right\}$$

Step 1.2 $n_{ix} = \lceil A_{ix} / B_i \rceil$

Step 1.3 Revising production quantities

$$A_{ix} = n_{ix} B_i$$

Step 2 Determining due dates of products at all levels

Step 2.1 for $l = 0$ (finished goods)

for $i = 1,2,\ldots,nl_0$

$d_{ix} = t_{ix}$, $x = 1,2,\ldots D_i$

Step 2.2 for $l = 1$ to L

for $i = 1$ to nl_l

for $x = 1$ to D_i

for $k1 = 1$ to $|A_i|$

for $k2 = 1$ to D_{k1}

for $k3 = 1$ to $n_{k1.k2}$

$$d_{i.x} = C_{k31k2k1} - p_{1k1}$$

Step 3 Sorting due dates at all levels

for $l = 0$ to L

for $i = 1$ to nl_l

for $x = 1$ to D_i

Create set $SD_l = \{u_{[k]}, v_{[k]}\}$ by sorting d_{ix} in non-increasing order. $u_{[k]}$ is the product at k^{th} position and $v_{[k]}$ is its corresponding due date.

$$|SD_l| = \sum_{i=1}^{nl_l} \sum_{x=1}^{D_i} n_{ix} \text{ , } k = 1 \text{ to } |SD_l| \text{ ,}$$

Step 4 Schedule level 0 products (finished goods) through solution procedure described in section 4.3 of this chapter.

Step 5 Scheduling the products at all levels from 1 to L

for l = 1 to L

Step 5.1 Schedule the products at all machines

for $j = 1$ to $|SD_l|$

$i = u_{[j]}$, $u_{[j]} \in SD_l$

$x = v_{[j]}$, $v_{[j]} \in SD_l$

$a = n_{ix}$,

Step 5.2 Schedule starting from the last operation

$b = q_i$,

$e = eq_{ib}$,

$c_e = 0$,

for $k = 1$ to $c_e + 1$

Step 5.2.1 Conflict checking

if ($e_c = 0$)

$C_{abxi} = d_{ix}$

if ($n = r\, m + 1$)

$S_{abxi} = C_{abxi} – n\, p_{iq} - M_i$

else

$S_{abxi} = C_{abxi} – n\, p_{iq}$

else

$x = \text{arg max}_{k}\ e1_{ek} \geq d_{ix}$

if $(e1_{ex} – e2_{ex} \leq d_{ix}$ and $d_{ix} – n\ p_{iq} – M_{i} \geq e1_{ex} – e2_{ex})$

$C_{abxi} = d_{ix}$

if $(e1_{ex} – e2_{ex} \leq d_{ix}$ and $d_{ix} – n\ p_{iq} – M < e1_{ex} – e2_{ex})$

for $y = k+1$ to $c_{e} + 1$

$z = \text{arg min}_{y}\ e2_{ey} \geq n\ p_{iq} + M_{i}$

$C_{abxi} = e1_{ez}$

if $(e1_{ex} – e2_{ex} > d_{ix})$

for $y = k+1$ to C_{e+1}

$z = \text{arg max}_{y}\ e2_{ey} \geq n\ p_{iq} + M_{i}$

$C_{abxi} = e1_{ez}$

if $(n = r\ m + 1)$

$S_{abxi} = C_{abxi} – n\ p_{iq} - M_{i}$

else

$S_{abxi} = C_{abxi} – n\ p_{iq}$

while $(a > 1)$

$a = a – 1$

$C_{abxi} = S_{a+1.b.x.i}$

if $(n = r\ m + 1)$

$S_{abxi} = C_{abxi} – n\ p_{iq} - M_{i}$

else

$S_{abxi} = C_{abxi} – n\ p_{iq}$

Step 5.3 Update machine status

$c_e = c_e + 1;$

if ($c_e = 1$)

$e1_{ce} = Ta$

$e2_{ce} = 0$

$te1_{ce} = Ta$

$te2_{ce} = 0$

for $z = 1$ to c_e

$y = \arg\max_z \; C_{nixbxi} \le e1_{ez}$

for $k = 1$ to y-1

$e1_{ek} = e1_{ek}$

$e2_{ek} = e2_{ek}$

$e1_{ey} = e1_{ey}$

$e2_{ey} = e1_{ey} - C_{nixbxi}$

$e1_{ey+1} = S_{1bxi}$

$e2_{ey+1} = te2_{ey} - n_{ix}.p_{ib} - M_i - e2_{ey}$

for $z = y+2$ to c_e+1

$e1_{ez} = te1_{ez}-1$

$e2_{ez} = te2_{ez}-1$

for $z = 1$ to c_e+1

$te1_{ez} = e1_{ez}$

$te2_{ez} = e2_{ez}$

Step 5.4 Scheduling previous operation till first operation is scheduled

$b = b - 1;$

while ($q_i > 1$)

$e = eq_{ib},$

$a = n_{ix},$

if ($p_{ib} \le p_{ib+1}$)

$$C_{n_x b x i} = C_{n_x b+1 x i} - \{n_{ix}(p_{ib+1} - p_{ib})\} + p_{ib}$$

if ($p_{ib} > p_{ib+1}$)

$$C_{n_{ix} b x i} = C_{n_{ix} b+1 x i} - p_{ib+1}$$

Go to step 5.3

If $b = 1$, STOP, else repeat Step 5.4.

Step 1 determines the number of batches of each product in one production run. The importance of this step is that it makes use of the available inventory while scheduling a product. A finished good could have many due dates, i.e., several customer orders. Orders that are in beginning of the scheduling period may be fulfilled from the inventory. However, if the entire quantity proposed by the production-planning model has to be scheduled, the availability of inventory gives the scheduler some degree of flexibility. In step 2.1, the due dates of finished good are specified. These are based on the customer orders. Finished goods have several due dates (customer orders). In step 2.2, we determine the due dates of intermediate products. At the time of determining due date of an intermediate product at a particular level, its higher-level product (where the intermediate product is input) is already scheduled. Based on this schedule, the due date of an intermediate product is determined. As in the case of finished goods, intermediate products would also have several due dates depending on how many times an intermediate product is required. In step 3, due dates are sorted in non-increasing order. Step 4 schedules the finished products according to the solution procedure described in section 4.3 of this chapter. We have to apply this procedure as many times as there are due dates of finished goods. Scheduling problem is decomposed into as many problems as there are common due dates in the scheduling horizon. In step 5, we schedule the intermediate products at all levels starting from level 1. The products are selected on the basis of sorted due dates at a level and beginning from the last operation, all operations of a product are scheduled till first operation is scheduled. It is ensured in step 5.2 that there is no overlapping of products on a machine. The explanation of this step is also provided in sub-section 4.4.1. In step 5.3, after any operation is scheduled on a machine, the availability status of the machine is updated. Step 5.4 ensures that all operations of a product are scheduled. In the next section, we describe the solution procedure for solving dedicated plants scheduling problem.

4.6 Dedicated Plant Scheduling Heuristic

There are some production plants in the production environment that produce only one type of product. These are called as dedicated production plants. We develop heuristic algorithm to schedule the products on dedicated production plants. The procedure is explained below.

Parameters

N = number of batches to be produced

q = index of machines, $q = 1, 2, 3, \ldots..k, \ldots..K$

n_q = number of machines in machine q

p_q = processing time of machine q

e = index of machines, $e = 1,2, \ldots..,E$

m = number of batches after which set-up is required

M = set-up time (in hours)

T_a = time available in a scheduling period

S_{qne} = start time of q^{th} machine of n^{th} batch on machine e

C_{qne} = completion time of q^{th} machine of n^{th} batch on machine e

<u>Dedicated Plant Scheduling Procedure (H3)</u>

Step 0 Determination of Bottleneck Operation

Let 'k' be the bottleneck operation

1 to $k-1$: Pre-bottleneck operation

$k+1$ to K: Post-Bottleneck operation

Bottleneck operation capacity:

$$k = \arg\max_{q} \{ [p_q \lceil N/n_q \rceil] + (\left\lceil \frac{\lceil N/n_q \rceil}{m} \right\rceil . M) \}$$

Step 1 Scheduling Bottleneck Operation

$Q = k$

$N = n$

$E = 1$

Step 1.1 $$C_{kne} = T_a - \sum_{q=k+1}^{K} p_q$$

Step 1.2 If $\lfloor n/nk \rfloor = r\,m+1$ for $r = 0,1,2,,,,,\lceil N/m \rceil$

Yes→ $x = 1$

No→ $x = 0$

Step 1.3 $$S_{kne} = C_{kne} - p_k - M.x$$

Step 1.4 Is $n \leq n_k$

Yes→ Check is $e = n_k$

Yes→ STOP

No→ $e = e + 1$

$n = n - e + 1$

Goto Step 1.2 and get $S_{kne} = C_{kne} - p_k - M.x$

No→ $n = n - n_k$

$C_{kne} = S_{kn+n_k e}$

Check Step 1.2 and get $S_{kne} = C_{kne} - p_k - Mx$

Goto Step 1.4

Step 2 Scheduling Pre-Bottleneck Operations

$q = k - 1$

$n = N$

$e = 1$

Step 2.1 $C_{qne} = \min(C_{q+1ne} - p_{q+1}, S_{qn+n_q e})$

Step 2.2 If $\lfloor n/n_k \rfloor = rm+1$ for $r = 0,1,2,\ldots,\lceil N/m \rceil$

Yes→ $x = 1$

No→ $x = 0$

Step 2.3 $S_{qne} = C_{qne} - p_q - Mx$

Step 2.4 Is $n = 1$

Yes→ Go to Step 2.5

No→ check is $e = n_q$

Yes→ $e = 1$

No → $e = e + 1$

$n = n - 1$

Repeat Step 2.1

Step 2.5 Is $q = 1$

Yes→ STOP

No→ $q = q - 1$

$n = N$

$e = 1$

Goto Step 2.1

Step 3.0 Scheduling Post-Bottleneck Operation

$q = k + 1$

$n = 1$

$e = 1$

Step 3.1 $C_{qne} = C_{q-1ne} + p_q$

Step 3.2 If $\lfloor n/n_k \rfloor = rm+1$ for $r = 0,1,2,,,,,\lceil N/m \rceil$

Yes→ $x = 1$

No→ $x = 0$

Step 3.3 $S_{qne} = C_{qne} - p_q - Mx$

Step 3.4 $d = S_{qne} - C_{qn-1e}$

Step 3.5 Is d ³ 0

Yes→ Check if $n = N$

Yes→ Goto Step 3.6

No→ Is $e = n_k$

Yes→ $e = 1$

No→ $e = e + 1$

$n = n + 1$

Repeat Step 3.1

No→ $S_{qne} = C_{qn-1e}$

Check Step 3.2 and get $C_{qne} = S_{qne} + p_q + Mx$

Repeat Step 3.4

Step 3.6 Is $q \neq K$

Yes→ $q = q + 1$

$n = 1$

$e = 1$

Repeat Step 3.1

No→ STOP

Step 0 determines the bottleneck operation in the dedicated production plant. There are several machines available for an operation. The bottleneck operation is the operation with maximum sum of processing time and setup time required for a product amongst all operation. Step 1 schedules the bottleneck operation. Step 2 and step 3 schedule the pre-bottleneck operations and post-bottleneck operations respectively.

4.7 Summary

In this chapter, we developed solution procedures for solving the production planning and scheduling problem. We use branch and bound algorithm to solve the production-planning problem. We have two models for scheduling problem, one of the finished good scheduling problem and the other model of intermediate products scheduling problem. The finished goods scheduling problem can be decomposed into three sub-problems based on the value of common due date. The three sub-problems are called as flowshop E/T problems with unrestricted due date, intermediate due date and restricted due date respectively. Due date is common for all jobs in all three sub-problems. We derive analytical results and obtain optimal schedule of sub-problem 1. For sub-problem 2, we develop a heuristic algorithm and derive permutation flowshop sequence. We derive an analytical result for solving sub-problem 3 in the restricted due date range. We also propose heuristic algorithm for obtaining permutation flowshop sequence for sub-problem 3. In the next chapter, we report computational results of the solution procedure for production planning and scheduling problem.

5 Results of Production Planning and Scheduling Problem

5.1 Introduction

In this chapter, we provide the results of solution procedures used for solving the production planning and scheduling problem. We also report the sensitivity analysis on the results. The data for studying the results of production planning and scheduling problem, is provided by a pharmaceutical company in India.

The solution procedures for production planning and scheduling problems were described in chapter 4. We solve the production-planning problem using the branch and bound algorithm from a commercial solver. We develop analytical results for sub-problem 1 of finished goods scheduling problem. Before applying the solution procedure to the overall production planning and scheduling problem, we test the performance of heuristic algorithms for solving sub-problems 2 and 3 on some benchmark problems in literature on flowshop scheduling.

The rest of this chapter is organized as follows. We have the optimal solution for sub-problem 1 in chapter 4. In the next section, we describe the experiment design and lower bound of sub-problem 2, and computational performance of heuristic algorithms for solving sub-problem 2. In section 5.3, we discuss the lower bound of sub-problems 3, some of the existing heuristic algorithms for solving sub-problem 3 and computational performance of the proposed heuristics for solving sub-problem 3. In section 5.4, we study the results of production planning and scheduling problem. The summary of this chapter is provided in section 5.6. We begin by studying the results of sub-problem 2 in the next section.

5.2 Results of Sub Problem 2

Sub-problem 2 is the flowshop E/T problem with *intermediate* common due date, i.e., problems where the due date falls in between restricted and unrestricted due dates for flowshop problems. In this section, we describe a valid lower bound of sub-problem 2. We also describe the experiment design to test the computation performance of the heuristic. Subsequently, we discuss the results of the solution procedure for sub-problem 2.

5.2.1 Lower Bound of Sub Problem 2

In this section, we develop the lower bound of sub-problem 2. Our objective is to get a valid lower bound of a job on its earliness and tardiness. We begin with some definitions.

Notation

i =	index of jobs,	$i = 1,2,\ldots n$.
j =	index of machines,	$j = 1,2\ldots.m$.
d =	common due date of all jobs	
p_{ij} =	processing time of job i on machine j	
$O_j(i)$ =	sum of i shortest processing times on machine j amongst all jobs	
LBC_i =	lower bound on the completion time of job i on machine m.	
C_{im} =	completion time of job i on machine m	
$LBET_i$ =	lower bound on earliness and tardiness of job i	

In a permutation flowshop, the completion time of the i^{th} job on the last stage m, *i.e.*, (LBC_i) of any sequence is not less than $\max_{1 \le j \le m}\left\{O_j(i) + \min_i \sum_{l=1}^{m} p_{il} - \min_i p_{ij}\right\}$. $O_j(i)$ is a lower bound on the time needed to process i jobs on machine j. Therefore, C_{im} is not less than the sum of $O_j(i)$ and the minimum processing times among all jobs on machine 1 through m except machine j. Since this is true for all machines, the LBC_i is a valid lower bound on completion time of i^{th} job on last machine of any sequence. LBC_i is provided by Kim (1995). The lower bound on earliness and tardiness of job i is given by: $LBET_i = \max\{d - LBC_i, 0\} +$

max{LBC_i –d, 0}. The first sum is the lower bound on earliness, and the second sum is lower bound on tardiness. It is difficult to determine the lower bound on earliness. Hence, we consider $LBET_i$ = max{$LBCi$ –d, 0}. Next, we describe the experiment design of sub-problem 2.

5.2.2 Experiment Design of Sub Problem 2

The procedures described in the heuristic solution of sub-problem 2 are applied to benchmark problems in the literature on flowshop scheduling (Taillard, 1993). The parameters used in the experiments are shown in the table 5.1 below.

Number of jobs *n*	*n = 5, 10, 20, 50, 80, 100*
Number of machines *m*	*m = 5, 10, 15, 20*
Number of instances *I* of test problems	*I = 50*
Processing time of a job on a machine in each instance.	Random number uniformly distribution between 1 and 99.
Number of tabu iterations	50, 60, 70, 80
Tabu tenure	Random number between 5 and 10

Table 5.1: Parameters in Experiment Design of Sub-Problem 2

For small problems, optimal solution is obtained using Branch and Bound algorithm from a commercial solver. The performance of the heuristic for small problems is compared with optimal solution. For large problems, the heuristic solution is compared with the lower bound. The performance measure (P_H) used for the heuristic is 'Average percentage deviation from the optimal solution in small problems, and lower bound in large problems. '.

We define,

Z_{HI}: Objective value of heuristic solution of instance *I*

Z_{OI}: Objective value of optimal solution of instance *I*

Z_{LBI}: Lower bound of the instance *I*

For smaller problems ($n = 5, 10; m = 5$)

$$P_H = \frac{1}{I}\left(\sum_I \frac{Z_{HI} - Z_{OI}}{Z_{OI}}\right)100$$

For large problems ($n > 10$)

$$P_H = \frac{1}{I}\left(\sum_I \frac{Z_{HI} - Z_{LBI}}{Z_{LBI}}\right)100$$

LBC_i is a weak lower bound (Kim, 1995). As mentioned above, it is difficult to estimate the lower bound on earliness. Thus, $LBET_i$ is a very weak lower bound on earliness and tardiness. This is verified for small problems ($n = 5, 10; m = 5$), as the average percentage deviation of optimal solution from the lower bound is found to very high. In case of n = 5;m = 5; 50 instances, average percentage deviation of optimal solution from lower bound is 326 percent and in case of n = 10; m = 5, it is found to be 284 percent. The average percentage deviation of heuristic solution from lower bound for small and large problems for 5-machines problem is shown in figure 5.1 and for 10-machines problem in figure 5.2. The deviation is again high but this is expected, as the deviation of lower bound is high from optimal solution itself. Since both heuristic solution and optimal solution deviate by almost same percentage from the lower bound for smaller problems, it is obvious that, at least for small problems, heuristic solution and optimal solution are close to each other. For (n = 5, 10; m =5), the average percentage deviation of optimal solution from heuristic solution is 0.894 percent and 1.126 percent for 5 jobs and 10 jobs respectively. The common due date considered for this analysis is $d = (d_1+d_2)/2$. The observations are encouraging for measuring heuristic performance, as the optimal solution also has large deviation from the lower bound.

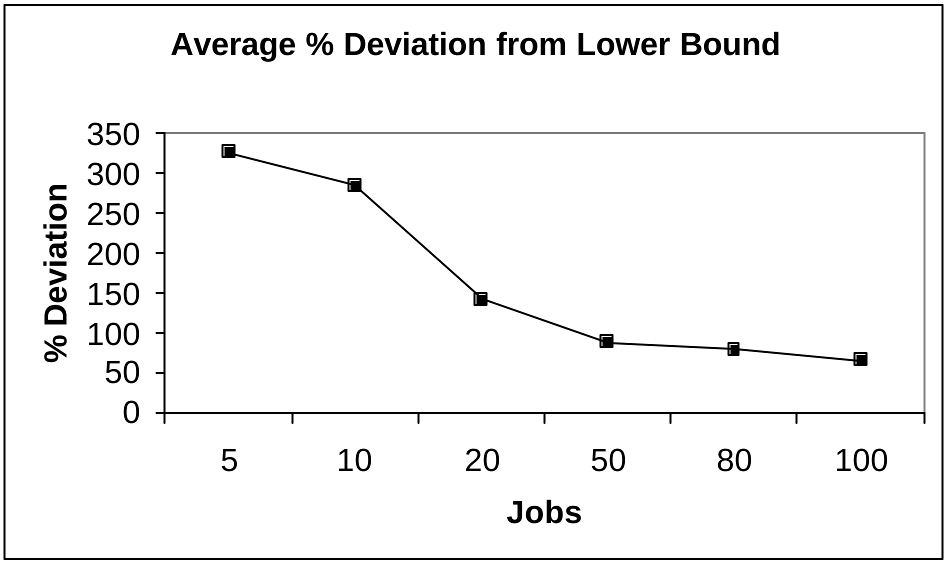

Figure 5.1: Average % Deviation of Heuristic Solution from Lower Bound: 5 Machines

The performance of the heuristic for smaller problems is also compared with optimal solution with a random common due date between d_1 and d_2. This is done to evaluate the quality of heuristic solution in the entire range of intermediate due date. The results of n = 5;m = 5; 50 instances with random due date between d_1 and d_2 were 0.846 percent average deviation of heuristic solution from the optimal solution. For of n = 10;m = 5; 50 instances, the average deviation of heuristic solution from the optimal solution is 1.247 percent.

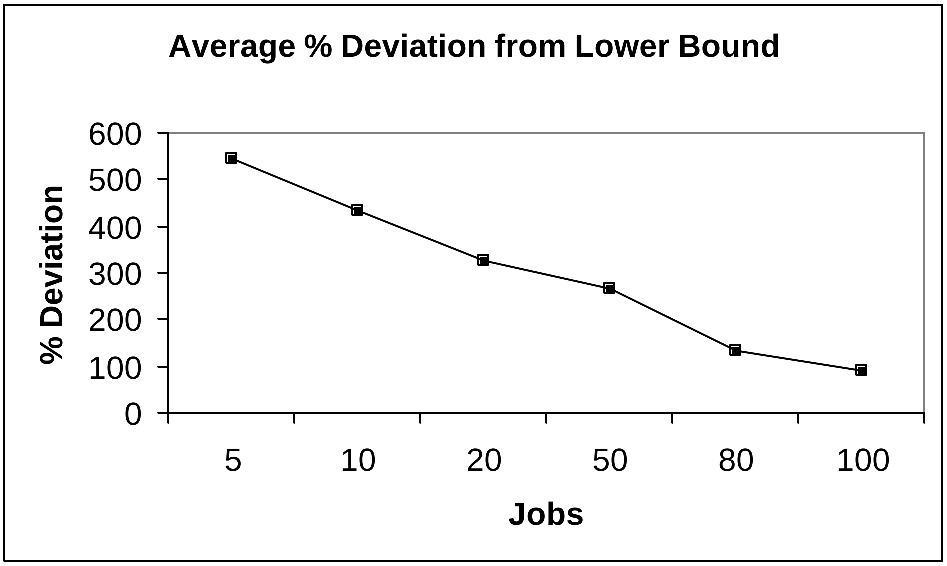

Figure 5.2: Average of Deviation of Heuristic Solution from Lower Bound: 10 Machines

As discussed above that the lower bound of sub-problem 2 is very weak, the performance measure of the heuristic for larger problems is tested for common due date value d_1(obtained in sub problem 1). This is because we have optimal solution of flowshop E/T problem for common due date d_1, obtainable in polynomial time. The results of this comparison are indicated in table 5.2. The results in table 5.2 indicate the average percentage deviation of optimal solution at $d = d_1$ from the heuristic solution. Each job and machine combination discussed in the experiment design is shown in table 5.2. The results of table 5.2 indicate that the performance of heuristic H1 is good, as the maximum average percent deviation of the optimal solution from lower bound is found to be 1.744 percent. The results in table 5.2 indicate that the average percentage deviation of jobs for a particular machine follow a non-linear pattern. This is indicated for 5-machine problem in figure 5.3. The non-linear pattern is observed for m = 10, 15 and 20 also.

	Machines			
Jobs	**5**	**10**	**15**	**20**
5	0.000	0.000	0.235	0.000
10	0.084	0.081	0.099	0.276
20	0.074	0.020	0.012	0.023
50	0.323	0.153	0.152	0.146
80	0.865	0.642	0.617	0.644
100	1.744	1.168	1.175	1.129

Table 5.2: Average Percentage of Deviation of Optimal Solution from Heuristic Solution

As it is seen in the figure 5.3, with increase in the number of jobs, the average percentage deviation follows a square ordered pattern. The square root of the average percentage deviation follows a linear pattern. These results are with 50 tabu iterations in each of the 50 instances solved for a particular job-machine combination

Figure 5.3: Average % Deviation from Optimal Solution and its Square Root

When number of tabu iterations is increased, the results improve as the average percentage deviation is reducing. This however, would increase the computational time to solve the problem. The improvement in results with increase in number of tabu iterations is shown in figure 5.4 for $n = 50, m = 5$. As seen in figure 5.4, the solution at 100 tabu iterations is around 70 percent better than the solution at 50 tabu iterations.

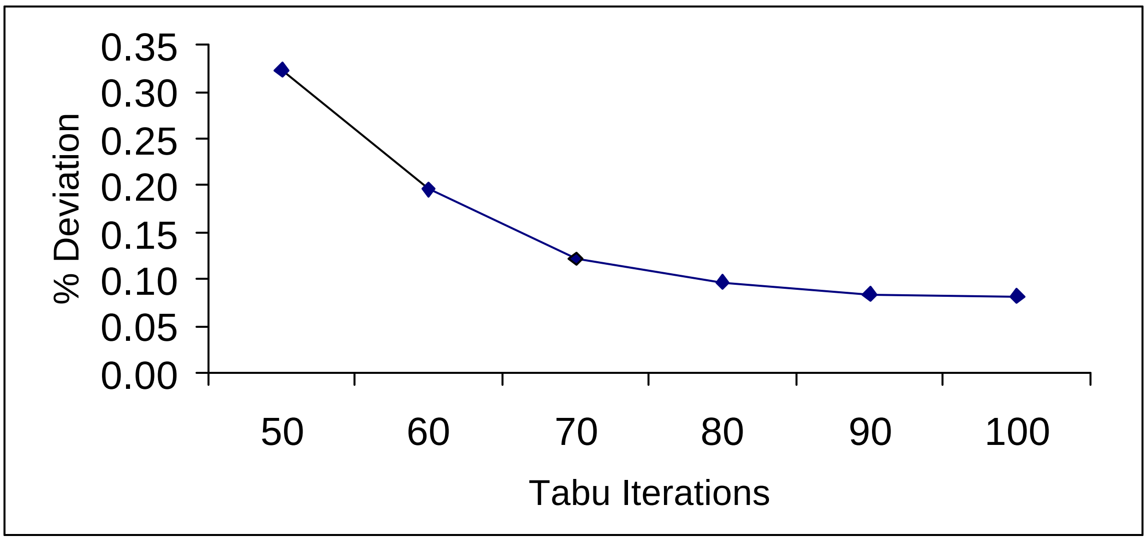

Figure 5.4: Improvement in the solution with Increase in Number of Tabu Iterations

5-machine case is analyzed in detail to observe the pattern of the results. At 100 tabu iterations, the average percentage deviation follows an almost linear pattern as compared to 50-tabu iterations. This phenomenon is shown in figure 5.5. The figure indicates for m = 5, and n = 5, 10, 20, 50, 80 and 100, the average percentage deviation of heuristic solution from optimal solution for 50 and 100 tabu iterations. In the next section, we discuss the results of sub problem 3.

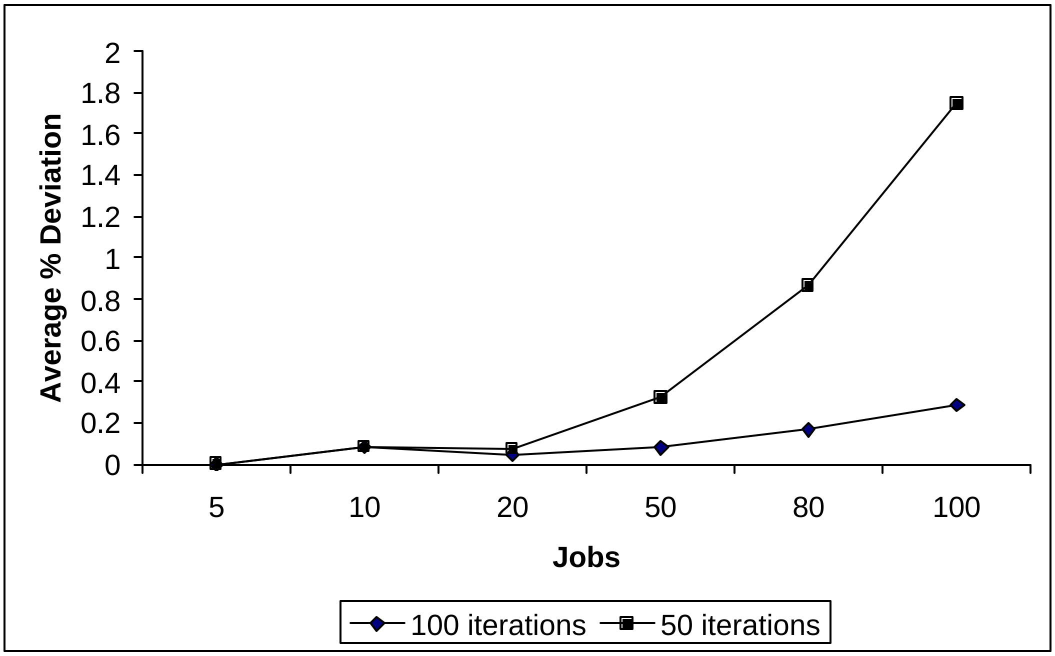

Figure 5.5: Comparison of Results with Different Tabu Iterations

5.3 Results of Sub Problem 3

In this section, we discuss the results of flowshop E/T problem with restricted common due date, i.e., $d < d_2$. The special structure of sub-problem 3 was discussed in chapter 4. The objective of this problem is to minimize earliness and tardiness. Because of the common due date and the property that no job is early, the objective of the problem is same as that of minimizing flowtime and minimizing completion time. As a result, we use one of the better-known lower bounds in literature, of flowshop completion time problem, as the lower bound of sub-problem 3. Lower bound of flowshop completion time problem is due to Ahmadi and Bagchi (1990). We describe this lower bound in the next sub-section.

5.3.1 Lower Bound of Sub Problem 3 (Ahmadi and Bagchi, 1990)

Notation

N =	set of n jobs, $\{i \mid i = 1,2, \ldots,n\}$.
M =	set of m machines in a flowshop, $\{j \mid j = 1,2,\ldots, m\}$
p_{ij} =	processing time of job i on machine j
π =	set of r jobs constituting a partial schedule which specifies completion times of the jobs in π on all machines
π' =	set of n-r jobs such that $\pi'_r = N - \pi_r$
$C_{\pi.j}$ =	completion time of the partial schedule π on machine j, or the earliest time machine j is available for processing a job in π'
C_{ij} =	completion time of job i on machine j
σ =	a complete sequence of n jobs
C_σ =	sum of completion times on last machine m of all jobs in σ

C_σ can be written as:

$$C_\sigma = \sum_{i \in \pi} C_{im} + \sum_{i \in \pi'} C_{im} \qquad (1)$$

The first sum on the right hand side in (1) is a constant. The optimal value of the second sum is the solution to the following mathematical programming problem P1 where the minimization is taken over V, the set of all possible sequences of the jobs in π':

Let $C_{i0} = 0$ for all $i \in N$, and $[i]$ is the job in the i^{th} position in a sequence, and $C_{[0].j} = 0$ for all $j \in M$.

Problem P1 $\quad \min_{V} \sum_{i \in \pi'} C_{im}$

s.t.

$$C_{ij} \geq C_{\pi j} + p_{ij}, \quad i \in \pi', \; j \in M \qquad (2)$$

$$C_{ij} \geq C_{ij-1} + p_{ij}, \quad i \in \pi', \; j \in M \qquad (3)$$

$$C_{[i]j} \geq C_{[i-1]j} + p_{[i]j}, \quad i=r+1, r+2,\ldots,n, \; j \in M \qquad (4)$$

Consider any one machine, say machine s, and let $M' = \{j \in M \mid j < s\}$ and $M'' = \{j \in M \mid j > s\}$. Furthermore, let $i_1, i_2, \ldots, i_{n-r}$ denote a permutation of the n-r jobs in p' such that

$p_{i1j} \leq p_{i2j} \leq \ldots \leq p_{in-rj}.$

Clearly for any complete schedule we have,

$$\sum_{i \in p'} C_{im} \geq \sum_{i \in p'} C_{is} + \sum_{x=1}^{x=n-r} \sum_{k=s+1}^{m} p_{i_x k} \qquad (5)$$

Consider now the following problem P2:

Problem P2

$$\min_{V} \sum_{i \in p'} C_{is} + \sum_{x=1}^{n-r} \sum_{k=s+1}^{m} p_{i_x.k}$$

Subject to:

$$C_{ij} \geq C_{pj} + p_{ij}, \quad i \in p', \ j \in M - M'' \qquad (6)$$

$$C_{ij} \geq C_{pj} + p_{ij}, \quad i \in p', \ j \in M'' \qquad (7)$$

$$C_{ij} \geq C_{ij-1} + p_{ij}, \quad i \in p', \ j \in M - M'' \qquad (8)$$

$$C_{ij} \geq C_{ij-1} + p_{ij}, \quad i \in p', \ j \in M'' \qquad (9)$$

$$C_{[i]j} \geq C_{[i-1]j} + p_{ij} \quad i = r+1, r+2, \ldots. n, \ j \in M' \qquad (10)$$

$$C_{[i]s} \geq C_{[i-1]s} + p_{is}, \quad i = r+1, r+2, \ldots. n \qquad (11)$$

The constraints of problems (P1) and (P2) are identical. It follows from (5) that the optimal solution to (P2) is a lower bound on the optimal solution to (P1). Suppose that in (P2), constraints (7), (9), (10) and (12) are relaxed and the constraints (6) and (8) are replaced by the following constraint:

$$C_{is} \geq EST(p', i, s) + p_{is}, \quad i \in p' \qquad (13)$$

where,

$$EST(p',i,s) = \max\left[C_{ps}, \max_{1\le k\le s-1}\left\{C_{pk} + \sum_{x=k}^{x=s-1} p_{ix}\right\}\right] \quad (14)$$

The resulting problem P3 is:

$$\min_{V} \sum_{i\in p'} C_{i.s} + \sum_{x=1}^{n-r} \sum_{k=s+1}^{m} p_{ix.k} \quad (15)$$

subject to (11) and (13).

Clearly, the optimal solution to (P3) is a lower bound on (P2). Problem (P3) is NP-Hard and is single machine problem of minimizing the sum of completion times subject to release times. The second sum in the objective function of (P3) is a constant, and the release time of job i Î p' is given by $EST(p', i, j)$. If pre-emption is allowed, (P3) can be optimally solved by the shortest remaining processing time (SRPT) rule.

Let the objective value of P3 with SRPT schedule be z^s. It follows that z^s is a lower bound on problems P3, P2 and P1. The overall lower bound on P1 is: $\max(z^1, z^2, \ldots\ldots z^m)$.

5.3.2 Existing Results of Sub Problem 3

There are several results in the literature on flowshop problems with an objective of minimizing tardiness, flowtime or completion time of jobs. We apply the best results of these problems on the instances generated from our experiment design, and compare the solution of the existing heuristics with our heuristic. We consider following three heuristics existing in the literature:

1. NEH Nawaz et al. (1983)

2. RZ Rajendran and Ziegler (1997)

3. WY Woo and Yim (1998)

We determine average percentage deviation from lower bound on each of the three heuristics. On the same instances we test our heuristic (H2), which was described in chapter 4. We also propose two more heuristics by applying tabu search procedure on heuristics RZ and WY. These heuristics are RZT and WYT. Heuristic NEH is not considered for tabu search, as its performance was worse than other existing heuristics. Figures 5.6 to 5.9 below shows the comparison of performance measure of existing heuristics and the proposed heuristics for various jobs and machine combinations.

Figure 5.6 indicates the comparison of three existing heuristics and three new heuristics developed to solve sub-problem 3. As seen in figure 5.6, the average percentage deviation of heuristic solution is compared for jobs (n = 5, 10, 20, 50, 80, 100) and 5 machines. Similar jobs are considered for m = 10, m = 15 and m = 20 in subsequent figures. The average percentage deviation of heuristic solution from lower bound for all jobs is minimum in H2, the heuristic we developed in chapter 4. For higher number of machines also (figures 5.7 to 5.9), it is seen that H2 is performing better. In all the heuristics, the average percentage deviation from lower bound increases with the number of jobs.

It is seen in all the cases that heuristic H2 performs better than the existing heuristics. The average percentage deviation from lower bound is minimum for H2. Other heuristics on which tabu is performed (WYT, RZT) also perform close to H2. In the next section, we discuss the results of the production planning and scheduling problem.

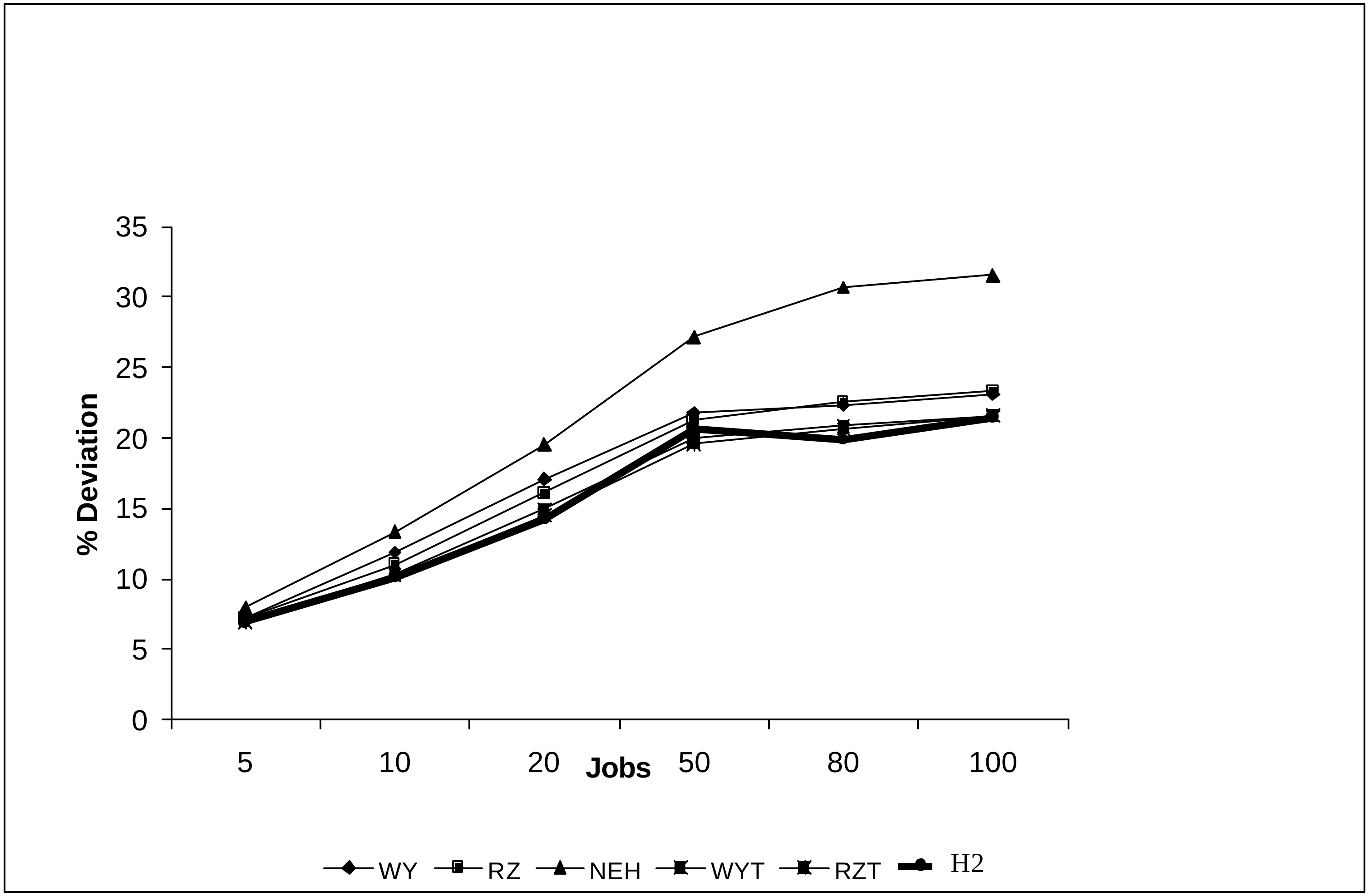

Figure 5.6: Average % Deviation of Heuristic Solution from Lower Bound: 5 Machines, Sub-Problem 3

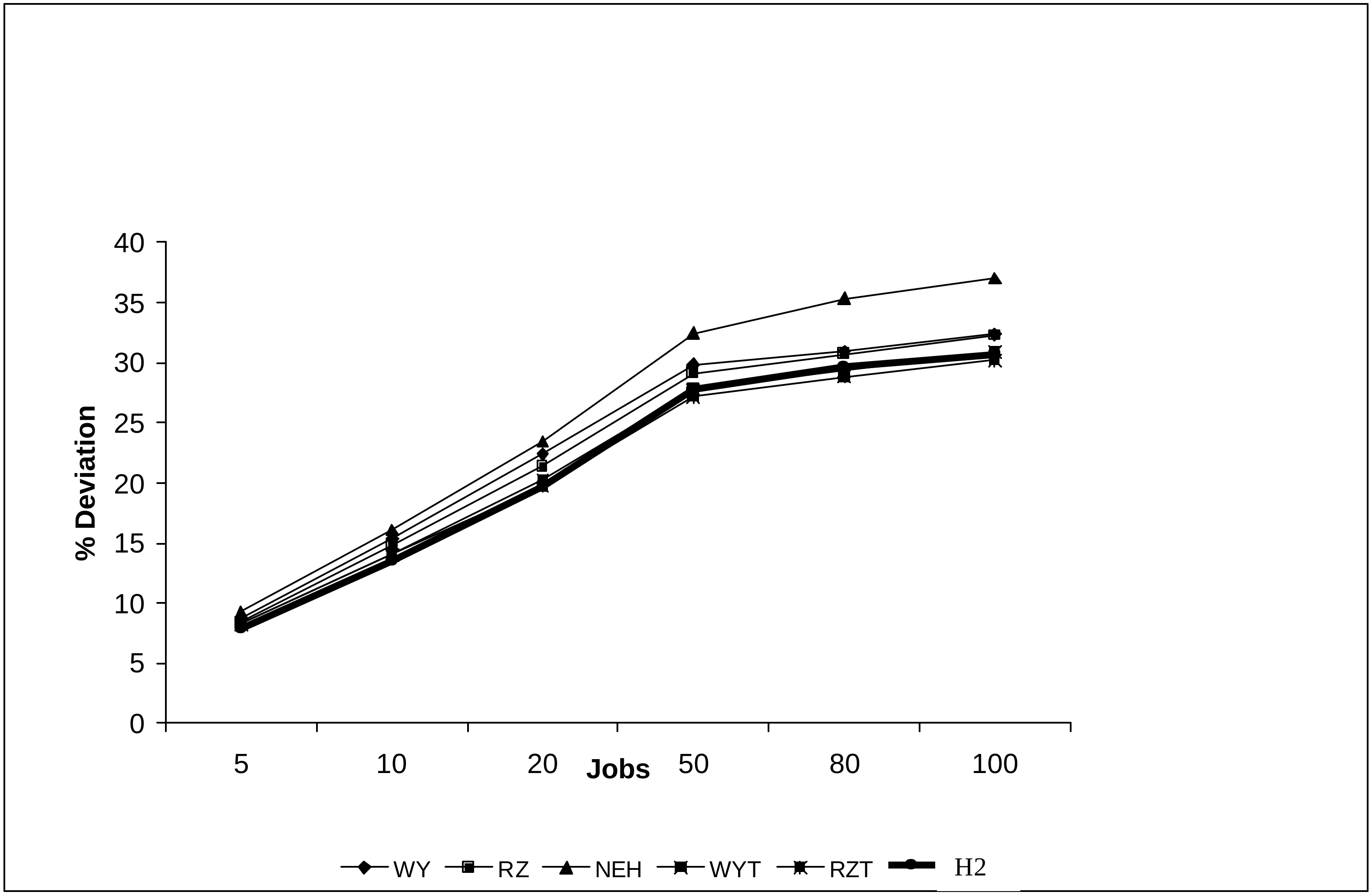

Figure 5.7: Average % Deviation of Heuristic Solution from Lower Bound: 10 Machines, Sub-Problem 3

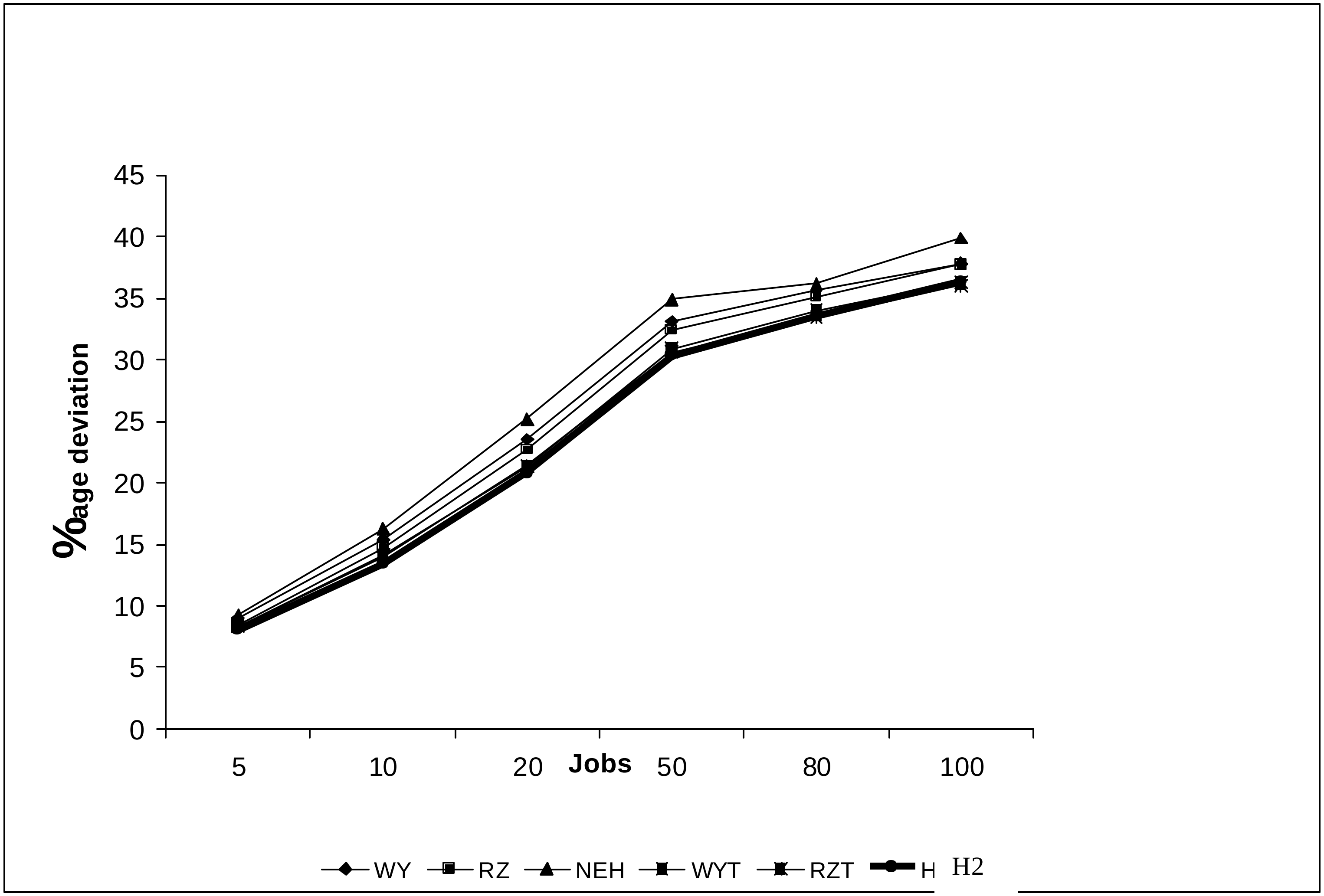

Figure 5.8: Average % Deviation of Heuristic Solution from Lower Bound: 15 Machines, Sub-Problem 3

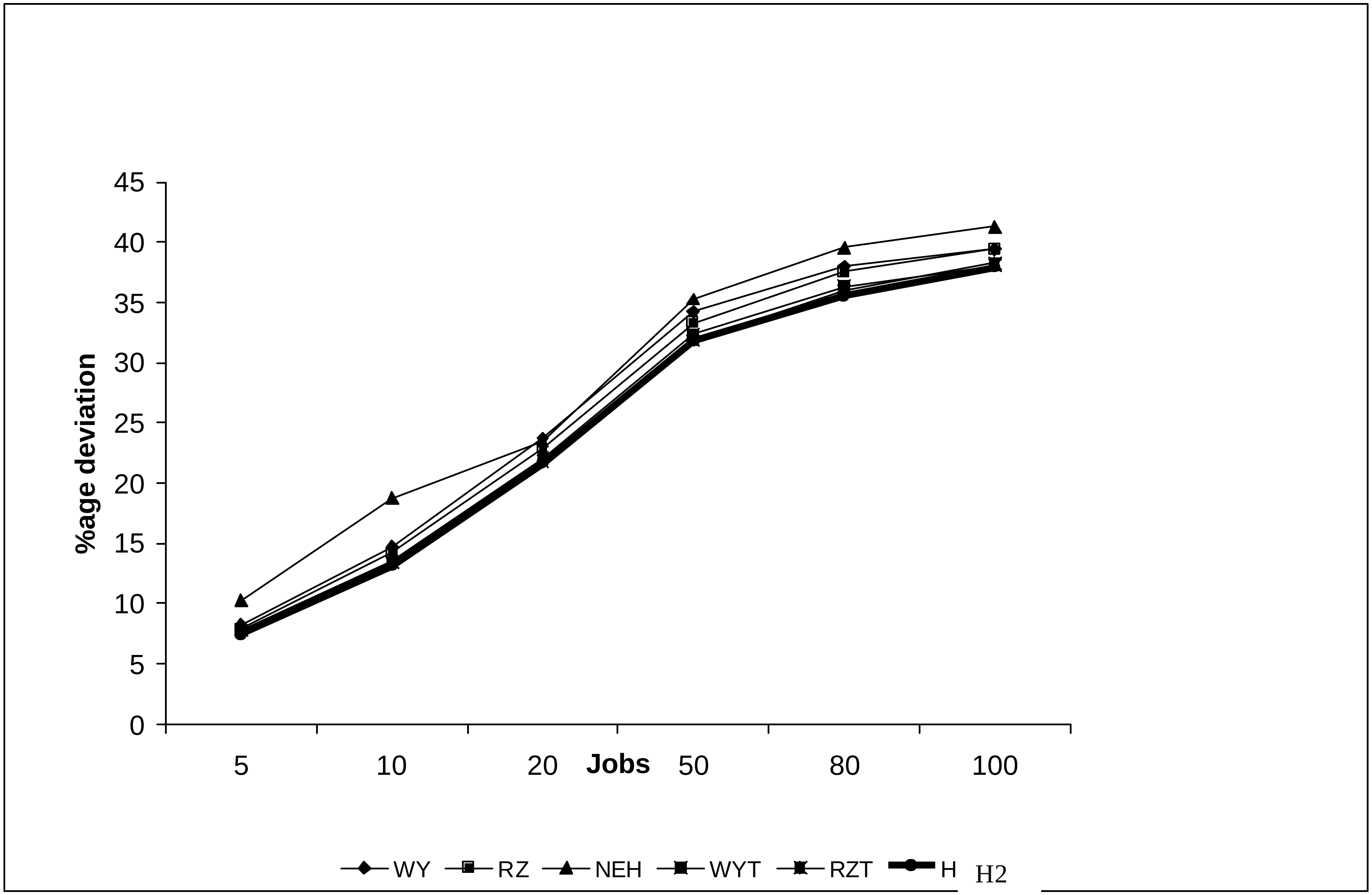

Figure 5.9: Average % Deviation of Heuristic Solution from Lower Bound: 20 Machines, Sub-Problem 3

5.4 Production Planning and Scheduling Results

In this section, we study the results of production planning and scheduling problem. The data for solving the problem is provided by a pharmaceutical company in India. The company has multi-stage, multi-product, multi-machine, batch processing environment. The problem instance solved with 5-month data has 10 finished products, 30 intermediate products, 50 by-products and 40 reusable raw materials. There are 15 production plants in this instance. Out of 15 production plants, 8 plants are dedicated production plants and remaining seven are flexible production plants. In appendix 1, product structure diagram (panel A) and process flow diagrams (panel B) of each product are shown. The instance solved is called as the 'base case'.

The results of the production-planning model are for monthly time period of the 5-month planning horizon. The decisions obtained are production quantities of finished goods and intermediate products, number of setups of finished goods and intermediate products, and inventory levels of finished goods and intermediate products. For illustration, table 5.2 below shows the production quantity of finished goods in each time of the planning horizon. Column 'Product' in table 5.3 indicates the finished goods and column 'Plant' indicates the corresponding production plants of finished goods. Remaining five columns indicate the production quantities of finished goods in each time period (1,2,…5) of the planning horizon. Details of the entire production plan and schedule of this base case are provided in appendix 2. For each time period of the planning horizon, appendix 3 consists of production quantities and number of setups of finished goods and intermediate products (panel A), inventory levels of finished goods and intermediate products (panel B), capacity utilization of dedicated plants (panel C), capacity utilization of flexible machines (panel D), and schedule of the plants (panel E). Production planning model gives the total cost of the production plan. Scheduling results are the start time and completion time of each product on each machine in each time period of the planning horizon. The overall production planning and scheduling costs in he instance solved are Rupees 54,127,000.

		Time Period				
Product	Plant	1	2	3	4	5
E1	1	13720.00	27000.00	27000.00	27000.00	27000.00
E1	2	0.00	0.00	0.00	0.00	0.00
E2	3	1292.13	7000.00	7000.00	7000.00	7000.00
E3	4	661.63	2464.93	0.00	2173.44	1820.00
E4	5	0.00	1256.39	4053.26	776.05	1723.95
E5	6	1474.15	529.79	1024.50	947.57	0.00
E6	7	0.00	0.00	0.00	0.00	740.00
E7	8	2710.00	2200.00	0.00	1785.58	3801.04
E7	9	0.00	0.00	7949.42	8200.00	6198.96
E8	10	168.29	556.22	1100.00	1000.00	1000.00

Table 5.3: Production Plan of Finished Goods

We also perform sensitivity analysis on the base case results of production planning and scheduling problem. We report the results on sensitivity analysis in chapter 6, where we describe a case study of application of production planning and scheduling models. Next, we provide summary of this chapter.

5.5 Summary

In this chapter, we discussed the computational performance of the heuristic algorithms used for solving the production planning and scheduling problem. We discussed in chapter 4 that the finished goods flowshop E/T problem can be decomposed in three sub-problems on the basis of common due dates. We have reported optimal solution for flowshop E/T problem with unrestricted due date (sub-problem 1) in chapter 4. Heuristic algorithms were reported for flowshop E/T problem with intermediate due date (sub-problem 2). We described the experiment design of testing the computational performance of the heuristic algorithm in this chapter. We also described a valid lower bound of sub-problem 2 and discussed the quality of the lower bound. We discussed that the lower bound of sub-problem 2 is very weak. The optimal solution of sub-problem 2 was obtained for small problems (n=5, 10; m=5) using branch and bound algorithm. We tested the computational performance of the heuristic algorithm for sub-problem 2 by determining the average percentage deviation of heuristic solution from optimal solution. In small problems, for $d = (d_1+d_2)/2$, the average percentage deviation of heuristic solution from optimal solution is 0.894 percent (n =5, m = 5) and 1.126 percent (n =10, m = 5). For a random due date between d_1 and d_2, the average percentage deviation of heuristic solution from optimal solution is 0.846 percent. We also obtained, for large problems, optimal solution of sub-problem 2 at $d = d_1$, from analytical results of sub-problem 1. The heuristic of sub-problem 2 is compared with optimal solution at $d = d_1$. The heuristic solution obtained is very close to the optimal solution at $d = d_1$.

We also developed a valid lower bound for flowshop E/T problem for restricted due date (sub-problem 3). The computational performance of the proposed heuristic algorithm for sub-problem 3 was compared with some of the exiting heuristic algorithms of sub-problem 3. The average percentage deviation of heuristic solution from lower bound is found to be better in our heuristic as compared to the existing heuristics.

We have reported results of the production planning and scheduling problem. We studied the production plan of the production planning problem and machine wise schedules of the scheduling problem. The data for solving the problem is from a pharmaceutical company in India.

In the next chapter, we apply the production planning and scheduling models to a pharmaceutical company in India. We discuss the results of the solution procedure used to solve production planning and scheduling problem in this application. We also provide sensitivity analysis on the results.

6 Case Study: Application of Production Planning and Scheduling Models

6.1 Introduction

In this chapter, we apply the production planning and scheduling models developed in chapter 3 to a real life application. The models are applied to a pharmaceutical company in India. The company was facing the problem of excess inventories, stockouts and low capacity utilization in order to meet the demand forecast. Demand fluctuation of the products was resulting in frequent changes in production plans and schedules on the shop floor. Also the process to change the products schedule to satisfy changing marketing requirements was time consuming and tedious. We develop a decision support system to solve the production planning and scheduling problem of this company.

The plan of this chapter is as follows. We briefly describe the production planning and scheduling problem in this application in section 6.2. We solve the production planning and scheduling problem in two steps, as discussed earlier in chapter 3. First, we solve the production-planning problem. The computational results of solving the production-planning problem are described in section 6.3. In section 6.4, we develop a variant of the production-planning model with additional market constraints. This model is used for jointly planning sales and production. We discuss results of sales and production planning model in this section. In section 6.5, we solve the scheduling problem. We apply the solution procedure of scheduling problem developed in chapter 4. The results of the application are discussed in this section. To provide some managerial insights from the application of production planning and scheduling models, we provide sensitivity analysis on results in section 6.6. We discuss some implementation issues in section 6.7. The summary of this chapter is provided in section 6.8.

6.2 Production Planning and Scheduling Problem

In this section, we describe the production planning and scheduling problem in this application. We first discuss the production environment, and then we discuss the decisions of the production planning and scheduling problem.

Chapter 1 provides the detailed description of the terminology used here in the production environment. The environment in this application is multi-product, multi-machine, multi-stage batch production. The environment produces finished goods and intermediate products. The production stage in the environment corresponds to production of an intermediate product or finished good. As shown in figure 6.1 below, there is a multi-level product structure, where a level is equivalent to production stage.

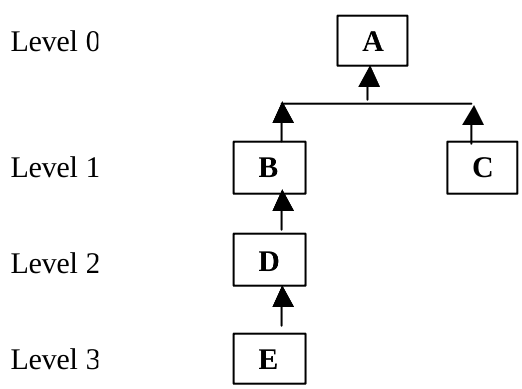

Figure 6.1: Multi-Level Product Structure

Finished goods (product A) are at higher level (level 0) followed by intermediate products (B, C, D, E) at different levels. Finished goods and intermediate products are produced in production plants. Each production plant comprises number of machines. Intermediate products and finished goods are processed on machines in a specific, pre-determined sequence, called as route. Machines are shared in plants producing intermediate products and finished goods. There are dedicated plants also in the production process. These plants produce only one type of product. Intermediate products are stored as work-in-process inventory.

By-products are generated from intermediate products and finished goods. By-products are recycled in recycling plants to extract reusable raw materials. The outputs of a production plant are intermediate products, finished goods and by-products. Inputs to a production plant are fresh raw materials, reusable raw materials and intermediate products.

Prior to this study, production planning in the company was done on the basis of the annual demand of the finished goods. Demand is combination of firm orders and forecast. The production planning method used is similar to the Material Requirements Planning (MRP) structure. Master schedule for end products is generated first and it is exploded to determine the intermediate products and raw material requirements A production schedule is then derived manually based on availability of raw materials and machines, raw material procurement and manufacturing lead times. Production plans are made with an objective of maximizing the capacity utilization of machines. This often results in high inventory of intermediate and end products. The production schedule is forced to undergo frequent changes due to demand variability, raw materials unavailability, shop-floor uncertainties like machine breakdowns etc.

Production planning and scheduling problem is to determine the decisions at minimum cost. We developed a decision support system in order to derive production planning and scheduling decisions and manage the above-mentioned complexities. We have discussed the production planning and scheduling decisions in detail in chapter 1. We would briefly revisit them as we are implementing the models in this application. In the next section, we describe the application of production planning model and its results.

6.3 Application of Production Planning Model

We model the production environment described in section 6.1 in two steps. In the first step, we develop a mixed integer linear programming production-planning model. Demand is forecast over the planning horizon. Aggregate available capacity of dedicated plants and shared machines in flexible plants is considered in the planning model. For plants where there is no sharing of machines, monthly available capacity of plant is considered. For plants where machines are shared by multiple products, machine wise monthly available capacity is considered. The decisions of the planning model are:

- Quantity of each product to be produced on each plant in each time period of the planning horizon
- Inventory levels of end products, intermediate products, solvents and by-products in each time period of the planning horizon
- Quantity of fresh raw material consumed in each time period of the planning horizon.

The planning model would also determine the capacity utilization of each plant and machine in each time period.

In the second step, we develop scheduling model for detailed machine wise scheduling decisions in each time period. Scheduling decisions comprise of start time and completion time of each product on each machine. The decisions of the planning model impose constraints within which the decisions for detailed scheduling are taken. Application of scheduling model is described in the next section.

The production-planning problem is solved using the branch and bound algorithm. The production-planning model is developed in GAMS modeling system and the branch and bound algorithm is applied from CPLEX solver. We report the results of the application of production-planning model in the sub-section below.

6.3.1 Results of Production Planning Model

Now, we compare the cost of actual production plan developed by the company against the production plan proposed by the cost minimization model (production-planning model) for a given period. The size of problem instance solved in this application is as follows. There are 10 finished goods, 30 intermediate products, 50 by-products and 40 reusable raw materials. There are 15 production plants, 8 dedicated plants, and 7 flexible plants. The planning model in this instance has 576 discrete variables, 5974 continuous variables and 3016 constraints. For solving the problem, 5-month data from January 2002 to May 2002 is considered. The unit of time period in the five-month planning horizon is one month. The execution time in this instance on a Pentium 4, 1.6 GHz workstation is 1240 seconds.

Actual production plan of the company to meet firm orders and demand forecast, from January 2002 to May 2002, is considered for comparing the model results. The production plan and schedule of this instance (base case) was discussed in chapter 5. The results of the production-planning model show considerable savings when compared to the actual production plan followed by the company during the five-month period.

Refer table 6.1 for the results on production planning model. We study two scenarios of results. In scenario 1, the demand forecast is used to solve the problem. We solve the production-planning model and determine production costs. We also calculate the cost of the production plan developed be the company to meet the demand forecast. In table 6.1, scenario 1 results show 61.20 percent reduction in inventory carrying cost of intermediate products and finished goods, 38.46 percent reduction in setup cost of intermediate products and finished goods, 20.50 percent reduction in cost of fresh raw materials, 8.58 percent reduction in cost of by-products and recovered raw material inventory. The production plan proposed by model in scenario 1is Rupees 2.60 crores. In model 2, the demand is set equal to the actual production of finished goods in the plant during the 5-month planning horizon. Table 6.1 summarizes the cost difference between the actual production plan and model results.

The results of scenario 2 suggest that to meet the demand equal to the actual production quantity produced of the company, the plan suggested by the model results in savings due to better production planning. The production plan proposed by model in scenario 1 is Rupees 1.90 crores. In the next section, we develop a variant of the production-planning model, which is the contribution maximization model.

	Cost Difference (%) *(Actual Production Plan – Production Plan Proposed by the Model)*	
Cost	**Scenario 1**	**Scenario 2**
Inventory Carrying Cost of Intermediates and End Products.	61.20	60.90
Setup Cost of Intermediates and End Products.	38.46	24.79
Fresh Raw Materials Cost	20.50	6.38
Inventory Carrying Cost of By-Products and Reusable Raw Materials	8.58	6.69
Total Cost	33.87	24.65

Table 6.1: Comparison of Model Results with Actual Production Plan Costs

6.4 Contribution Maximization Model

In this section, we develop a variant of the production-planning model with additional market constraints and call it contribution[2] maximization model. The contribution maximization model is used for jointly planning sales and production. The model determines the best sales and production plan and maximizes the total contribution. The model is based on minimum and maximum monthly demand provided by the company (typically 75 percent and 120 percent respectively of actual demand). As compared to the cost minimization model, following changes are made in the contribution maximization model:

1. As compared to the cost minimization production-planning model developed in chapter 3, the contribution maximization model has one additional variable. The variable is SD_{it}: quantity of finished good sold in time period t.

[2] Contribution = revenue net of material Cost – total production costs.
Revenue net of material cost = sales-material cost of goods sold
Material costs of goods sold = Cost of raw materials (excluding cost of reusable raw materials) + cost of intermediates.
Production costs are inventory cost of products, inventory cost of by-products & reusable raw materials, and cost of fresh raw materials.

2. The objective function in cost minimization model minimizes ‘Total Production Costs’ whereas the objective function in contribution maximization model maximizes ‘Total Contribution’. The objective function of contribution maximization model is:

$$\max z = \sum_i \sum_t RM_i.SD_{it} - \sum_i \sum_t h_i.I_{it} - \sum_i \sum_j \sum_t S_{ij}.O_{ijt} - \sum_s \sum_t h_s.I_{st} - \sum_m \sum_t h_m.\mathrm{I}_{mt} - \sum_s \sum_t f_s.F_{st}$$

3. The inventory balance constraint of end products in contribution maximization model replaces the *demand* parameter by variable SD_{it},

$$I_{it} = I_{it-1} + \sum_j X_{ijt} - SD_{it}$$

4. There are three additional parameters in contribution maximization model. Two parameters are minimum demand and maximum demand of end products. This is to provide lower and upper bounds on SD_{it}. The purpose of providing bounds is to satisfy the constraint of meeting minimum and maximum demand of end products in each time period. The constraints providing these bounds are:

$$SD_{it} \geq DMIN_{it}$$
$$SD_{it} \leq DMAX_{it}$$

The third parameter is the *contribution* of each end product. Table 6.2 presents the results of contribution maximization model. The table shows the percentage increase in revenue net of material cost proposed by the model.

Percentage Increase in *Sales*	11.45
Percentage Increase in *Materials Cost of Goods Sold*	5.44
Percentage Increase in Revenue Net of Material Cost	24.82

Table 6.2: Percentage Increase in 'Revenue Net of Material Cost' in Contribution Maximization Model as Compared to the Actual Sales and Production Plan.

To meet the sales plan, table 6.3 below shows the improvement in production costs in the contribution maximization model.

Inventory Carrying Cost of Intermediates and End Products.	60.90
Set-up Cost of Intermediates and End Products.	24.79
Fresh Raw Material cost	6.38
Inventory Carrying Cost of By-Products and Raw Materials	6.69
Percentage Increase in Contribution	42.54

Table 6.3: Production Costs Difference In Percentage: (Actual Production Plan– Production Plan Proposed by the Model)

Savings due to jointly planning sales and production is Rupees 9.92 crores. This is 42.54 percent increase in contribution. Significantly higher benefits are realized in the case of jointly planning sales and production over the production plan to meet the demand forecast.

It is interesting to see that with only 11.45 percent increase in model sales plan as compared to actual sales (Table 6.2), reduction in production costs due to improved production-planning (Table 6.3), results in 42.54 percent increase in contribution. There are few issues to be analyzed for considerable increase in contribution proposed by the contribution maximization model. The results indicate the operating philosophy of the company. The notion followed by the company is to maximize the capacity utilization, rather then to plan for the demand forecast. This leads to excess inventories of most of the products. It is seen in results, that to meet the demand, reduction in inventory costs is a major component of savings. This is one of the important insights for managers that producing to capacity can lead to very high operational costs. The other reason for considerable improvement in contribution is, obviously, more sales in the model results. There are upper bounds on the demand, and it is assumed that the company would be able to realize the sales suggested by the model. The contribution maximization model guides the marketing people to sell a certain product mix, which will maximize the contribution of the firm. It is possible that due to high demand variability, price competitiveness, and other market constraints, sales plan suggested by the model may not be realized. There are other uncertainties in the environment like machine breakdowns, rejections due to poor quality etc., which would affect the actual contribution realization. In the next section, we describe the application of scheduling model.

6.5 Application of Scheduling Model

In this section, we solve the scheduling problem. We apply the solution procedures developed for solving scheduling problem in chapter 4. The production-planning model is an input to the detailed scheduling model. As discussed in the product structure diagram, products at level 0 are the finished goods. Finished goods are scheduled by applying the solution procedure of flowshop E/T problems described in chapter 4. Finished goods derived their due dates on the basis of customer orders and demand forecast. In this application, finished goods have shipments several times in a month based on customer orders. Hence, we apply the flowshop E/T scheduling problem in each week of the month. The common due dates of finished goods is end of each week. The objective of the finished goods scheduling is to minimize earliness and tardiness penalties. Intermediate products (Level 1 onwards) derive their due dates from the schedule of higher-level products. The objective of intermediate products scheduling is to minimize earliness penalties. As discussed in chapter 4, in solution procedure for intermediate products scheduling, tardiness is not allowed in the intermediate products scheduling to maintain the feasibility of the schedule. This is because while scheduling intermediate products at any level of product structure, the higher-level products are already scheduled. Intermediate products have due date based on the start time of higher-level products. Allowing tardiness would lend the schedule of higher-level product infeasible. To solve intermediate products scheduling problem in this application, we apply the solution procedure for solving the intermediate goods scheduling problem described in chapter 4. For products produced on dedicated production plants, we apply the solution procedure for scheduling dedicated plants discussed in chapter 4. A product has a standard batch size. The number of batches to be produced is determined from the batch size of products. Scheduling decisions determine the start time and completion time of each batch of product on each machine. The detailed schedule of products and its earliness and tardiness costs for each time period of the planning horizon are provided in appendix 3. In the next section, we discuss sensitivity analysis on the results of the production planning ands scheduling problem.

6.6 Sensitivity Analysis on Production Planning and Scheduling Results

In this section, we perform sensitivity analysis on the production planning and scheduling results obtained in the previous section. Sensitivity analysis is done on demand of finished goods, initial inventory of finished goods and intermediate products, capacity of dedicated and flexible plants, and ratio of setup cost to inventory cost of intermediate products and finished goods. Demand is chosen for sensitivity analysis as the environment has demand variability, and sensitivity on demand will help in coordinating the marketing decisions in a better way. Sensitivity on initial inventory will help in evaluating the cost of purchasing the intermediate products as compared to in-house production to avoid production delays. The instance solved in the previous section is considered as the base case. We are not analyzing the sensitivity of parameters on scheduling costs, as they are very less as compared to the production costs.

As shown in table 6.4, we observe the impact of change in parameters on inventory costs and setup costs of intermediate products and finished goods, inventory costs of by-products and reusable raw materials and cost of fresh raw material used. The detailed results (production quantities of products and number of setups) of all the cases are provided in appendix 3. We also observe the impact on capacity utilization of dedicated plants, and of each machine in the flexible plants. Table 6.4 shows for each case, its production costs and the change in costs from the base case. Demand of finished goods is varied from 80 percent of the base case demand to 120 percent of the base case demand. The major factor that reduces the cost at 80 percent of the base case demand is the cost of fresh raw material. The inventory of intermediate products and finished goods are higher in case 2 as compared to case 1, due to the high initial inventory of products. With increase in demand, the cost of fresh raw material also goes up. Table 6.5 and table 6.6 indicate the capacity utilization in percentage of dedicated plants and of machines in flexible plants respectively. In table 6.5, for each case, the average capacity utilization of dedicated plant is shown. The average capacity utilization is determined over the 5-month planning horizon. The average capacity utilization of dedicated plants is reduced by around 30 percent at 80 percent base case demand. In table 6.6, the average capacity utilization over a 5-month period is determined for each machine in the flexible plant. The average capacity utilization of machines is reduced by around 25 percent in the case of 80 percent base case demand. With increase in demand, the capacity utilization of dedicated plants goes up by 20 percent and 15 percent in case of flexible plants. The production plan is infeasible at 120 percent of the base case demand of finished goods. At 120 percent of the base case demand, the capacity constraint of one of the flexible machines gets violated.

The aggregate capacity of dedicated plants and machines of flexible plants is varied from 80 percent to 120 percent of the base case capacity. It is seen that at 80 percent of the base case capacity, the production plan is infeasible and is not able to meet the base case demand. Reduction in capacity is resulting in high inventory costs. At 110 percent and 120 percent of the base case capacity, the production costs are decreasing. This is due to the reduction in inventory costs and setup costs. Since capacity is more, the model is suggesting to produce when required, resulting in less build up of inventory. It is seen in table 6.5 that the capacity utilization of some of the dedicated plants is not very high. This is an important observation to the management for capacity planning related issues. One of the reasons for low capacity utilization could be the seasonality in the demand of products produced in these plants. In the five months instance solved, the products produced in low capacity utilization plants may have less demand. Another insight from this result is that some reallocation of the capacity is required to improve the overall capacity utilization of the production plants. Low capacity utilization is also an indication to the marketing department to enhance the sales of the products produced in these plants. Sensitivity on capacity is also useful for long-term strategic decisions for the company. Marginal value of the capacity is an useful indicator to the management for determining the appropriate capacity of the resources.

Impact of initial inventory is significant on the production plan costs and capacity. Initial inventory of intermediate products and finished goods is varied from 80 percent to 120 percent of the base case. With high initial inventory, reduction in total costs is seen in table 6.4. At 120 percent of the base case initial inventory, although the inventory costs go up, the cost of fresh raw material reduces (due to less production of products) considerably. This reduces the overall cost of case 14. The inventory costs are rising with increase in inventory due more to inventory being carried over in the planning horizon. The capacity utilization decreases with increase in initial inventory. With less initial inventory, the production costs increase due to more consumption of fresh raw materials. This is happening because more production is required with less initial inventory. Capacity utilization is also increasing with less initial inventory.

Sensitivity is also done on ratio of setup costs to inventory costs. It is seen that with more setup to inventory costs ratio (cases 15; 16), the number of setups decrease (as seen in reduced setup costs). The inventory costs in these cases increase resulting in overall increase of costs. This is because in the production plan, more inventory is carried due to high setup costs. The production plan changes in both the cases. In cases 17 and 18, there is no change in the production plan. However, the production costs reduce due to significantly less use of fresh raw material. The inventory of by-products and reusable raw material is also less in cases 17 and 18.

6.7 Implementation Issues

The benefits of production planning model were shown to the company from the results of five-month data. The benefits provided the motivation to the management for implementing the models. The extent of savings due to production planning model is presently difficult to estimate over a longer duration. The company is in the process of using the production planning and scheduling models for their complete operations. Presently, the implementation of scheduling model is not fully functional.

On-site training was provided to the personnel involved in planning and shop floor scheduling. The Decision Support System (DSS) developed was documented to include; production-planning and scheduling problem, key decisions in the problem, structure of production planning and scheduling models, interpretation of results, and sensitivity analysis on results. The planning model developed in GAMS was provided interface with Microsoft Excel to import the parameters of the model. This was also done to facilitate the change in parameters with ease.

One major observation from implementation of the models is that managers do not easily internalize the benefits of optimization tools. We faced difficulties in convincing the plant managers that producing just to increase capacity utilization often results in high operational costs. The results of the models helped managers to understand the importance of this issue. Another important issue we experienced in implementation of these models is that right training and competence is imperative to exploit maximum benefits of optimization tools. It is very important for the users to know the capabilities of such decisions support systems.

6.8 Summary

In this chapter, we have solved a real life large-scale complex production planning and scheduling problem of a pharmaceutical company. We have applied the mathematical models developed in chapter 3 to address the production planning and scheduling decisions of the problem. The solution procedure developed for solving production planning and scheduling problem were applied to solve the problem in this application. In section 6.3, the application of production planning model is described. The results of the production-planning model over the finite planning horizon have shown considerable savings in the production costs over the actual production plan of the company. A variant of the production-planning model is developed in section 6.4. This model is for jointly planning sales and production. It is shown that significant increase in the savings is realized in the sales and production plan over the plan to meet just the demand forecast. Application of scheduling model is discussed in section 6.5. The solution procedure and results of the scheduling model are described in this section. To provide managerial insights from the problem, sensitivity analysis on the production planning and scheduling results is provided in section 6.6. We also discussed

Case No.	Case	Total Costs	Inventory Costs of Finished Goods and Intermediates	Setup Costs of Finished Goods and Intermediates	Inventory Costs of By-Products and Reusable Raw Materials	Cost of Fresh Raw Materials
Case 1	Base Case	54,127,000	4,839,510	197,266	1,656,298	47,433,926
Case 2	80% Demand	40,034,000	5,042,543	163,748	1,461,187	33,366,522
Case 3	90% Demand	46,825,000	4,814,037	187,866	1,554,902	40,268,195
Case 4	110% Demand	61,873,000	5,176,059	211,520	1,787,368	54,698,053
Case 5	120% Demand	Infeasible	-	-	-	-
Case 6	80% Capacity	Infeasible	-	-	-	-
Case 7	90% Capacity	54,754,000	5,179,250	196,786	1,686,581	47,691,383
Case 8	110% Capacity	53,814,000	4,536,070	193,982	1,650,022	47,433,926
Case 9	120% Capacity	53,739,000	4,368,796	191,352	1,645,352	47,533,500
Case 11	80% Initial Inventory	57,225,000	3,568,487	204,782	1,752,560	51,699,171
Case 12	90% Initial Inventory	55,766,000	4,100,603	199,666	1,707,393	49,758,338
Case 13	110% Initial Inventory	52,777,000	5,652,213	194,792	1,628,691	45,301,304
Case 14	120% Initial Inventory	51,519,000	6,478,924	187,262	1,615,043	43,237,771
Case 15	Setup to Inventory Cost Ratio-25	55,458,000	6,488,321	169,722	5,562,186	43,237,771
Case 16	Setup to Inventory Cost Ratio-50	59,272,000	6,712,369	158,858	9,163,002	43,237,771
Case 17	Setup to Inventory Cost Ratio-0.1	51,209,000	6,357,042	188,086	1,426,101	43,237,771
Case 18	Setup to Inventory Cost Ratio-0.5	51,331,000	6,395,766	189,558	1,507,905	43,237,771
				Change in Costs (Case – Base Case)		
Case 2	80% Demand		203,033	-33,518	-195,111	-14,067,405
Case 3	90% Demand		-25,473	-9,400	-101,396	-7,165,731
Case 4	110% Demand		336,549	14,254	131,070	7,264,127
Case 5	120% Demand					
Case 6	80% Capacity					
Case 7	90% Capacity		339,740	-480	30,283	257,457
Case 8	110% Capacity		-303,440	-3,284	-6,276	0
Case 9	120% Capacity		-470,714	-5,914	-10,946	99,574
Case 10	80% Initial Inventory		-1,271,023	7,516	96,261	4,265,245
Case 11	90% Initial Inventory		-738,907	2,400	51,095	2,324,412
Case 12	110% Initial Inventory		812,704	-2,474	-27,607	-2,132,623
Case 13	120% Initial Inventory		1,639,414	-10,004	-41,255	-4,196,155
Case 14	Setup to Inventory Cost Ratio-25		1,648,811	-27,544	3,905,888	-4,196,155
Case 15	Setup to Inventory Cost Ratio-50		1,872,859	-38,408	7,506,704	-4,196,155
Case 16	Setup to Inventory Cost Ratio-0.1		1,517,532	-9,180	-230,197	-4,196,155
Case 17	Setup to Inventory Cost Ratio-0.5		1,556,256	-7,708	-148,393	-4,196,155

Table 6.4: Sensitivity Analysis on Production Planning and Scheduling Results

Plant	Case 1	Case 2	Case 3	Case 4	Case 7	Case 8	Case 9	Case 10	Case 11	Case 12	Case 13	Case 14	Case 15	Case 16	Case 17
1	87	67	87	96	87	87	87	88	88	85	84	82	82	85	85
2	51	38	51	57	51	51	51	55	53	48	46	46	46	46	46
3	39	20	48	39	48	51	50	21	52	38	38	51	20	51	38
4	58	54	51	66	51	50	50	70	49	59	59	49	69	49	59
5	62	44	62	72	62	62	62	69	66	59	56	56	56	56	56
6	56	39	56	65	56	56	56	61	59	53	50	50	50	50	50
7	54	41	54	60	54	54	54	56	55	52	50	50	50	50	50
8	85	67	49	54	49	85	85	87	85	84	83	83	83	83	83
9	39	30	74	83	74	39	39	40	39	38	37	37	37	37	37
10	43	17	43	56	43	43	43	60	52	34	26	26	26	26	26
11	61	34	61	75	61	61	61	76	69	53	46	46	46	46	46
12	31	21	31	35	31	31	31	34	32	29	27	26	26	27	27
13	36	33	36	40	36	36	40	37	37	35	34	34	34	34	34
14	22	20	22	24	22	22	24	23	22	21	21	21	21	21	21

Table 6.5: Capacity Utilization (in Percentage) of Dedicated Plants

EQ1	31	21	26	35	31	31	30	34	33	29	27	25	24	27	27
EQ2	23	14	19	27	23	23	23	28	26	21	18	18	17	19	19
EQ3	52	31	41	38	55	49	39	57	53	47	28	24	53	32	30
EQ4	63	40	47	59	60	52	53	57	53	51	49	49	49	59	49
EQ5	55	37	46	66	55	55	55	65	60	50	46	47	46	47	46
EQ6	32	33	39	73	32	45	56	48	46	42	55	58	28	43	54
EQ7	53	37	45	63	53	53	53	61	57	49	46	46	46	47	46
EQ8	68	50	60	78	68	68	68	73	70	66	64	63	63	64	64
EQ9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EQ10	51	37	44	58	51	51	50	56	54	48	44	44	43	46	45
EQ11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EQ12	57	40	48	68	57	57	58	65	61	54	51	50	50	51	51
EQ13	44	33	39	50	44	44	44	46	45	42	40	41	40	41	40
EQ14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EQ15	18	10	14	22	18	18	18	24	21	15	12	12	11	13	13
EQ16	56	46	51	61	56	56	56	56	56	56	56	56	55	56	56
EQ17	55	45	50	60	55	55	55	55	55	55	55	55	54	55	55
EQ18	50	41	45	54	49	49	49	50	50	50	50	49	49	51	50
EQ19	50	40	45	54	50	50	50	50	50	50	50	50	49	49	50
EQ20	3	2	2	3	3	3	3	3	3	3	3	3	2	2	3
EQ21	2	1	2	2	2	2	2	2	2	2	2	2	2	2	2
EQ22	38	31	35	41	38	38	38	38	38	39	39	39	37	38	39
EQ23	50	40	45	54	50	50	50	50	50	50	50	50	49	49	50
EQ24	58	47	52	63	57	57	57	58	58	58	58	57	56	58	58
EQ25	56	45	51	62	56	56	56	56	56	57	57	57	56	56	57
EQ26	49	40	45	54	49	49	49	50	50	50	50	50	49	49	50
EQ27	50	40	45	55	50	50	50	50	50	50	50	50	50	50	50
EQ28	58	47	52	63	57	57	57	58	58	58	58	57	56	58	58
EQ29	68	54	61	74	67	67	67	68	68	67	67	67	66	68	67
EQ30	53	41	47	58	52	52	52	53	53	53	53	52	52	53	53
EQ31	55	45	50	60	55	55	55	55	55	55	55	55	54	55	55

Table 6.6: Capacity Utilization (in Percentage) of Machines in Flexible Plants

some issues regarding implementation of the models. In the next chapter, we provide conclusions of this research. We also discuss issues relevant to future research from this thesis.

7 Summary, Contribution and Future Research

7.1 Summary

In this research, we investigate the potential of production planning and scheduling in reducing the operational costs of manufacturing firms with complex production environment. We discussed that manufacturing firms are facing complexities in form of increasing product variety, shrinking product volumes, demand variability, and increase in customer response times. As a result, firms are paying high attention to the operating costs. In this context, we discussed that production planning and scheduling can contribute significantly in reducing the operating costs of firms.

Motivated by the complex production environment of chemical plants, we consider the production planning and scheduling problem existing in process industries and discrete parts manufacturing industries. We consider multi-stage, multi-product, multi-machine and batch processing production environment. We model the production planning and scheduling decisions in two steps. In the first step, we develop production-planning model, which is mixed integer linear programme (MIP). The decisions of the production model are to determine production quantity of products, inventory levels of products, and to determine the aggregate capacity of resources required to meet the production plan. The objective of the production-planning model is to minimize the production costs over the planning horizon. In the second step, we model scheduling decisions. There are two scheduling MIP models, finished goods scheduling model and intermediate products scheduling model. This is because in the production environment, finished goods follow flowshop setting and intermediate products follow jobshop setting with re-entrant flows. The decisions of scheduling model are to determine start time and completion times of all products on each machine. The objective of scheduling model is to minimize the earliness and tardiness penalties. The production-planning decisions impose constraints, within which the detailed scheduling decisions are made.

We solve the production-planning problem using the branch and bound algorithm. For finished goods scheduling, we consider the permutation flowshop problem with common due date. The problem is NP-complete. Based on the common due dates, the finished goods scheduling problem can be decomposed in three sub-problems; sub-problem 1 with unrestricted due date, sub-problem 2 with intermediate due date, and sub-problem 3 with restricted due date. Using some of the known optimal results of single machine earliness and tardiness problem, we develop procedures to determine unrestricted, intermediate and restricted due dates in multi-machine environment. The objective of all the three sub-problems is to minimize the absolute deviation of job completion times from a common due date. We derive analytical results for solving sub-problem 1.

We develop heuristic algorithms to solve sub-problem 2. The heuristic derives a permutation flowshop sequence at the bottleneck machine. The pre-bottleneck bottleneck processing times are treated as release dates of jobs at the bottleneck. The post-bottleneck processing times are considered to derive due dates of jobs at the bottleneck machine. By solving single machine problem with release dates and due dates iteratively at the bottleneck machine, we derive a permutation flowshop sequence. Then, we apply tabu search methods to improve the solution. There are no previous results available in the literature of sub-problems 1 and 2, i.e., flowshop scheduling problems minimizing absolute deviation of jobs from a common due date. For small problem instances, we determine optimal solution of sub-problem 2, using branch and bound algorithm. We show that the lower bound of sub-problem 2 is very weak, as the average percentage deviation of optimal solution from lower bound is very high (326 percent for n = 5, m = 5, 50 instances; 284 percent for n = 10; m = 5, 50 instances). For small problems, we compare the performance of heuristic solution with the optimal solution. The average percentage deviation of heuristic solution from optimal solution is 0.894 percent for n = 5, m = 5, 50 instances and 1.126 percent for n = 10, m = 5, 50 instances respectively. For large problems, we compare the heuristic performance with the analytical solution of sub-problem 1 obtained in polynomial time. The average percentage deviation of the largest problem instance solved (n = 100, m = 20, 50 instances) is 1.129 percent.

We develop heuristic algorithm for sub-problem 3. Sub-problem 3 is the flowshop problem of minimizing earliness and tardiness penalties, with restricted common due date. Sub-problem 3 has a special structure, that by definition of restricted due date, no job is early. The objective of sub-problem 3 reduces to minimize tardiness. If all jobs are tardy and simultaneously available with non-positive release dates, minimizing tardiness is same as minimizing flow time or minimizing completion time. There are results available in the literature on minimizing tardiness, minimizing flow time and minimizing completion time in permutation flowshops. We compare our heuristic results with some of the best-known results of these problems and found our heuristic to perform better. We applied our heuristic results and the existing results on some benchmark problems in the literature on flowshop scheduling. The average percentage deviation of our heuristic solution from the lower bound of sub-problem 3 is found to be less than the solution of existing heuristics.

Then, we solve the intermediate products scheduling problem. The production process of intermediate products is a general job shop setting wit re-entrant flows. Intermediate products have derived demand. To maintain feasibility of scheduling decisions, we do not allow tardiness in the intermediate products schedule. This is because intermediate products derive their due dates from the existing schedule of the higher-level products, where the intermediate products are consumed. We develop heuristics to solve the intermediate products scheduling problem. The heuristic is developed with an objective of minimizing earliness costs, i.e., the completion time of a job is as close as possible to the due date. As discussed above, the completion time cannot exceed the due date to maintain feasibility. The heuristic determines the schedule of intermediate products at all levels in the product structure.

We report the results of production planning and scheduling problem. The data is provided by a large pharmaceutical company in India. We obtain optimal production plan over a finite planning horizon, consisting of production quantity and inventory levels of intermediate products and finished goods, inventory levels of by-products and reusable raw materials and amount of fresh raw materials used in the production process. We determine aggregate capacity of resources required to meet the production plan. We obtain scheduling decisions; start time and completion times of each intermediate product and each end product on each machine. We also determine the earliness and tardiness costs in the schedule.

We apply the production planning and scheduling models in a pharmaceutical company in India. This application has multi-stage, multi-product, multi-machine, batch production environment. We implement the solution procedure developed for solving production planning and scheduling problem on 5-month data (January, 2002 to May, 2002).

In this instance, there are 10 finished goods, 30 intermediate products and 15 production lines. We compare the results of the production-planning model with the actual production plan followed by the company during the 5-month period. The total savings due to improved production planning using the production planning model over the actual production plan followed by company are Rupees.2.60 crores. The savings in the plan proposed by the production-planning model while maintaining the supply of end products at par with actual production (i.e., cumulative demand in this case is same as that in actual production case) are Rupees 1.90 crores. We also develop a variant of the production-planning model with additional market constraints. The model is used for jointly planning sales and production, and it maximizes contribution. Based on the lower and upper bounds on demand, this model suggests the best sales mix that will maximize contribution. Savings in contribution using the contribution maximization model sales and production plan are Rupees 9.92 crores. The results show that savings can be significantly increased by using the model for joint planning of sales and production over the production plan to meet the demand forecast. We report results on scheduling the production plan proposed by the production-planning model. We determine start time and completion time of each product in the production plan on each machine. Earliness and tardiness costs of the products are determined in the schedule. We also perform sensitivity analysis on the production planning and scheduling results in this application. In sensitivity analysis, we study the impact of aggregate capacity, demand, initial inventory and ratio of setup to inventory costs. The production planning and scheduling models are presently being used by the company.

In this research, we have shown the tangible benefits of production planning and scheduling in complex manufacturing environment. We have solved large and difficult production planning and scheduling problems faced by manufacturing firms today. We have derived some new analytical results and proposed new heuristic algorithms for a class of production planning and scheduling problems. We expect that models developed in this research would form basis for production planning and scheduling decisions in complex production environments. Sensitivity analysis on the results would help the managers to evaluate alternate production plans and schedules and manage the complexities in the environment in a better way. In the next section, we discuss the contribution of this research.

7.2 Contribution

- In this research, we address the decisions of complex production planning and scheduling problems existing in discrete parts manufacturing industries and process industries. We consider a multi-stage, multi-product, multi-machine batch-processing environment.

- We consider some new complexities in the production environment that have not been addressed in the literature on production planning and scheduling. In our problem, the production environment produces finished goods and intermediate products. By-products are generated from finished goods and intermediate products, and are recycled to recover reusable raw materials. Traditional models on multi-stage production planning and scheduling, are primarily based on assembly and fabrication types of product structures. We consider the complexity of recycling process in the product structures. There are flexible machines in the production environment. The machines are used for processing both finished goods and intermediate products, which complicates the scheduling decisions considerably. The production environment faces demand forecast over the finite planning horizon. Finished goods in the environment follow flowshop type of production process, and intermediate products follow general jobshop type of production process with re-entrant flows.

- We model the production planning and scheduling decisions through sequence of hierarchical models. First, we develop a mixed integer program (MIP) for production planning decisions. The objective of the production-planning model is to minimize inventory costs and setup costs of intermediate products and finished goods, inventory costs of by-products and reusable raw materials, and cost of fresh raw material. In the next step, we develop an MIP for finished goods scheduling decisions. Then, we develop an MIP for intermediate products scheduling problem decisions. The objective of the scheduling problem is to minimize the absolute deviation of job completion times from a common due date.

- The production-planning problem is solved using the branch and bound algorithm. We report some new results for solving the scheduling problem. Finished goods scheduling (flowshop) problem of minimizing absolute deviation of job completion times from a common due date is not addressed in the literature. We develop analytical results for solving flowshop scheduling problem in certain ranges of common due dates. We also develop new heuristic algorithms for flowshop problem, where obtaining analytical solution is difficult. For flowshop earliness and tardiness problems with special structure (minimizing tardiness, flowtime, completion time), we develop new heuristic algorithm for the problems. We compare the heuristic algorithms for this class of problems with some of the best existing results. The computational performance of our heuristics is found to be better that those of existing heuristics. We develop heuristic algorithms for solving the intermediate products scheduling problem.

- We report implementation of the production planning and scheduling models in a real life case of a pharmaceuticals company in India. The results of the models indicate substantial savings over the actual company performance. Sensitivity analysis on the results is provided evaluate various production plans and schedules. In the next section, we identify certain issues which we have not considered in this research, and which deserve attention in future research from this work.

7.3 Future Research

In this research, we have decomposed the overall production planning and scheduling problem and developed sequential models to address the decisions in the problem. One possible natural extension of this research would be to exploit further the benefits of decision-making in hierarchical production planning. We have not considered the aggregation of products, as mentioned in Bitran and Tirupati (1993). Products may be aggregated into *families*, and families into *types*. A type is a collection of products with similar demand patterns and production rates. A family is a set of products within a type such that products in the family share a common setup. Advantages of aggregation of products will be less dimensionality of mathematical programs, less detailed demand forecast will be required (for product types only). The next process will be to develop disaggregation models to determine the production plans for product families. Further disaggregation of the family production lots to determine the quantities of product will be required.

Another useful extension of this work could be to analyze uncertainties in production situations. Some of the uncertainties in the form of machine failures and demand forecast errors often exist in real life situations. Production planning and scheduling with stochastic demand would be a good generalization of this research. Situations like variability in processing times of jobs and variability in process yields are very common, especially in process industries. We have focused primarily on deterministic scheduling in this research. The information about the problem instances is known with certainty in deterministic scheduling. Modeling scheduling decisions with stochastic parameters would help in capturing some common uncertainties in scheduling.

We have considered permutation flowshop environment for finished goods scheduling, and common due dates of finished goods. Analysis of general flowshop setting and considering distinct due dates of products will be very useful extension of this work.

For flowshop problems where it was difficult to obtain analytical solution, we have developed heuristic algorithms. It will be a useful contribution to determine the theoretical performance guarantee of the heuristics.

Finally, one very important extension of this research would be to study the issues involved in solving the production planning and scheduling problem, as an integrated problem. A monolithic model for addressing production planning and scheduling decisions is difficult to solve. The computational effort required to solve is also enormous. Another issue in an integrated problem solving approach will be to ensure consistency between the production planning decisions, and scheduling decisions. A useful comparison will be the quality of the solution in hierarchical problems as compared to the integrated production planning and scheduling problem. To prove the decomposition approach of solving the production planning and scheduling problem, as a valid lower bound on the overall problem will be another important and interesting issue of research.

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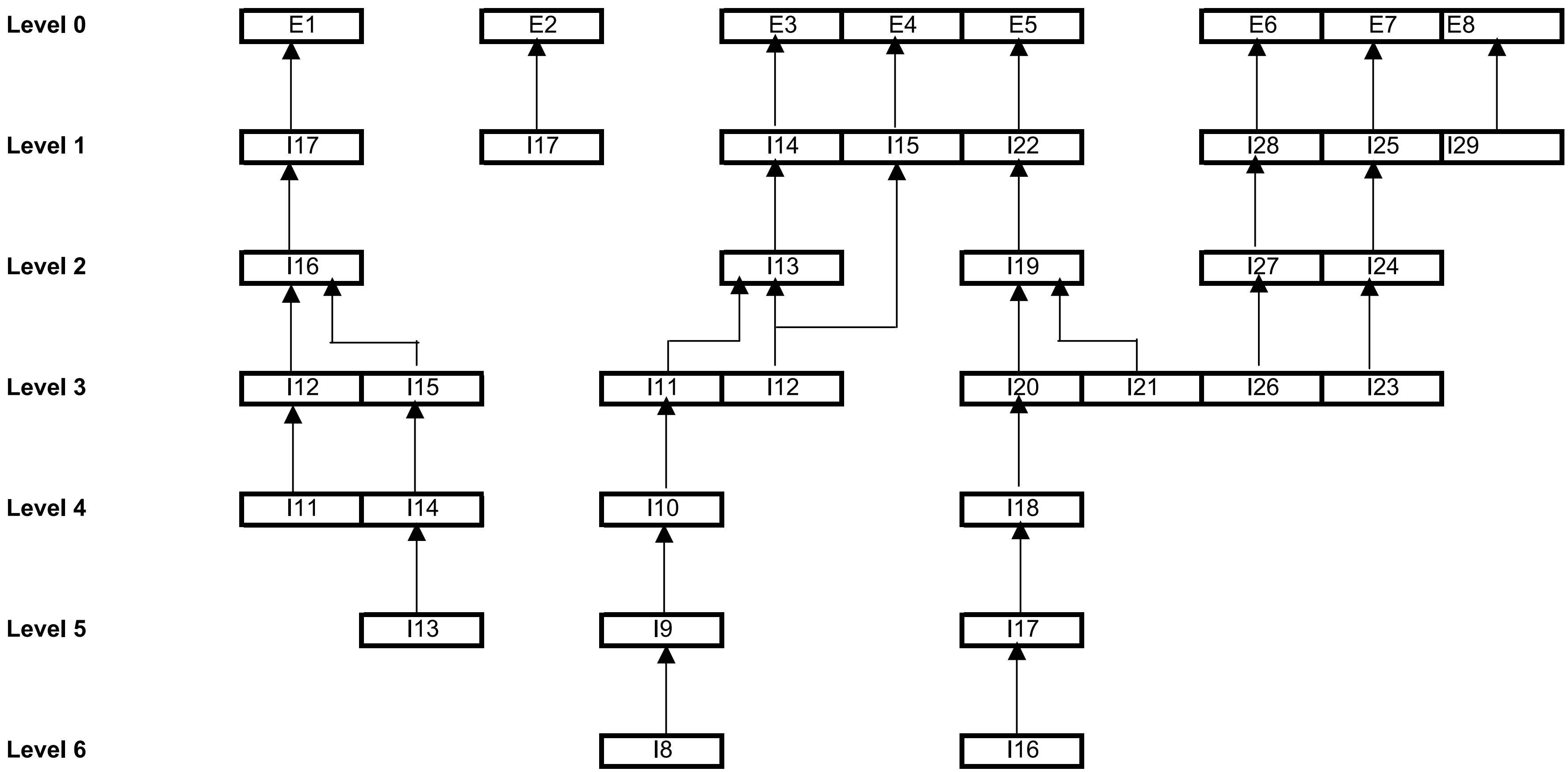
Appendix 1: Product Structure Diagram and Process Flow Diagrams

Panel A: **Product Structure Diagram**

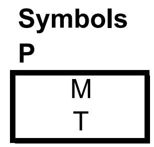
E1 to E8 **Finished Goods**
I1 to I29 **Intermediate Products**

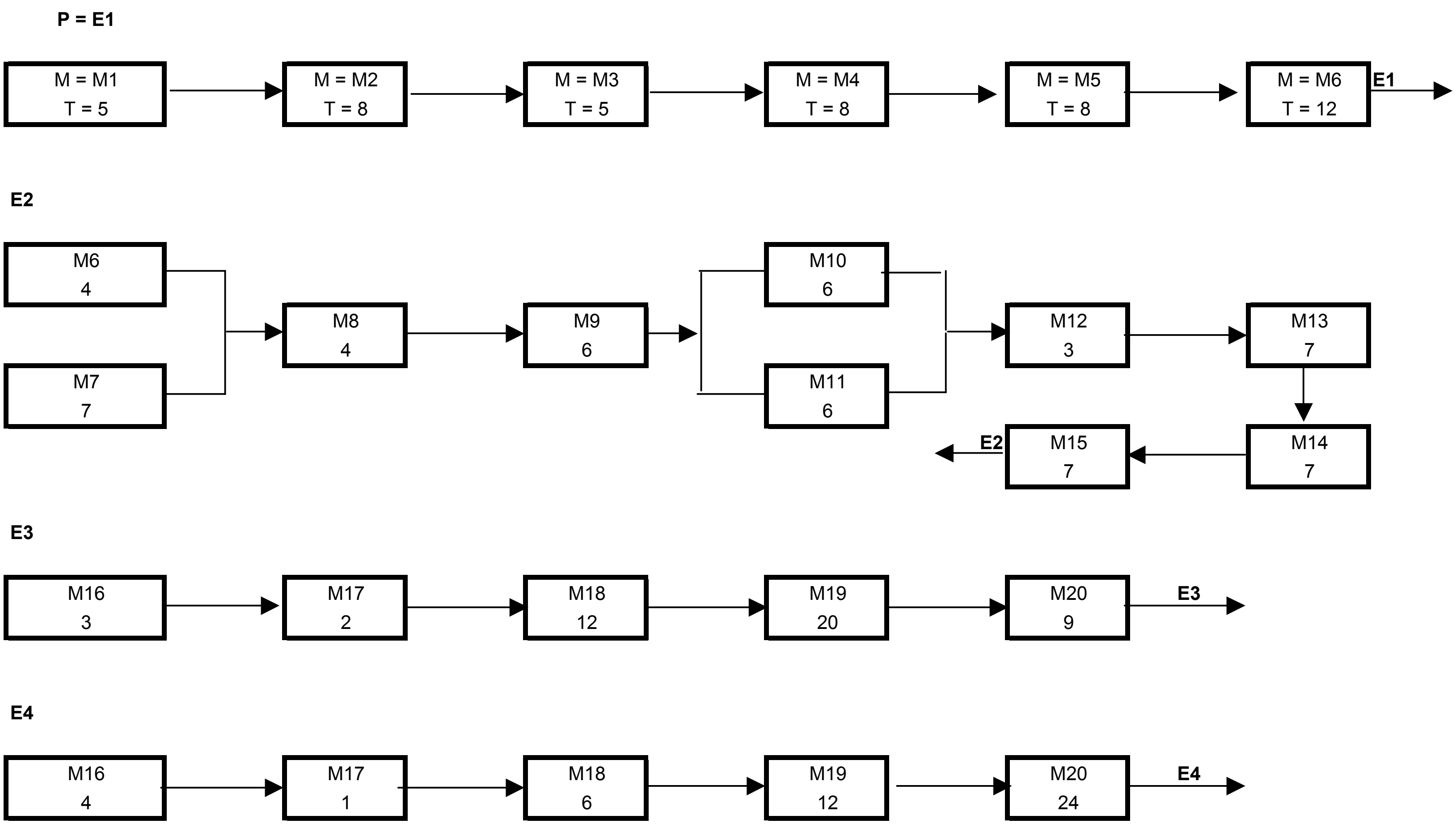


Appendix 1: Product Structure Diagram and Process Flow Diagrams

Panel B Process Flow Diagrams of Finished Goods and Intermediate Products

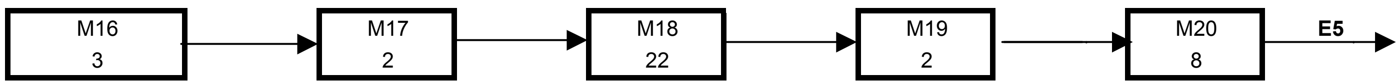
Legend	P	Product
	M	Machine
	T	Processing Time in hours.





Panel B Process Flow Diagrams of Finished Goods and Intermediate Products

E5



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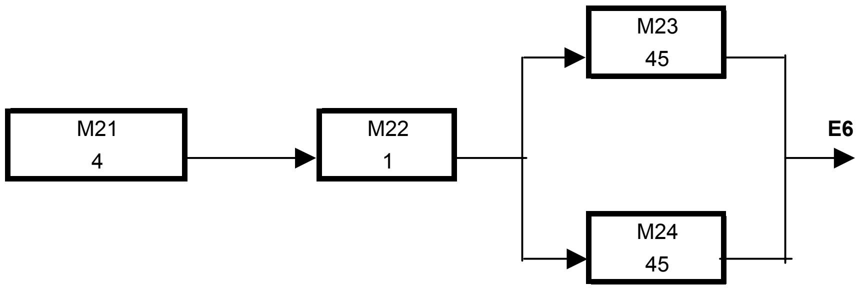

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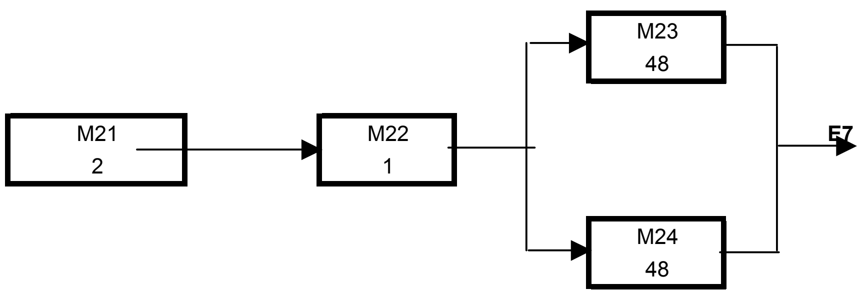
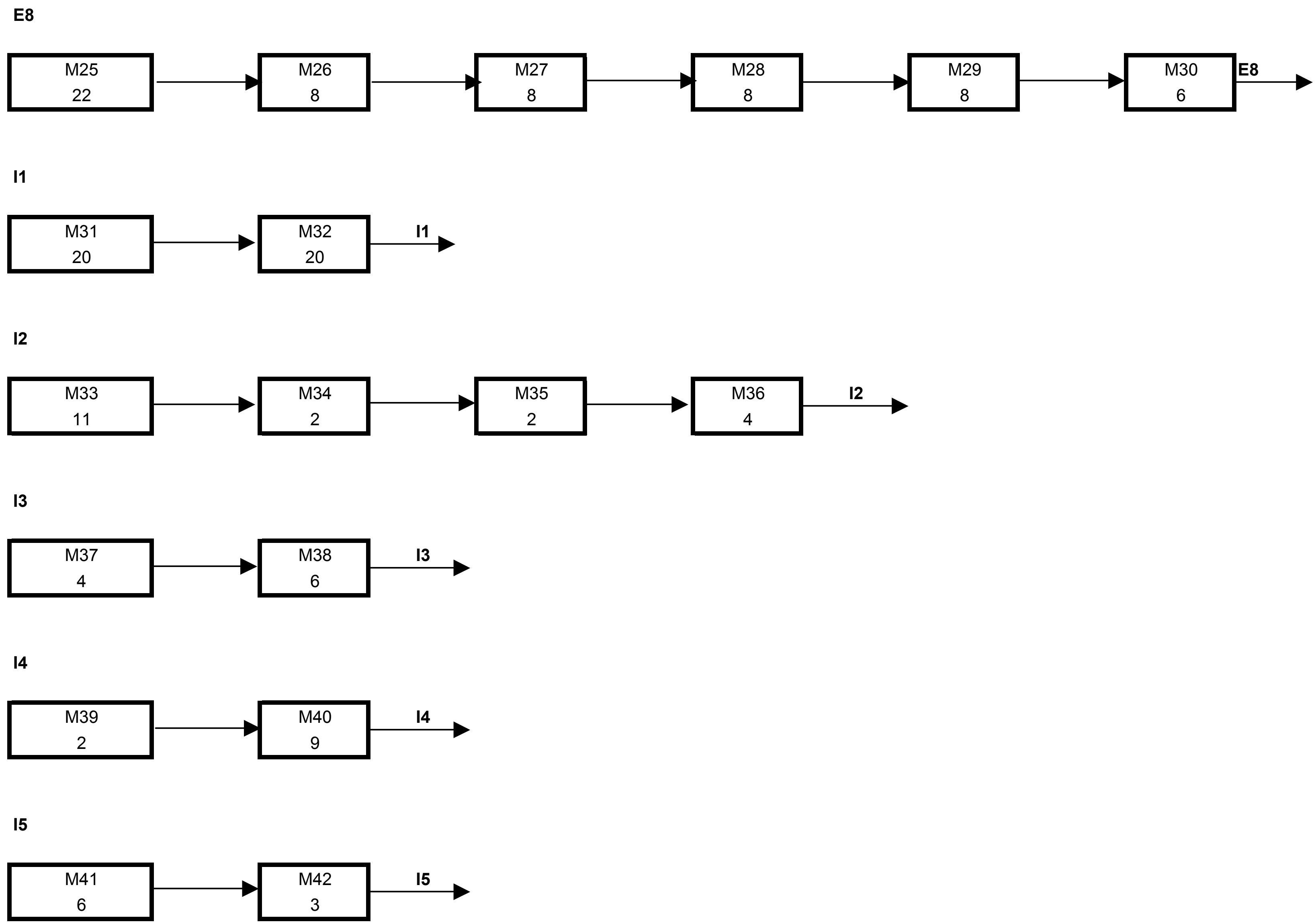
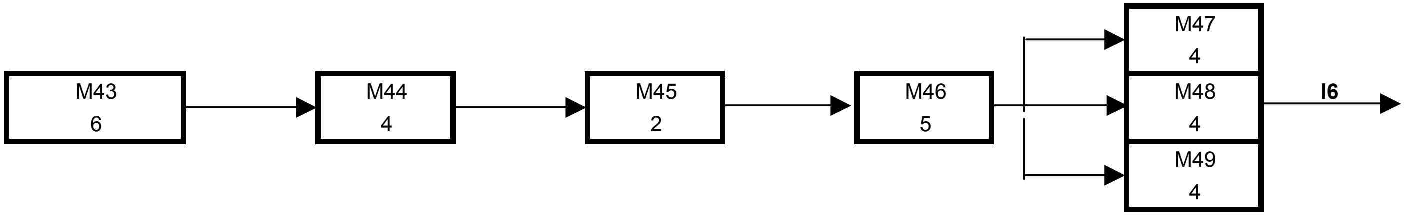


Panel B Process Flow Diagrams of Finished Goods and Intermediate Products



Panel B Process Flow Diagrams of Finished Goods and Intermediate Products

I6



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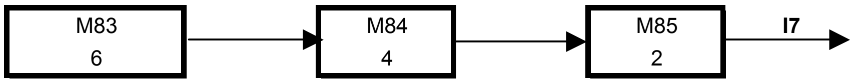

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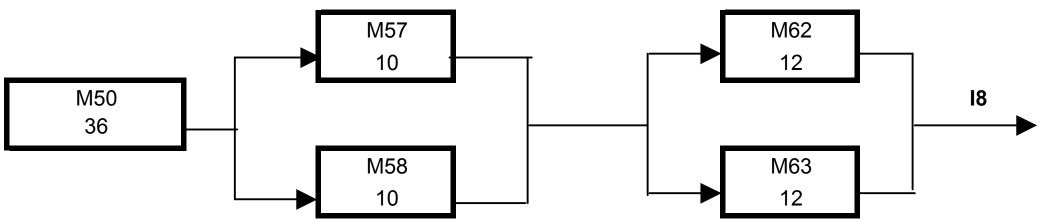
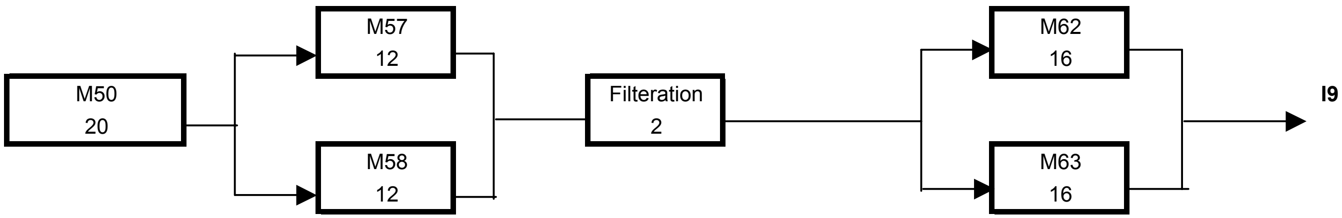


Panel B Process Flow Diagrams of Finished Goods and Intermediate Products

I9



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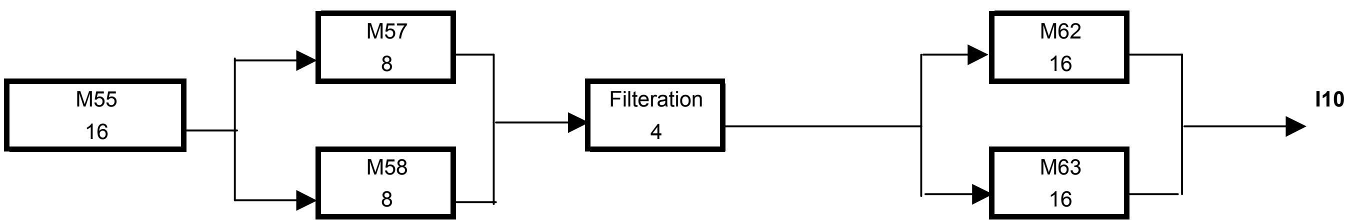

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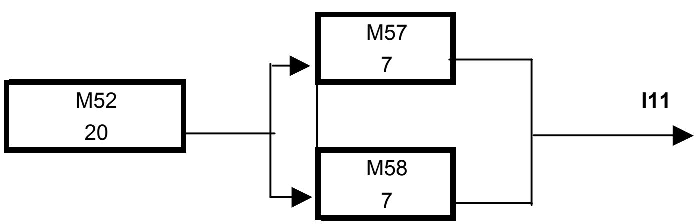

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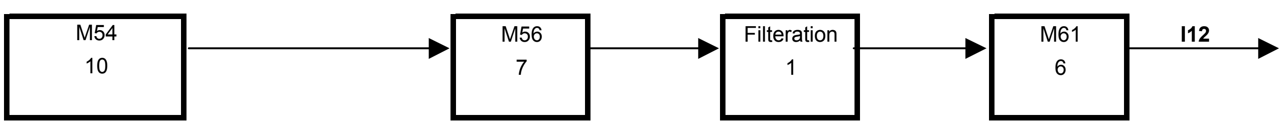
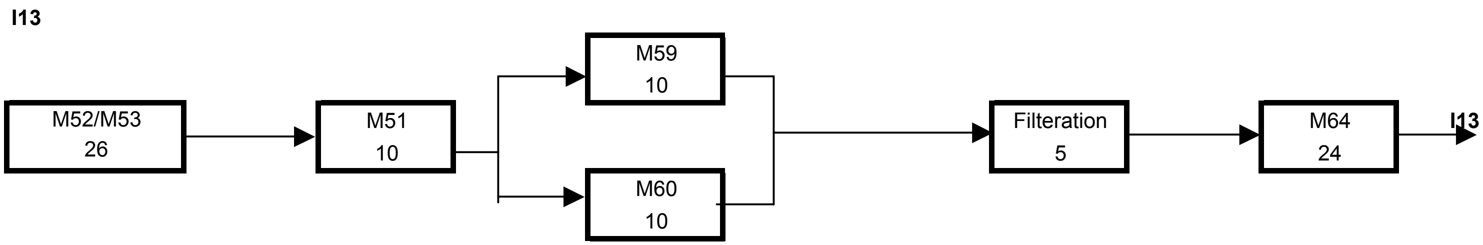
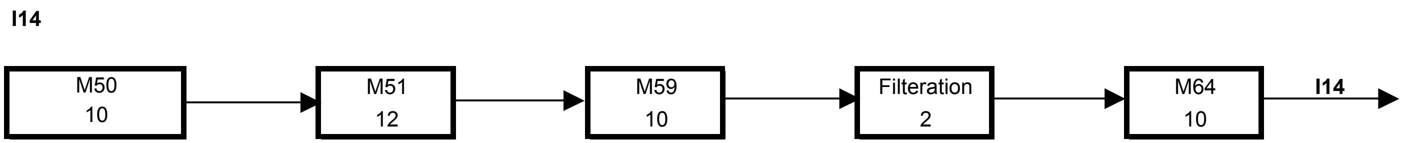
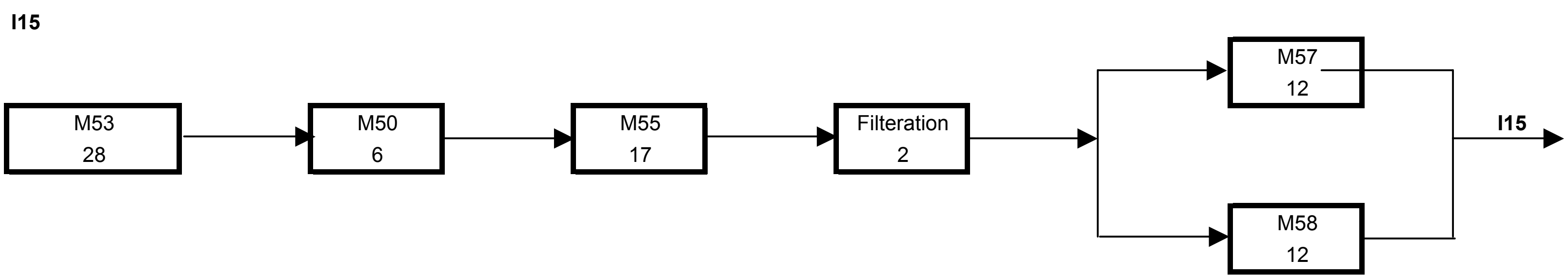


Panel B Process Flow Diagrams of Finished Goods and Intermediate Products

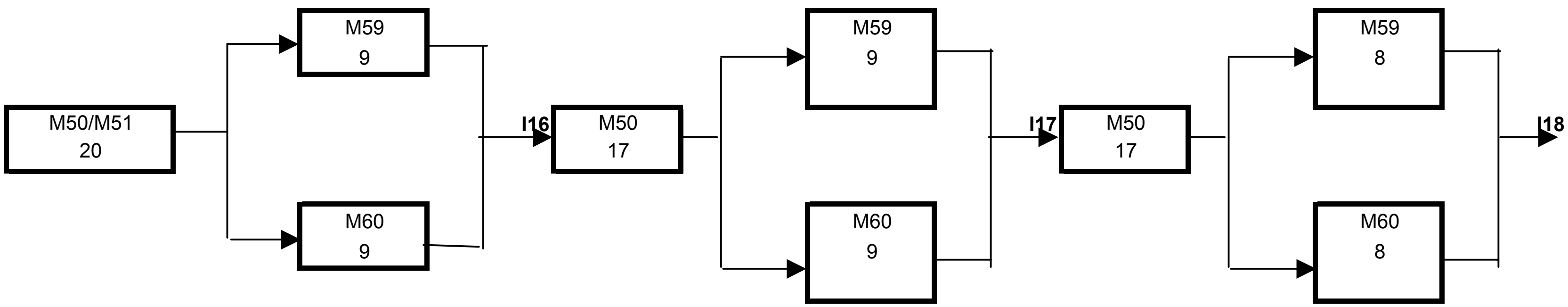






Panel B Process Flow Diagrams of Finished Goods and Intermediate Products

I16 **I17** **I18**



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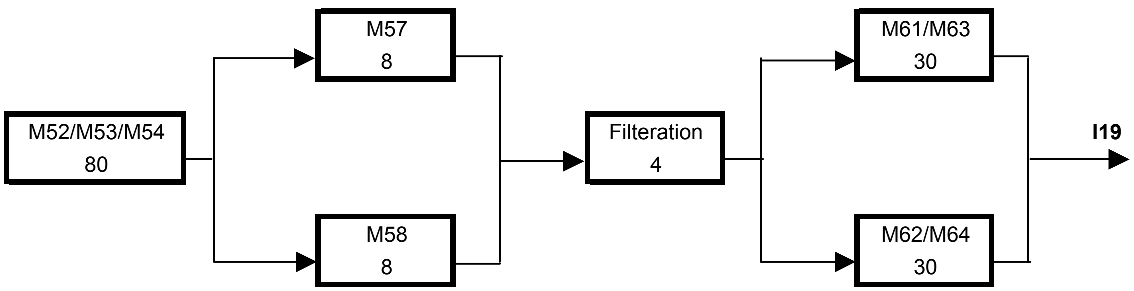

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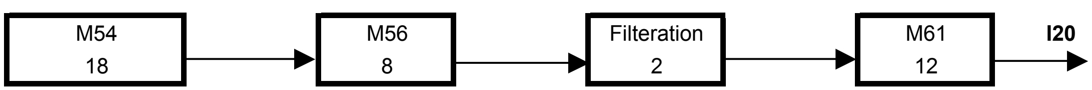
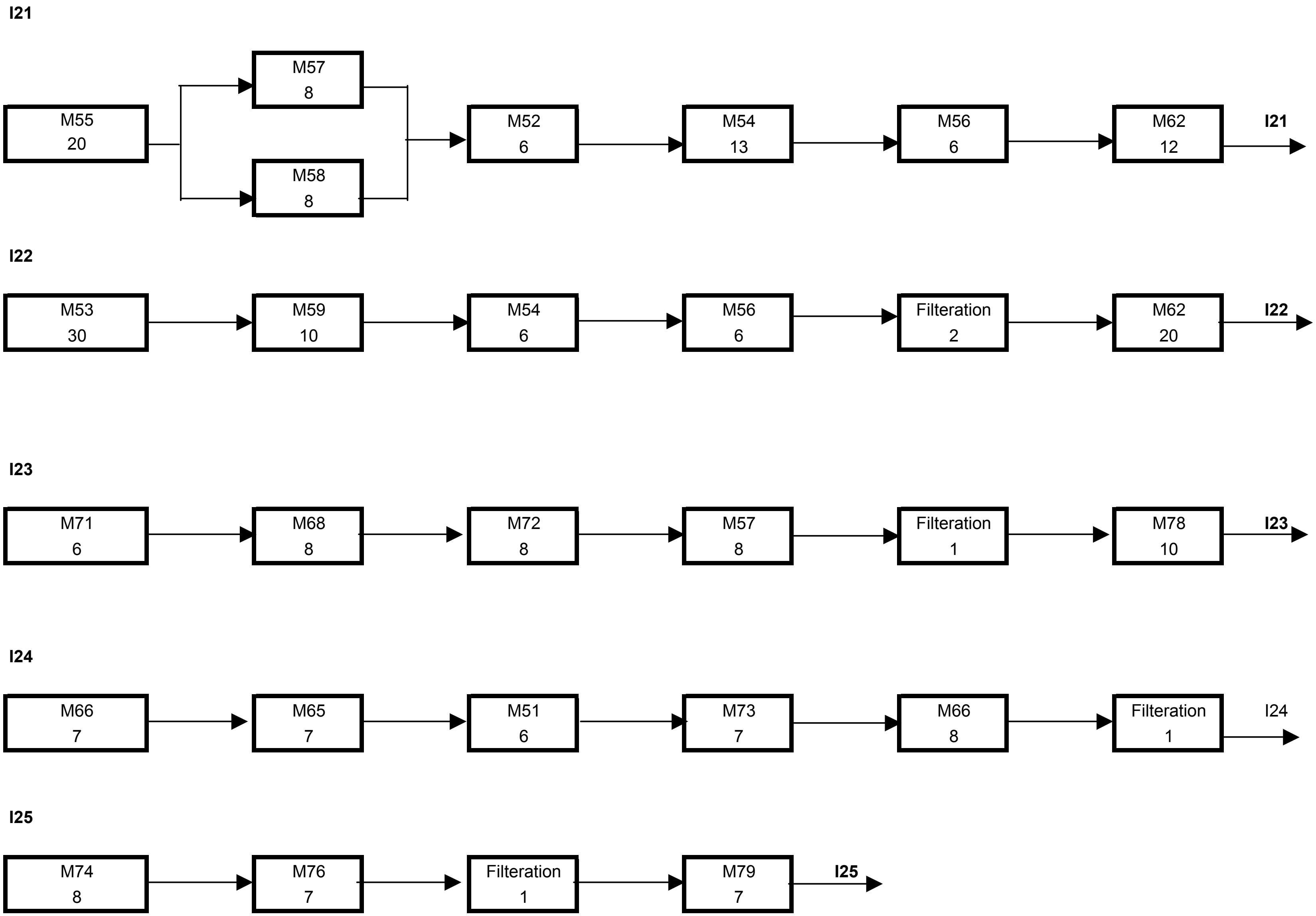
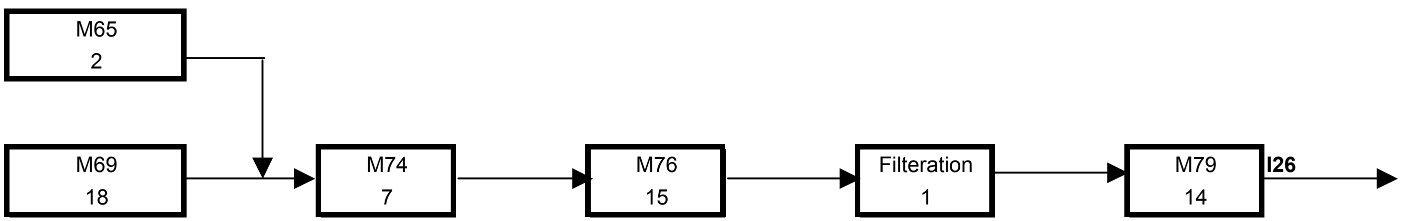


Panel B Process Flow Diagrams of Finished Goods and Intermediate Products



Panel B Process Flow Diagrams of Finished Goods and Intermediate Products

I26



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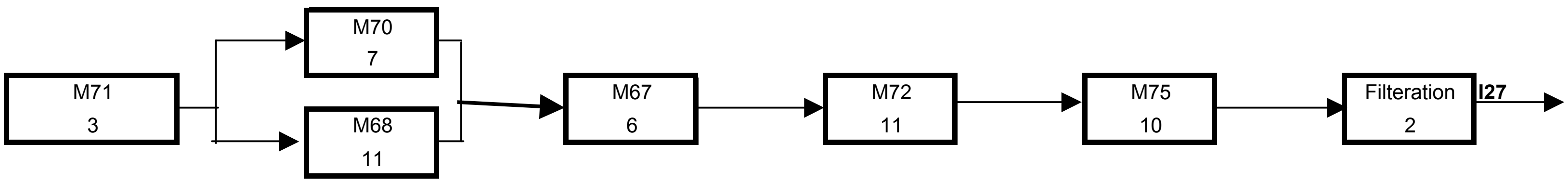

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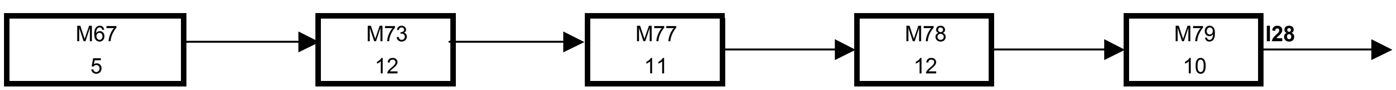
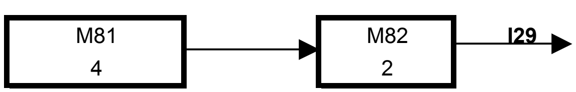


I29



Appendix 2: Base Case Production Plan and Schedule

Panel A: Prodction Quantities and Number of Setups

		Production Quantity of Finished Goods					Number of Setups				
		Time Period					Time Period				
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	13720.00	27000.00	27000.00	27000.00	27000.00	1.00	2.00	2.00	2.00	2.00
E1	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E2	3	1292.13	7000.00	7000.00	7000.00	7000.00	1.00	1.00	1.00	1.00	1.00
E3	4	661.63	2464.93	0.00	2173.44	1820.00	1.00	1.00	0.00	1.00	1.00
E4	5	0.00	1256.39	4053.26	776.05	1723.95	0.00	1.00	1.00	1.00	1.00
E5	6	1474.15	529.79	1024.50	947.57	0.00	1.00	1.00	1.00	1.00	0.00
E6	7	0.00	0.00	0.00	0.00	740.00	0.00	0.00	0.00	0.00	1.00
E7	8	2710.00	2200.00	0.00	1785.58	3801.04	1.00	1.00	0.00	1.00	1.00
E7	9	0.00	0.00	7949.42	8200.00	6198.96	0.00	0.00	1.00	1.00	1.00
E8	10	168.29	556.22	1100.00	1000.00	1000.00	1.00	1.00	1.00	1.00	1.00

		Production Quantity of Intermediate Products					Number of Setups				
		Time Period					Time Period				
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	17262.39	52452.26	52452.26	52452.26	0.00	1.00	2.00	2.00	2.00
I2	1	0.00	18247.77	55446.37	55446.37	55446.37	0.00	1.00	3.00	3.00	3.00
I3	2	0.00	0.00	17491.74	64501.43	64501.43	0.00	0.00	1.00	2.00	2.00
I4	2	0.00	2159.07	46571.43	46571.43	46571.43	0.00	1.00	4.00	4.00	4.00
I5	2	0.00	12096.79	70240.50	70240.50	70240.50	0.00	1.00	2.00	2.00	2.00
I6	3	0.00	27089.82	63222.77	63222.77	63222.77	0.00	1.00	2.00	2.00	2.00
I7	3	447.23	19740.12	22356.00	22356.00	22356.00	1.00	2.00	2.00	2.00	2.00
I8	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I9	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I10	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I11	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I12	3	0.00	0.00	0.00	0.00	2209.73	0.00	0.00	0.00	0.00	1.00
I13	3	0.00	0.00	0.00	0.00	1391.71	0.00	0.00	0.00	0.00	1.00
I14	3	0.00	824.70	0.00	2349.49	1967.42	0.00	1.00	0.00	1.00	1.00
I15	3	0.00	1439.17	5330.04	1020.50	2267.00	0.00	1.00	1.00	1.00	1.00
I16	3	515.80	0.00	1270.59	1617.40	0.00	1.00	0.00	1.00	1.00	0.00
I17	3	392.84	0.00	967.70	1231.83	0.00	1.00	0.00	1.00	1.00	0.00
I18	3	0.00	352.01	867.12	1103.79	0.00	0.00	1.00	1.00	1.00	0.00
I19	3	1150.00	899.16	1040.96	1325.08	0.00	1.00	1.00	1.00	2.00	0.00
I20	3	0.00	782.89	1040.96	1325.08	0.00	0.00	1.00	1.00	1.00	0.00
I21	3	0.00	666.44	814.03	1036.22	0.00	0.00	1.00	1.00	1.00	0.00
I22	3	1608.00	577.89	1117.52	1033.61	0.00	1.00	1.00	1.00	1.00	0.00
I23	4	2543.33	2064.69	7460.51	9371.43	9384.96	1.00	1.00	2.00	2.00	2.00
I24	4	2978.13	2417.67	8735.96	10973.57	10989.42	1.00	1.00	2.00	2.00	2.00
I25	4	3138.18	2547.60	9205.43	11563.30	11580.00	1.00	1.00	2.00	3.00	3.00
I26	4	0.00	0.00	454.05	0.00	0.00	0.00	0.00	1.00	0.00	0.00
I27	4	0.00	0.00	1121.10	0.00	0.00	0.00	0.00	1.00	0.00	0.00
I28	4	0.00	0.00	740.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00
I29	4	0.00	574.58	1136.30	1033.00	1033.00	0.00	1.00	1.00	1.00	1.00

Panel B: Inventory Levels of Products

	Inventory of Finished Goods					
	Initial Inventory	Time Period				
	0	1	2	3	4	5
E1	13280.00	0.00	0.00	0.00	0.00	0.00
E2	5707.87	0.00	0.00	0.00	0.00	0.00
E3	0.00	661.63	2736.56	476.56	0.00	0.00
E4	3861.35	3476.35	532.74	0.00	776.05	0.00
E5	0.00	1474.15	967.93	92.43	0.00	0.00
E6	510.00	510.00	510.00	510.00	510.00	0.00
E7	0.00	0.00	0.00	14.42	0.00	0.00
E8	875.49	543.78	0.00	0.00	0.00	0.00

	Inventory of Intermediate Products					
	Initial Inventory	Time Period				
	0	1	2	3	4	5
I1	0.00	0.00	0.00	0.00	0.00	0.00
I2	5510.00	5510.00	0.00	0.00	0.00	0.00
I3	50000.00	50000.00	47009.69	0.00	0.00	0.00
I4	60000.00	60000.00	51199.38	34132.92	17066.46	0.00
I5	18000.00	18000.00	0.00	0.00	0.00	0.00
I6	30000.00	28735.23	0.00	0.00	0.00	0.00
I7	12000.00	2615.88	0.00	0.00	0.00	0.00
I8	9.48	9.48	9.48	9.48	9.48	9.48
I9	2500.00	2500.00	2500.00	2500.00	2500.00	2500.00
I10	2500.00	2500.00	2500.00	2500.00	2500.00	2500.00
I11	5010.00	5010.00	5010.00	5010.00	5010.00	3509.74
I12	5664.00	5664.00	4673.85	1006.78	304.68	0.00
I13	4516.00	4516.00	3568.42	3568.42	868.86	0.00
I14	2555.11	1839.89	0.00	0.00	0.00	0.00
I15	212.98	212.98	0.00	0.00	0.00	0.00
I16	0.00	0.00	0.00	0.00	0.00	0.00
I17	0.00	392.84	0.00	0.00	0.00	0.00
I18	300.14	300.14	0.00	0.00	0.00	0.00
I19	1144.86	233.40	391.71	0.00	0.00	0.00
I20	1266.27	116.27	0.00	0.00	0.00	0.00
I21	936.00	36.70	0.00	0.00	0.00	0.00
I22	0.00	0.00	0.00	0.00	0.00	0.00
I23	0.00	0.00	0.00	0.00	0.00	0.00
I24	0.00	0.00	0.00	0.00	0.00	0.00
I25	0.00	0.00	0.00	0.00	0.00	0.00
I26	0.00	0.00	0.00	0.00	0.00	0.00
I27	0.00	0.00	0.00	0.00	0.00	0.00
I28	0.00	0.00	0.00	740.00	740.00	0.00
I29	173.84	0.00	0.00	0.00	0.00	0.00

Panel C: Capacity Utilized (in hours) in Dedicated Plants

	Time Period				
Plant	**1**	**2**	**3**	**4**	**5**
1	457.63	684.00	684.00	684.00	478.77
2	0.00	219.63	502.56	502.56	502.56
4	341.07	281.40	0.00	232.91	468.72
5	0.00	0.00	636.11	655.40	684.00
6	0.00	213.15	641.43	641.43	641.43
8	0.00	188.48	572.46	572.46	572.46
9	28.50	415.06	464.76	464.76	464.76
10	325.84	642.00	642.00	642.00	642.00
11	76.98	311.00	311.00	311.00	311.00
12	0.00	0.00	175.92	647.01	647.01
13	0.00	38.23	684.00	684.00	684.00
14	0.00	72.39	328.96	328.96	328.96
15	72.63	184.75	341.90	313.00	313.00
16	0.00	131.89	213.90	198.82	198.82

Available Capacity : 684 hours in a month

Panel D: Capacity utilized (in hours) of Machines in Flexible Plants

		Time Period				
Machine	**Plant**	**1**	**2**	**3**	**4**	**5**
M50	**3**	44.03	171.64	247.32	419.28	247.31
M51	**3**	35.21	105.25	69.18	340.26	292.41
M52	**3**	412.00	460.45	534.50	312.71	161.90
M53	**3**	345.23	283.07	689.47	720.00	220.32
M54	**3**	320.40	374.27	529.90	586.47	165.31
M55	**3**	12.00	243.86	453.27	300.40	131.05
M56	**3**	320.40	369.45	523.73	578.62	120.51
M57	**3**	393.33	518.42	720.00	720.00	99.57
M58	**3**	0.00	0.00	0.00	0.00	0.00
M59	**3**	367.46	254.40	366.80	595.23	260.11
M60	**3**	0.00	0.00	0.00	0.00	0.00
M61	**3**	399.03	418.93	497.09	638.22	105.58
M62	**3**	327.88	302.11	457.61	492.64	0.00
M63	**3**	0.00	0.00	0.00	0.00	0.00
M64	**3**	0.00	91.71	0.00	216.90	336.51
M65	**7**	171.12	143.43	505.73	590.09	590.88
M66	**7**	171.12	143.43	479.56	590.09	590.88
M67	**7**	150.27	126.51	470.52	537.28	513.95
M68	**7**	147.17	123.23	498.69	508.57	509.25
M69	**7**	0.00	0.00	92.01	0.00	0.00
M70	**7**	0.00	0.00	63.24	0.00	0.00
M71	**7**	116.65	98.46	364.32	396.11	396.63
M72	**7**	147.17	123.23	498.69	508.57	509.25
M73	**7**	171.12	143.43	570.90	614.09	590.88
M74	**7**	164.91	135.38	523.33	602.16	603.00
M75	**7**	147.17	123.23	493.08	508.57	509.25
M76	**7**	142.94	117.55	491.58	521.22	521.94
M77	**7**	171.12	143.43	564.98	614.09	590.88
M78	**7**	198.03	164.53	653.58	720.00	696.95
M79	**7**	142.94	117.55	567.74	545.22	521.94
M80	**7**	171.12	143.43	479.56	590.09	590.88

Available Capacity : 720 hours in a month

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

Legend	
O	Operation #
IS	Setup Start Time
IC	Setup Completion Time
S	Production Start Time of Product
C	Production Completion Time of Product
E1 to E8 and I1 to I29	Products
M1 to M80	Machines

Schedule Month 1

	I7								
	O	IS	IC	S	C	S	C	S	C
M83	1	6	26	26	32	32	38	38	44
M84	2	12	32	32	36	38	42	44	48
M85	3	16	36	36	38	42	44	48	50

	I22																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M53	1	112	136	136	166	166	196	196	226	226	256	256	286	286	316	316	346	346	376	376	406	406	436
M54	3	183	207	207	213	227	233	248	254	270	276	314	320	336	342	358	364	402	408	424	430	446	452
M56	4	189	213	213	219	233	239	254	260	276	282	320	326	342	348	364	370	408	414	430	436	452	458
M59	2	173	197	197	207	217	227	238	248	260	270	304	314	326	336	348	358	392	402	414	424	436	446
M62	5	195	219	219	239	239	259	260	280	282	302	326	346	348	368	370	390	414	434	436	456	458	478

	I19													I17						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	O	S	C	S	C	S	C
M50																				
M52	1	0	12	12	92	92	172	172	252	252	332	332	412	1	769	781	781	798	798	815
M57	2	236	248	248	256	278	286	338	346	368	376	412	420							
M59																				
M61	3	244	256	256	286	286	316	346	376	376	406	420	450	2	794	806	806	815	815	824

	E3														
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C
M16	1	104	116	116	119	131	134	146	149	161	164	176	179	191	194
M17	2	107	119	119	121	134	136	149	151	164	166	179	181	194	196
M18	3	109	121	121	133	136	148	151	163	166	178	181	193	196	208
M19	4	121	133	133	148	148	163	163	178	178	193	193	208	208	223
M20	5	136	148	148	157	163	172	178	187	193	202	208	217	223	232

	E5																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M16	1	203	251	251	254	259	262	280	283	302	305	324	327	346	349	368	371	390	393	412	415	434	437
M17	2	206	254	254	256	262	264	283	285	305	307	327	329	349	351	371	373	393	395	415	417	437	439
M18	3	208	256	256	278	264	286	285	307	307	329	329	351	351	373	373	395	395	417	417	439	439	461
M19	4	230	278	278	280	286	288	307	309	329	331	351	353	373	375	395	397	417	419	439	441	461	463
M20	5	232	280	280	288	288	296	309	317	331	339	353	361	375	383	397	405	419	427	441	449	463	471

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

	E5					
	O	C	S	C	S	C
M16	1	481	500	503	522	525
M17	2	483	503	505	525	527
M18	3	505	505	527	527	549
M19	4	507	527	529	549	551
M20	5	515	529	537	551	559

Schedule Month 2

	I1																					
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M31	1	424	430	430	450	450	470	470	490	490	510	510	530	530	550	550	570	570	590	590	610	610	630
M32	2	444	450	450	470	470	490	490	510	510	530	530	550	550	570	570	590	590	610	610	630	630	650

	I2																					
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M33	1	130	136	136	147	147	158	158	169	169	180	180	191	191	202	202	213	213	224	224	235	235	246
M34	2	141	147	147	149	158	160	169	171	180	182	191	193	202	204	213	215	224	226	235	237	246	248
M35	3	143	149	149	151	160	162	171	173	182	184	193	195	204	206	215	217	226	228	237	239	248	250
M36	4	145	151	151	155	162	166	173	177	184	188	195	199	206	210	217	221	228	232	239	243	250	254

	I4								
	O	IS	IC	S	C	S	C	S	C
M39	1	683	691	691	693	700	702	709	711
M40	2	685	693	693	702	702	711	711	720

	I5																					
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M41	1	621	645	645	651	651	657	657	663	663	669	669	675	675	681	681	687	687	693	693	699	699	705
M42	2	627	651	651	654	657	660	663	666	669	672	675	678	681	684	687	690	693	696	699	702	705	708

	I6																					
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M43	1	315	345	345	351	351	357	357	363	363	369	369	375	375	381	381	387	387	393	393	399	399	405
M44	2	321	351	351	355	357	361	363	367	369	373	375	379	381	385	387	391	393	397	399	403	405	409
M45	3	325	355	355	357	361	363	367	369	373	375	379	381	385	387	391	393	397	399	403	405	409	411
M46	4	327	357	357	362	363	368	369	374	375	380	381	386	387	392	393	398	399	404	405	410	411	416
M47	5	332	362	362	366	380	384	398	402	416	420												
M48	5	338	368	368	372	386	390	404	408														
M49	5	344	374	374	378	392	396	410	414														

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

I7

	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M83	1	0	20	20	26	26	32	32	38	38	44	44	50	50	56	56	62	62	68	68	74	74	80
M84	2	6	26	26	30	32	36	38	42	44	48	50	54	56	60	62	66	68	72	74	78	80	84
M85	3	8	28	28	30	36	38	42	44	48	50	54	56	60	62	66	68	72	74	78	80	84	86

I29

	O	IS	IC	S	C	S	C	S	C	S	C
M81	1	654	702	702	706	706	710	710	714	714	718
M82	2	658	706	706	708	710	712	714	716	718	720

I14

	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M50	1	260	284	284	294	299	309	314	324	329	339	344	354	359	369	374	384	389	399
M51	2	270	294	294	306	309	321	324	336	339	351	354	366	369	381	384	396	399	411
M59	3	282	306	306	316	321	331	336	346	351	361	366	376	381	391	396	406	411	421
M64	4	292	316	316	326	331	341	346	356	361	371	376	386	391	401	406	416	421	431

I15

	O	IS	IC	S	C	S	C	S	C	S	C	S	C
M50	2	425	449	449	455	466	472	483	489	500	506	517	523
M53	1	322	346	346	374	374	402	402	430	430	458	458	486
M55	3	431	455	455	472	472	489	489	506	506	523	523	540
M57	4	468	492	492	504	504	516	516	528	528	540	540	552

I18

	O	IS	IC	S	C	S	C
M50	1	202	214	214	231	231	260
M59	2	230	242	242	250	484	504
M60	3	238	250	250	258	492	512

I19

	O	IS	IC	S	C	S	C	S	C	S	C
M52	1	208	220	220	300	300	380	380	460	679	771
M57	2	432	444	444	452	452	460	460	468	759	779
M61	3	498	510	510	540	540	570	570	600	767	809

I20

	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C
M54	1	116	128	128	146	240	258	258	276	312	330	500	530	530	548
M56	2	146	158	158	166	268	276	276	284	342	350	528	548	548	556
M61	3	196	208	208	220	276	288	288	300	368	380	655	679	679	691

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

	I21												
	O	IS	IC	S	C	S	C	S	C	S	C	S	C
M52	3	140	164	164	170	170	176	190	196	196	202	202	208
M54	4	146	170	170	183	183	196	227	240	330	343	343	356
M55	1	70	94	94	114	114	134	134	154	154	174	174	194
M56	5	166	190	190	196	196	202	262	268	350	356	356	362
M57	2	130	154	154	162	162	170	178	186	186	194	194	202
M62	6	172	196	196	208	208	220	288	300	356	368	368	380

	I22										
	O	IS	IC	S	C	S	C	S	C	S	C
M53	1	486	510	510	540	540	570	570	600	600	630
M54	3	550	574	574	580	596	602	618	624	640	646
M56	4	556	580	580	586	602	608	624	630	646	652
M59	2	540	564	564	574	586	596	608	618	630	640
M62	5	562	586	586	606	608	628	630	650	652	672

	E3																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M16	1	89	101	101	104	116	119	131	134	146	149	161	164	176	179	191	194	206	209	221	224	236	239
M17	2	92	104	104	106	119	121	134	136	149	151	164	166	179	181	194	196	209	211	224	226	239	241
M18	3	94	106	106	118	121	133	136	148	151	163	166	178	181	193	196	208	211	223	226	238	241	253
M19	4	106	118	118	133	133	148	148	163	163	178	178	193	193	208	208	223	223	238	238	253	253	268
M20	5	124	136	136	145	151	160	166	175	181	190	196	205	211	220	226	235	241	250	256	265	271	280

	E3																						
	O	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M16	1	266	269	281	284	296	299	311	314	326	329	341	344	356	359	371	374	386	389	401	404	416	419
M17	2	269	271	284	286	299	301	314	316	329	331	344	346	359	361	374	376	389	391	404	406	419	421
M18	3	271	283	286	298	301	313	316	328	331	343	346	358	361	373	376	388	391	403	406	418	421	433
M19	4	283	298	298	313	313	328	328	343	343	358	358	373	373	388	388	403	403	418	418	433	433	448
M20	5	301	310	316	325	331	340	346	355	361	370	376	385	391	400	406	415	421	430	436	445	451	460

	E4																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M16	1	450	498	498	502	504	508	510	514	516	520	522	526	528	532	534	538	540	544	546	550	552	556
M17	2	451	499	499	503	508	509	514	515	520	521	526	527	532	533	538	539	544	545	550	551	556	557
M18	3	455	503	503	509	509	515	515	521	521	527	527	533	533	539	539	545	545	551	551	557	557	563
M19	4	463	511	511	523	523	535	535	547	547	559	559	571	571	583	583	595	595	607	607	619	619	631
M20	5	475	523	523	535	535	547	547	559	559	571	571	583	583	595	595	607	607	619	619	631	631	643

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

	E5												
	O	IS	IC	S	C	S	C	S	C	S	C	S	C
M16	1	558	606	606	609	628	631	650	653	672	675	694	697
M17	2	561	609	609	611	631	633	653	655	675	677	697	699
M18	3	563	611	611	633	633	655	655	677	677	699	699	721
M19	4	641	689	689	691	697	699	705	707	713	715	721	723
M20	5	643	691	691	699	699	707	707	715	715	723	723	731

Schedule Month 3

	I1																				
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M31	1	18	24	24	44	44	64	64	84	84	104	104	124	124	144	144	164	164	184	184	204
M32	2	38	44	44	64	64	84	84	104	104	124	124	144	144	164	164	184	184	204	204	224

	I1												
	O	S	C	S	C	S	C	S	C	S	C	S	C
M31	1	204	224	224	244	244	264	264	284	284	304	304	324
M32	2	224	244	244	264	264	284	284	304	304	324	324	344

	I2																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M33	1	0	6	6	17	17	28	28	39	39	50	50	61	61	72	72	83	83	94	94	105	105	116
M34	2	11	17	17	19	28	30	39	41	50	52	61	63	72	74	83	85	94	96	105	107	116	118
M35	3	13	19	19	21	30	32	41	43	52	54	63	65	74	76	85	87	96	98	107	109	118	120
M36	4	17	23	21	25	32	36	43	47	50	54	61	65	71	75	82	86	93	97	104	108	120	124

	I3																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M37	1	625	626	626	630	632	636	638	642	644	648	650	654	656	660	662	666	668	672	674	678	680	684
M38	2	629	630	630	636	636	642	642	648	648	654	654	660	660	666	666	672	672	678	678	684	684	690
				S	C	S	C	S	C	S	C	S	C										
M37				686	690	692	696	698	702	704	708	710	714										
M38				690	696	696	702	702	708	708	714	714	720										

	I4																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M39	1	182	190	190	192	199	201	208	210	217	219	226	228	235	237	244	246	253	255	262	264	271	273
M40	2	184	192	192	201	201	210	210	219	219	228	228	237	237	246	246	255	255	264	264	273	273	282

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

	I5																					
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M41	1	255	279	279	285	285	291	291	297	297	303	303	309	309	315	315	321	321	327	327	333	333	339
M42	2	261	285	285	288	291	294	297	300	303	306	309	312	315	318	321	324	327	330	333	336	339	342

	I6																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M43	1	0	30	30	36	36	42	42	48	48	54	54	60	60	66	66	72	72	78	78	84	84	90
M44	2	6	36	36	40	42	46	48	52	54	58	60	64	66	70	72	76	78	82	84	88	90	94
M45	3	10	40	40	42	46	48	52	54	58	60	64	66	70	72	76	78	82	84	88	90	94	96
M46	4	12	32	42	47	48	53	54	59	60	65	66	71	72	77	78	83	84	89	90	95	96	101
M47	5	17	47	47	51	65	69	83	87	101	105												
M48	5	23	53	53	57	71	75	89	93														
M49	5	29	59	59	63	77	81	95	99														

	I7																				
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M83	1	25	31	31	37	37	43	43	49	49	55	55	61	61	67	67	73	73	79	79	85
M84	2	31	35	37	41	43	47	49	53	55	59	61	65	67	71	73	77	79	83	85	89
M85	3	33	35	41	43	47	49	53	55	59	61	65	67	71	73	77	79	83	85	89	91

	I29																		
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M81	1	638	686	686	690	690	694	694	698	698	702	702	706	706	710	710	714	714	718
M82	2	642	690	690	692	694	696	698	700	702	704	706	708	710	712	714	716	718	720

	I15																
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M50	2	185	209	209	215	233	239	257	263	281	287	305	311	329	335	353	359
M53	1	0	24	24	52	52	80	80	108	108	136	136	164	164	192	192	220
M55	3	191	215	215	232	239	256	263	280	287	304	311	328	335	352	359	376
M57	4	208	232	232	244	256	268	280	292	304	316	328	340	352	364	376	388

	I17								
	O	IS	IC	S	C	S	C	S	C
M50	1	582	594	594	611	611	628	628	645
M59	2	636	648	648	657	662	671	671	680

	I18								
	O	IS	IC	S	C	S	C	S	C
M50	1	645	657	657	674	674	691	691	708
M59	2	680	692	692	700	700	708	708	716
M60	3	688	700	700	708	708	716	716	724

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

I22

	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M53	1	220	244	244	274	274	304	304	334	334	364	364	394	394	424	424	454	454	484
M54	3	330	354	354	360	374	380	394	400	414	420	434	440	454	460	474	480	494	500
M56	4	336	360	360	366	380	386	400	406	420	426	440	446	460	466	480	486	500	506
M59	2	320	344	344	354	364	374	384	394	404	414	424	434	444	454	464	474	484	494
M62	5	342	366	366	386	386	406	406	426	426	446	446	466	466	486	486	506	506	526

E4

	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M16	1	4	52	52	56	64	68	76	80	88	92	100	104	112	116	124	128	136	140	148	152	160	164
M17	2	8	56	56	57	68	69	80	81	92	93	104	105	116	117	128	129	140	141	152	153	164	165
M18	3	9	57	57	63	69	75	81	87	93	99	105	111	117	123	129	135	141	147	153	159	165	171
M19	4	15	63	63	75	75	87	87	99	99	111	111	123	123	135	135	147	147	159	159	171	171	183
M20	5	27	75	75	87	87	99	99	111	111	123	123	135	135	147	147	159	159	171	171	183	183	195

E4

	O	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M16	1	184	188	196	200	208	212	220	224	232	236	244	248	256	260	268	272	280	284	292	296	304	308
M17	2	188	189	200	201	212	213	224	225	236	237	248	249	260	261	272	273	284	285	296	297	308	309
M18	3	189	195	201	207	213	219	225	231	237	243	249	255	261	267	273	279	285	291	297	303	309	315
M19	4	195	207	207	219	219	231	231	243	243	255	255	267	267	279	279	291	291	303	303	315	315	327
M20	5	207	219	219	231	231	243	243	255	255	267	267	279	279	291	291	303	303	315	315	327	327	339

E4

	O	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M16	1	328	332	340	344	352	356	364	368	376	380	388	392	400	404
M17	2	332	333	344	345	356	357	368	369	380	381	392	393	404	405
M18	3	333	339	345	351	357	363	369	375	381	387	393	399	405	411
M19	4	339	351	351	363	363	375	375	387	387	399	399	411	411	423
M20	5	351	363	363	375	375	387	387	399	399	411	411	423	423	435

E5

	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M16	1	406	454	454	457	462	465	470	473	478	481	486	489	494	497	502	505	510	513	518	521	526	529
M17	2	409	457	457	459	465	467	473	475	481	483	489	491	497	499	505	507	513	515	521	523	529	531
M18	3	411	459	459	481	467	489	475	497	483	505	491	513	499	521	507	529	515	537	523	545	531	553
M19	4	433	481	481	483	489	491	497	499	505	507	513	515	521	523	529	531	537	539	545	547	553	555
M20	5	435	483	483	491	491	499	499	507	507	515	515	523	523	531	531	539	539	547	547	555	555	563

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

Schedule Month 4																							
	I1																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C		
M31	1	28	34	34	54	54	74	74	94	94	114	114	134	134	154	154	174	174	194	194	214		
M32	2	58	64	64	74	74	94	94	114	114	134	134	154	154	174	174	194	194	214	214	234		
	I1																						
	O	S	C	S	C	S	C	S	C	S	C	S	C										
M31	1	214	234	234	254	254	274	274	294	294	314	314	334										
M32	2	234	254	254	274	274	294	294	314	314	334	334	354										
	I2																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M33	1	10	16	16	27	27	38	38	49	49	60	60	71	71	82	82	93	93	104	104	115	115	126
M34	2	21	27	27	29	38	40	49	51	60	62	71	73	82	84	93	95	104	106	115	117	126	128
M35	3	23	29	29	31	40	42	51	53	62	64	73	75	84	86	95	97	106	108	117	119	128	130
M36	4	27	33	31	35	42	46	53	57	60	64	71	75	81	85	92	96	103	107	114	118	130	134
	I3																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M37	1	384	385	385	389	391	395	397	401	403	407	409	413	415	419	421	425	427	431	433	437	439	443
M38	2	388	389	389	395	395	401	401	407	407	413	413	419	419	425	425	431	431	437	437	443	443	449
	I4																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M39	1	192	200	200	202	209	211	218	220	227	229	236	238	245	247	254	256	263	265	272	274	281	28.3
M40	2	194	202	202	211	211	220	220	229	229	238	238	247	247	256	256	265	265	274	274	283	283	292
	I5																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M41	1	265	289	289	295	295	301	301	307	317	313	313	319	319	325	325	331	331	337	337	343	343	349
M42	2	271	305	295	298	301	304	307	310	313	316	319	322	325	328	331	334	337	340	343	346	349	352
	I6																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M43	1	10	40	40	46	46	52	52	58	58	64	64	70	70	76	76	82	82	88	88	94	94	100
M44	2	16	46	46	50	52	56	58	62	64	68	70	74	76	80	82	86	88	92	94	98	100	104
M45	3	20	50	50	52	56	58	62	64	68	70	74	76	80	82	86	88	92	94	98	100	104	106
M46	4	22	42	52	57	58	63	64	69	70	75	76	81	82	87	88	93	94	99	100	105	106	111
M47	5	27	57	57	61	75	79	93	97	111	115												
M48	5	33	63	63	67	81	85	99	103														
M49	5	39	69	69	73	87	91	105	109														

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

	I7																				
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M83	1	30	36	36	42	42	48	48	54	54	60	60	66	66	72	72	78	78	84	84	90
M84	2	36	40	42	46	48	52	54	58	60	64	66	70	72	76	78	82	84	88	90	94
M85	3	38	40	46	48	52	54	58	60	64	66	70	72	76	78	82	84	88	90	94	96

	I29																
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M81	1	642	690	690	694	694	698	698	702	702	706	706	710	710	714	714	718
M82	2	646	694	694	696	698	700	702	704	706	708	710	712	714	716	718	720

	I14																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M50	1	195	219	219	229	231	241	243	253	255	265	267	277	279	289	291	301	303	313	315	325	327	337
M51	2	205	229	229	241	241	253	253	265	265	277	277	289	289	301	301	313	313	325	325	337	337	349
M59	3	217	241	241	251	253	263	265	275	277	287	289	299	301	311	313	323	325	335	337	347	349	359
M64	4	227	251	251	261	263	273	275	285	287	297	299	309	311	321	323	333	335	345	347	357	359	369

	I14																		
	O	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M50	1	351	361	363	373	375	385	387	397	399	409	411	421	423	433	435	445	447	457
M51	2	361	373	373	385	385	397	397	409	409	421	421	433	433	445	445	457	457	469
M59	3	373	383	385	395	397	407	409	419	421	431	433	443	445	455	457	467	469	479
M64	4	383	393	395	405	407	417	419	429	431	441	443	453	455	465	467	477	479	489

	I15								
	O	IS	IC	S	C	S	C	S	C
M50	2	480	504	504	510	521	527	538	544
M53	1	403	427	427	455	455	483	483	511
M55	3	486	510	510	527	527	544	544	561
M57	4	513	537	537	549	549	561	561	573

	I16										
	O	IS	IC	S	C	S	C	S	C	S	C
M50	1	835	847	847	867	867	899	899	919	919	939
M59	2	888	900	900	909	909	930	930	939	939	948

	I17				
	O	IS	IC	S	C
M50	1	931	943	943	960
M59	2	948	960	960	969

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

I18

	O	IS	IC	S	C	S	C	S	C	S	C	S	C
M50	1	0	12	12	29	92	109	144	161	161	178	178	195
M59	2	17	25	25	37	109	117	193	201	201	209	209	217
M60	3	25	37	37	45	117	125	219	227	379	387	417	425

I19

	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M52	1	93	105	105	185	185	265	265	345	345	425	425	505	505	597	597	677
M57	2	469	481	481	489	489	497	497	505	505	513	605	613	635	655	677	685
M61	3	481	493	493	523	523	553	553	583	583	613	613	643	643	685	685	715

I20

	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M54	1	33	45	45	63	63	81	125	143	143	161	227	245	285	303	303	321	387	405	407	425	425	443
M56	2	61	73	73	81	81	89	153	161	161	169	245	253	313	321	321	329	405	413	437	445	445	453
M61	3	69	81	81	93	93	105	161	173	173	185	253	265	321	333	333	345	413	425	445	457	457	469

I21

	O	IS	IC	S	C	S	C	S	C
M52	3	870	894	894	900	900	906	906	912
M54	4	876	900	900	913	913	926	926	939
M55	1	814	838	838	858	858	878	878	898
M56	5	903	927	927	933	933	939	939	945
M57	2	858	882	882	890	890	898	898	906
M62	6	909	933	933	945	945	957	957	969

I22

	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M53	1	511	535	535	565	565	595	595	625	625	655	655	685	685	715	715	745
M54	3	577	601	601	607	623	629	645	651	667	673	711	717	733	739	755	761
M56	4	583	607	607	613	629	635	651	657	673	679	717	723	739	745	761	767
M59	2	567	591	591	601	613	623	635	645	657	667	701	711	723	733	745	755
M62	5	589	613	613	633	635	655	657	677	679	699	723	743	745	765	767	787

E3

	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M16	1	249	261	261	264	273	276	285	288	297	300	309	312	321	324	333	336	345	348	357	360	369	372
M17	2	252	264	264	266	276	278	288	290	300	302	312	314	324	326	336	338	348	350	360	362	372	374
M18	3	254	266	266	278	278	290	290	302	302	314	314	326	326	338	338	350	350	362	362	374	374	386
M19	4	266	278	278	293	293	308	308	323	323	338	338	353	353	368	368	383	383	398	398	413	413	428
M20	5	284	296	296	305	311	320	326	335	341	350	356	365	371	380	386	395	401	410	416	425	431	440

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

	E3										
	O	S	C	S	C	S	C	S	C	S	C
M16	1	393	396	405	408	414	419.4	425	430	435.6	441
M17	2	396	398	408	410	419	421.2	430	432	441	443
M18	3	398	410	410	422	421	435.2	432	450	443	464
M19	4	443	458	458	473	475	489.5	450	508	508	526
M20	5	461	470	476	485	493	500.3	508	516	526	532

	E4								
	O	IS	IC	S	C	S	C	S	C
M16	1	501	549	549	553	555	559	561	565
M17	2	505	553	553	554	559	560	565	566
M18	3	506	554	554	560	560	566	566	572
M19	4	578	626	626	638	638	650	650	662
M20	5	590	638	638	650	650	662	662	674

	E5										
	O	IS	IC	S	C	S	C	S	C	S	C
M16	1	585	633	633	636	655	658	677	680	691	700.4
M17	2	588	636	636	638	658	660	680	682	700	702.2
M18	3	590	638	638	660	660	682	682	704	704	728.2
M19	4	708	756	756	758	764	766	772	774	774	781.6
M20	5	710	758	758	766	766	774	774	782	782	790.8

Schedule Month 5

	I1																				
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M31	1	23	29	29	49	49	69	69	89	89	109	109	129	129	149	149	169	169	189	189	209
M32	2	43	49	49	69	69	89	89	109	109	129	129	149	149	169	169	189	189	209	209	229

	I1												
	O	S	C	S	C	S	C	S	C	S	C	S	C
M31	1	209	229	229	249	249	269	269	289	289	309	309	329
M32	2	229	249	249	269	269	289	289	309	309	329	329	349

	I2																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M33	1	5	11	11	22	22	33	33	44	44	55	55	66	66	77	77	88	88	99	99	110	110	121
M34	2	16	22	22	24	33	35	44	46	55	57	66	68	77	79	88	90	94	101	110	112	121	123
M35	3	18	24	24	26	35	37	46	48	57	59	68	70	79	81	93	92	101	103	112	114	123	125
M36	4	22	28	26	30	37	41	48	52	55	59	66	75	76	80	87	91	98	102	109	113	125	129

	I3																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M37	1	379	380	380	384	386	390	392	396	398	402	404	408	410	414	416	420	422	426	428	432	434	438
M38	2	383	384	384	390	390	396	396	402	402	408	408	414	414	420	420	426	426	432	432	438	438	444

Panel E: Monthly Schedule of Intermediate Products and Finished Goods

	I4																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M39	1	187	195	195	197	204	206	213	215	222	224	231	233	240	242	249	251	258	260	267	269	276	278
M40	2	189	197	197	206	206	215	215	224	224	233	233	242	242	251	251	260	260	269	269	278	278	287
	I5																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M41	1	260	284	284	290	290	296	296	302	302	308	308	314	314	320	320	326	326	332	332	338	338	344
M42	2	266	295	290	293	296	299	302	305	308	311	314	317	320	323	326	329	332	335	338	341	344	347
	I6																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C
M43	1	5	35	35	41	41	47	47	53	53	59	59	65	65	71	71	77	77	83	83	89	89	95
M44	2	11	41	41	45	47	51	53	57	59	63	65	69	71	75	77	81	83	87	89	93	95	99
M45	3	15	45	45	47	51	53	57	59	63	65	69	71	75	77	81	83	87	89	93	95	99	101
M46	4	17	37	47	52	53	58	59	64	65	70	71	76	77	82	83	88	89	94	95	100	101	106
M47	5	22	52	52	56	70	74	88	92	106	110												
M48	5	28	58	58	62	76	80	94	98														
M49	5	34	64	64	68	82	86	100	104														
	I7																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C		
M83	1	40	46	46	52	52	58	58	64	64	70	70	76	76	82	82	88	88	94	94	95		
M84	2	46	50	52	56	58	62	64	68	70	74	76	80	82	86	88	92	94	98	100	99		
M85	3	48	50	56	58	62	64	68	70	74	76	80	82	86	88	92	94	98	100	104	101		
	I29																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C						
M81	1	642	690	690	694	694	698	698	702	702	706	706	710	710	714	714	718						
M82	2	646	694	694	696	698	700	702	704	706	708	710	712	714	716	718	720						
	I14																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C				
M50	1	0	24	24	34	39	49	54	64	69	79	84	94	99	109	114	124	129	139				
M51	2	10	34	34	46	49	61	64	76	79	91	94	106	109	121	124	136	139	151				
M59	3	22	46	46	56	61	71	76	86	91	101	106	116	121	131	136	146	151	161				
M64	4	32	56	56	66	71	81	86	96	101	111	116	126	131	141	146	156	161	171				
	I15																						
	O	IS	IC	S	C	S	C	S	C	S	C	S	C										
M50	2	316	340	340	346	364	370	388	394	412	418	436	442										
M53	1	264	288	288	316	316	344	344	372	372	400	400	428										
M55	3	322	346	346	363	370	387	394	411	418	435	442	459										
M57	4	339	363	363	375	387	399	411	423	435	447	459	471										

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 1- Base Case											
			Production Quantity of Finished Goods				Number of Setups				
				Time Period				Time Period			
Product	**Plant**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
E1	1	13720.00	27000.00	27000.00	27000.00	27000.00	1	2	2	2	2
E1	2	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	3	1292.13	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	661.63	2464.93	0.00	2173.44	1820.00	1	1	0	1	1
E4	5	0.00	1256.39	4053.26	776.05	1723.95	0	1	1	1	1
E5	6	1474.15	529.79	1024.50	947.57	0.00	1	1	1	1	0
E6	7	0.00	0.00	0.00	0.00	740.00	0	0	0	0	1
E7	8	2710.00	2200.00	0.00	1785.58	3801.04	1	1	0	1	1
E7	9	0.00	0.00	7949.42	8200.00	6198.96	0	0	1	1	1
E8	10	168.29	556.22	1100.00	1000.00	1000.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
			Production Quantity of Intermediate Products				Number of Setups				
				Time Period				Time Period			
Product	**Plant**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
I1	1	0.00	17262.39	52452.26	52452.26	52452.26	0	1	2	2	2
I2	1	0.00	18247.77	55446.37	55446.37	55446.37	0	1	3	3	3
I3	2	0.00	0.00	17491.74	64501.43	64501.43	0	0	1	2	2
I4	2	0.00	2159.07	46571.43	46571.43	46571.43	0	1	4	4	4
I5	2	0.00	12096.79	70240.50	70240.50	70240.50	0	1	2	2	2
I6	3	0.00	27089.82	63222.77	63222.77	63222.77	0	1	2	2	2
I7	3	447.23	19740.12	22356.00	22356.00	22356.00	1	2	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	0.00	2209.73	0	0	0	0	1
I13	3	0.00	0.00	0.00	0.00	1391.71	0	0	0	0	1
I14	3	0.00	824.70	0.00	2349.49	1967.42	0	1	0	1	1
I15	3	0.00	1439.17	5330.04	1020.50	2267.00	0	1	1	1	1
I16	3	515.80	0.00	1270.59	1617.40	0.00	1	0	1	1	0
I17	3	392.84	0.00	967.70	1231.83	0.00	1	0	1	1	0
I18	3	0.00	352.01	867.12	1103.79	0.00	0	1	1	1	0
I19	3	1150.00	899.16	1040.96	1325.08	0.00	1	1	1	2	0
I20	3	0.00	782.89	1040.96	1325.08	0.00	0	1	1	1	0
I21	3	0.00	666.44	814.03	1036.22	0.00	0	1	1	1	0
I22	3	1608.00	577.89	1117.52	1033.61	0.00	1	1	1	1	0
I23	4	2543.33	2064.69	7460.51	9371.43	9384.96	1	1	2	2	2
I24	4	2978.13	2417.67	8735.96	10973.57	10989.42	1	1	2	2	2
I25	4	3138.18	2547.60	9205.43	11563.30	11580.00	1	1	2	3	3
I26	4	0.00	0.00	454.05	0.00	0.00	0	0	1	0	0
I27	4	0.00	0.00	1121.10	0.00	0.00	0	0	1	0	0
I28	4	0.00	0.00	740.00	0.00	0.00	0	0	1	1	0
I29	4	0.00	574.58	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 2- 80% Demand											
			Production Quantity of Finished Goods				Number of Setups				
				Time Period				Time Period			
Product	**Plant**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
E1	**1**	8320.00	21600.00	21600.00	21600.00	21600.00	1	2	2	2	2
E1	**2**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	**3**	0.00	5492.13	5600.00	5600.00	5600.00	0	1	1	1	1
E3	**4**	0.00	2150.85	0.00	2089.15	1456.00	0	1	0	1	1
E4	**5**	0.00	0.00	3475.45	1160.74	839.26	0	0	1	1	1
E5	**6**	0.00	1063.44	1285.36	832.00	0.00	0	1	1	1	0
E6	**7**	0.00	0.00	0.00	0.00	490.00	0	0	0	0	1
E7	**8**	2168.00	1760.00	0.00	0.00	1210.13	1	1	0	0	1
E7	**9**	0.00	0.00	6348.00	8000.00	6789.87	0	0	1	1	1
E8	**10**	168.29	880.00	880.00	800.00	800.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
			Production Quantity of Intermediate Products				Number of Setups				
				Time Period				Time Period			
Product	**Plant**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
I1	**1**	0.00	0.00	38243.30	41961.81	41961.81	0	0	2	2	2
I2	**1**	0.00	0.00	40426.32	44357.09	44357.09	0	0	3	3	3
I3	**2**	0.00	0.00	0.00	0.00	58356.12	0	0	0	0	2
I4	**2**	0.00	0.00	0.00	31664.04	46571.43	0	0	0	3	4
I5	**2**	0.00	0.00	40192.99	56192.40	56192.40	0	0	1	2	2
I6	**3**	0.00	1800.71	50578.21	50578.21	50578.21	0	1	2	2	2
I7	**3**	667.17	10577.78	17884.80	17884.80	17884.80	1	1	2	2	2
I8	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I13	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I14	**3**	0.00	0.00	0.00	2028.33	1573.94	0	0	0	1	1
I15	**3**	0.00	0.00	4357.24	1526.37	1103.63	0	0	1	1	1
I16	**3**	0.00	0.00	626.33	1420.14	0.00	0	0	1	1	0
I17	**3**	0.00	0.00	477.02	1081.60	0.00	0	0	1	1	0
I18	**3**	0.00	0.00	427.44	969.17	0.00	0	0	1	1	0
I19	**3**	1150.00	46.93	942.78	1163.47	0.00	1	1	1	2	0
I20	**3**	0.00	0.00	873.45	1163.47	0.00	0	0	1	1	0
I21	**3**	0.00	0.00	737.26	909.84	0.00	0	0	1	1	0
I22	**3**	0.00	1160.01	1402.07	907.55	0.00	0	2	1	1	0
I23	**4**	2034.66	1651.75	5957.58	7507.97	7507.97	1	1	2	2	2
I24	**4**	2382.51	1934.14	6976.08	8791.54	8791.54	1	1	2	2	2
I25	**4**	2510.54	2038.08	7350.98	9264.00	9264.00	1	1	2	2	2
I26	**4**	0.00	0.00	0.00	0.00	300.65	0	0	0	0	1
I27	**4**	0.00	0.00	0.00	0.00	742.35	0	0	0	0	1
I28	**4**	0.00	0.00	0.00	0.00	490.00	0	0	0	0	1
I29	**4**	0.00	909.04	909.04	826.40	826.40	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 3- 90% Demand											
		Production Quantity of Finished Goods					Number of Setups				
		Time Period					Time Period				
Product	**Plant**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
E1	**1**	11020.00	24300.00	24300.00	24300.00	24300.00	1	2	2	2	2
E1	**2**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	**3**	592.13	6300.00	6300.00	6300.00	6300.00	1	1	1	1	1
E3	**4**	0.00	2565.93	0.00	2204.07	1638.00	0	1	0	1	1
E4	**5**	0.00	416.69	3975.86	754.90	1495.10	1	1	1	1	1
E5	**6**	743.70	839.25	1059.44	936.00	0.00	1	1	1	1	0
E6	**7**	0.00	0.00	0.00	0.00	615.00	0	0	0	0	1
E7	**8**	0.00	1980.00	0.00	800.00	2505.58	0	1	0	1	1
E7	**9**	2439.00	0.00	7141.50	8200.00	6494.42	1	0	1	1	1
E8	**10**	168.29	396.22	990.00	900.00	900.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
		Production Quantity of Intermediate Products					Number of Setups				
		Time Period					Time Period				
Product	**Plant**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
I1	**1**	0.00	6771.94	47207.04	47207.04	47207.04	0	1	2	2	2
I2	**1**	0.00	7158.50	49901.73	49901.73	49901.73	0	1	3	3	3
I3	**2**	0.00	0.00	0.00	37923.93	64501.43	0	0	0	1	2
I4	**2**	0.00	0.00	16911.56	46571.43	46571.43	0	0	2	4	4
I5	**2**	0.00	0.00	61265.14	63216.45	63216.45	0	0	2	2	2
I6	**3**	0.00	14445.27	56900.49	56900.49	56900.49	0	1	2	2	2
I7	**3**	557.20	15158.95	20120.40	20120.40	20120.40	1	1	2	2	2
I8	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	**3**	0.00	0.00	0.00	0.00	547.16	0	0	0	0	1
I13	**3**	0.00	0.00	0.00	0.00	507.36	0	0	0	0	1
I14	**3**	218.66	0.00	0.00	2382.60	1770.68	1	0	0	1	1
I15	**3**	0.00	334.97	5228.26	992.69	1966.06	0	1	1	1	1
I16	**3**	0.00	0.00	1127.48	1597.65	0.00	0	0	1	1	0
I17	**3**	0.00	0.00	858.70	1216.80	0.00	0	0	1	1	0
I18	**3**	0.00	0.00	769.45	1090.32	0.00	0	0	1	1	0
I19	**3**	1150.00	352.01	1048.28	1308.91	0.00	1	1	1	2	0
I20	**3**	0.00	235.74	1048.28	1308.91	0.00	0	1	1	1	0
I21	**3**	0.00	238.57	819.75	1023.57	0.00	0	1	1	1	0
I22	**3**	811.23	915.46	1155.64	1020.99	0.00	1	1	1	1	0
I23	**4**	2288.99	1858.22	6702.27	8446.47	8446.47	1	1	2	2	2
I24	**4**	2680.32	2175.91	7848.09	9890.48	9890.48	1	1	2	2	2
I25	**4**	2824.36	2292.84	8269.86	10422.00	10422.00	1	1	2	3	3
I26	**4**	0.00	0.00	0.00	0.00	377.35	0	0	0	0	1
I27	**4**	0.00	0.00	0.00	0.00	931.73	0	0	0	0	1
I28	**4**	0.00	0.00	0.00	0.00	615.00	0	0	0	0	1
I29	**4**	0.00	409.30	1022.67	929.70	929.70	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 4- 110% Demand											
		Production Quantity of Finished Goods					Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	16420.00	15140.91	15140.91	15140.91	15140.91	2	1	1	1	1
E1	2	0.00	14559.09	14559.09	14559.09	14559.09	0	1	1	1	1
E2	3	1992.13	7700.00	7700.00	7700.00	7700.00	1	1	1	1	1
E3	4	2255.81	1304.46	0.00	2269.72	2002.00	1	1	0	1	1
E4	5	0.00	1987.34	4239.41	603.13	2146.87	0	1	1	1	1
E5	6	1474.15	999.73	940.46	959.27	0.00	1	1	1	1	0
E6	7	0.00	0.00	0.00	0.00	865.00	0	0	0	0	1
E7	8	0.00	4145.08	0.00	2566.09	4133.82	0	1	0	1	1
E7	9	2981.00	0.00	8200.00	8200.00	5903.51	1	1	1	1	1
E8	10	168.29	716.22	1210.00	1100.00	1100.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
		Production Quantity of Intermediate Products					Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	32845.31	56000.00	56000.00	56000.00	0	1	2	2	2
I2	1	0.00	29337.05	60991.00	60991.00	60991.00	0	2	4	4	4
I3	2	0.00	0.00	61560.98	64501.43	64501.43	0	0	2	2	2
I4	2	0.00	33978.02	46571.43	46571.43	46571.43	0	3	4	4	4
I5	2	0.00	26144.89	77264.54	77264.54	77264.54	0	1	2	2	2
I6	3	0.00	39734.37	69545.04	69545.04	69545.04	0	1	2	2	2
I7	3	324.49	24334.06	24591.60	24591.60	24591.60	1	2	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	935.89	2936.40	0	0	0	1	1
I13	3	0.00	0.00	0.00	0.00	2276.06	0	0	0	0	1
I14	3	0.00	1293.55	0.00	2453.57	2164.16	0	1	0	1	1
I15	3	0.00	2400.37	5574.82	793.11	2823.14	0	1	1	1	1
I16	3	0.00	1195.99	1249.10	1637.37	0.00	0	1	1	1	0
I17	3	0.00	910.88	951.33	1247.04	0.00	0	1	1	1	0
I18	3	0.00	816.20	852.45	1117.42	0.00	0	1	1	1	0
I19	3	1380.60	1225.82	1023.35	1341.44	0.00	2	2	1	2	0
I20	3	114.33	1225.82	1023.35	1341.44	0.00	1	1	1	1	0
I21	3	143.63	958.59	800.26	1049.01	0.00	1	1	1	1	0
I22	3	1608.00	1090.50	1025.85	1046.37	0.00	1	1	1	1	0
I23	4	2797.66	4071.39	8246.93	9371.43	9420.00	1	1	2	2	2
I24	4	3275.95	4555.20	9011.32	11831.31	11030.44	1	1	2	2	2
I25	4	3452.00	4800.00	9495.60	12467.13	11623.23	1	1	2	3	3
I26	4	0.00	0.00	530.74	0.00	0.00	0	0	1	0	0
I27	4	0.00	0.00	1310.48	0.00	0.00	0	0	1	0	0
I28	4	0.00	0.00	865.00	0.00	0.00	0	0	1	1	0
I29	4	0.00	739.86	1249.93	1136.30	1136.30	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 7- 90% Capacity											
			Production Quantity of Finished Goods				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	13720.00	14245.45	14245.45	14245.45	14245.45	1	1	1	1	1
E1	2	0.00	12754.55	12754.55	12754.55	12754.55	0	1	1	1	1
E2	3	1292.13	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	1774.12	1390.18	0.00	2135.69	1820.00	1	1	0	1	1
E4	5	0.00	1370.88	3938.77	268.24	2231.76	0	1	1	1	1
E5	6	1474.15	886.90	744.15	870.80	0.00	1	1	1	1	0
E6	7	0.00	0.00	0.00	0.00	740.00	0	0	0	0	1
E7	8	2710.00	2451.88	0.00	3061.96	5056.41	1	1	0	1	1
E7	9	0.00	0.00	7683.12	7683.12	4198.51	0	0	1	1	1
E8	10	168.29	556.22	1100.00	1000.00	1000.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
			Production Quantity of Intermediate Products				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	23719.18	50300.00	50300.00	50300.00	0	1	2	2	2
I2	1	0.00	18247.77	55446.37	55446.37	55446.37	0	1	3	3	3
I3	2	0.00	0.00	31025.17	57734.71	57734.71	0	0	1	2	2
I4	2	0.00	16816.21	41685.71	41685.71	41685.71	0	2	4	4	4
I5	2	0.00	12096.79	70240.50	70240.50	70240.50	0	1	2	2	2
I6	3	0.00	27089.82	63222.77	63222.77	63222.77	0	1	2	2	2
I7	3	447.23	19740.12	22356.00	22356.00	22356.00	1	2	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	0.00	2209.73	0	0	0	0	1
I13	3	0.00	0.00	0.00	0.00	1391.71	0	0	0	0	1
I14	3	0.00	865.51	0.00	2308.68	1967.42	0	1	0	1	1
I15	3	0.00	1589.73	5179.48	352.74	2934.76	0	1	1	1	1
I16	3	0.00	821.97	1095.45	1486.37	0.00	0	1	1	1	0
I17	3	0.00	626.03	834.31	1132.04	0.00	0	1	1	1	0
I18	3	0.00	560.96	747.59	1014.37	0.00	0	1	1	1	0
I19	3	1150.00	1150.00	897.47	1217.73	0.00	1	1	1	2	0
I20	3	0.00	1033.73	897.47	1217.73	0.00	0	1	1	1	0
I21	3	0.00	865.54	698.88	952.27	0.00	0	1	1	1	0
I22	3	1608.00	967.43	811.72	949.87	0.00	1	1	1	1	0
I23	4	2543.33	2301.08	8609.08	8685.71	8685.71	1	1	2	2	2
I24	4	2978.13	2694.48	8443.30	11808.21	10170.63	1	1	2	2	2
I25	4	3138.18	2839.28	8897.05	12442.80	10717.20	1	1	2	3	3
I26	4	0.00	454.05	0.00	0.00	0.00	0	1	0	0	0
I27	4	0.00	1121.10	0.00	0.00	0.00	0	1	0	0	0
I28	4	0.00	740.00	0.00	0.00	0.00	0	1	0	0	0
I29	4	0.00	574.58	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 8- 110% Capacity											
			Production Quantity of Finished Goods				Number of Setups				
				Time Period				Time Period			
Product	**Plant**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
E1	**1**	13720.00	27000.00	27000.00	27000.00	27000.00	1	2	2	2	2
E1	**2**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	**3**	1292.13	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	**4**	0.00	2778.96	0.00	2521.04	1820.00	0	1	0	1	1
E4	**5**	0.00	842.40	4467.25	776.05	1723.95	0	1	1	1	1
E5	**6**	934.11	832.25	1169.64	1040.00	0.00	1	1	1	1	0
E6	**7**	0.00	0.00	0.00	0.00	740.00	0	0	0	0	1
E7	**8**	2710.00	2200.00	0.00	6225.64	2912.73	1	1	0	1	1
E7	**9**	0.00	0.00	7935.00	3774.36	7087.27	0	0	1	1	1
E8	**10**	168.29	556.22	1100.00	1000.00	1000.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
			Production Quantity of Intermediate Products				Number of Setups				
				Time Period				Time Period			
Product	**Plant**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
I1	**1**	0.00	23719.18	50300.00	50300.00	50300.00	0	1	2	2	2
I2	**1**	0.00	18247.77	55446.37	55446.37	55446.37	0	1	3	3	3
I3	**2**	0.00	0.00	31025.17	57734.71	57734.71	0	0	1	2	2
I4	**2**	0.00	16816.21	41685.71	41685.71	41685.71	0	0	4	5	5
I5	**2**	0.00	12096.79	70240.50	70240.50	70240.50	0	1	2	2	2
I6	**3**	0.00	27089.82	63222.77	63222.77	63222.77	0	1	2	2	2
I7	**3**	447.23	19740.12	22356.00	22356.00	22356.00	1	2	2	2	2
I8	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	**3**	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	**3**	0.00	0.00	0.00	0.00	2209.73	0	0	0	0	1
I13	**3**	0.00	0.00	0.00	0.00	1391.71	0	0	0	0	1
I14	**3**	0.00	865.51	0.00	2308.68	1967.42	0	1	0	1	1
I15	**3**	0.00	1589.73	5179.48	352.74	2934.76	0	1	1	1	1
I16	**3**	0.00	821.97	1095.45	1486.37	0.00	1	0	1	1	0
I17	**3**	0.00	626.03	834.31	1132.04	0.00	1	0	1	1	0
I18	**3**	0.00	560.96	747.59	1014.37	0.00	0	1	1	1	0
I19	**3**	1150.00	1150.00	897.47	1217.73	0.00	1	1	1	2	0
I20	**3**	0.00	1033.73	897.47	1217.73	0.00	0	1	1	1	0
I21	**3**	0.00	865.54	698.88	952.27	0.00	0	1	1	1	0
I22	**3**	1608.00	967.43	811.72	949.87	0.00	1	1	1	1	0
I23	**4**	2543.33	2301.08	8609.08	8685.71	8685.71	1	1	2	2	2
I24	**4**	2978.13	2694.48	8443.30	11808.21	10170.63	1	1	2	2	2
I25	**4**	3138.18	2839.28	8897.05	12442.80	10717.20	1	1	2	3	3
I26	**4**	0.00	454.05	0.00	0.00	0.00	0	0	0	0	1
I27	**4**	0.00	1121.10	0.00	0.00	0.00	0	0	0	0	1
I28	**4**	0.00	740.00	0.00	0.00	0.00	0	0	0	0	1
I29	**4**	0.00	574.58	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 9- 120% Capacity											
			Production Quantity of Finished Goods				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	13720.00	27000.00	27000.00	27000.00	27000.00	1	2	2	2	2
E1	2	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	3	1292.13	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	0.00	2650.00	0.00	2650.00	1820.00	0	1	0	1	1
E4	5	0.00	723.65	4586.00	1112.81	1387.19	0	1	1	1	1
E5	6	181.13	1306.80	1448.07	1040.00	0.00	1	1	1	1	0
E6	7	0.00	0.00	0.00	0.00	740.00	0	0	0	0	1
E7	8	2710.00	2200.00	0.00	6810.26	2024.42	1	1	0	1	1
E7	9	0.00	0.00	7935.00	3189.74	7975.58	0	0	1	1	1
E8	10	168.29	1063.96	1100.00	1000.00	1000.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
			Production Quantity of Intermediate Products				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	17262.39	52452.26	52452.26	52452.26	0	1	2	2	2
I2	1	0.00	18247.77	55446.37	55446.37	55446.37	0	1	3	3	3
I3	2	0.00	0.00	0.00	69251.17	77243.43	0	0	0	2	2
I4	2	0.00	0.00	30330.50	55771.43	55771.43	0	0	3	5	5
I5	2	0.00	12096.79	70240.50	70240.50	70240.50	0	1	2	2	2
I6	3	0.00	27089.82	63222.77	63222.77	63222.77	0	1	2	2	2
I7	3	447.23	19740.12	22356.00	22356.00	22356.00	1	2	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	0.00	2209.73	0	0	0	0	1
I13	3	0.00	0.00	0.00	0.00	1391.71	0	0	0	0	1
I14	3	0.00	309.54	0.00	2864.65	1967.42	0	1	0	1	1
I15	3	0.00	738.62	6030.59	1463.35	1824.15	0	1	1	1	1
I16	3	0.00	0.00	1628.62	1775.17	0.00	0	0	1	1	0
I17	3	0.00	0.00	1240.38	1352.00	0.00	0	0	1	1	0
I18	3	0.00	0.00	1111.45	1211.47	0.00	0	0	1	1	0
I19	3	1150.00	476.58	1334.28	1454.34	0.00	1	1	2	2	0
I20	3	0.00	360.31	1334.28	1454.34	0.00	0	1	1	1	0
I21	3	0.00	497.78	881.61	1137.30	0.00	0	1	1	1	0
I22	3	197.58	1425.46	1579.55	1134.43	0.00	1	1	1	1	0
I23	4	2543.33	2064.69	7446.97	9384.96	9384.96	1	1	2	2	2
I24	4	2978.13	2417.67	8720.10	10989.42	10989.42	1	1	2	2	2
I25	4	3138.18	2547.60	9188.73	11580.00	11580.00	1	1	2	3	3
I26	4	0.00	0.00	0.00	0.00	454.05	0	0	0	0	1
I27	4	0.00	0.00	0.00	0.00	1121.10	0	0	0	0	1
I28	4	0.00	0.00	0.00	0.00	740.00	0	0	0	0	1
I29	4	0.00	1099.07	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 10- 80% Initial Inventory											
			Production Quantity of Finished Goods				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	16376.00	27000.00	27000.00	27000.00	27000.00	2	2	2	2	2
E1	2	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	3	2433.70	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	868.84	1781.16	0.00	2650.00	1820.00	1	1	0	1	1
E4	5	0.00	1666.45	4594.16	0.00	2321.31	0	1	1	0	1
E5	6	1339.69	816.00	780.31	1040.00	0.00	1	1	1	1	0
E6	7	0.00	0.00	0.00	0.00	842.00	0	0	0	0	1
E7	8	0.00	0.00	0.00	1785.58	4042.13	0	0	0	1	1
E7	9	2710.00	2200.00	7949.42	8200.00	5957.87	1	1	1	1	1
E8	10	134.63	764.98	1100.00	1000.00	1000.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
			Production Quantity of Intermediate Products				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	34790.82	52452.26	52452.26	52452.26	0	2	2	2	2
I2	1	0.00	36776.76	55446.37	55446.37	55446.37	0	2	3	3	3
I3	2	0.00	11829.87	64501.43	64501.43	64501.43	0	1	2	2	2
I4	2	0.00	37422.29	46571.43	46571.43	46571.43	0	4	4	4	4
I5	2	0.00	37773.63	70240.50	70240.50	70240.50	0	1	2	2	2
I6	3	0.00	46960.96	63222.77	63222.77	63222.77	0	2	2	2	2
I7	3	2736.28	22356.00	22356.00	22356.00	22356.00	1	2	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	1015.67	1100.93	2976.31	0	0	1	1	1
I13	3	0.00	0.00	0.00	621.51	2260.57	0	0	0	1	1
I14	3	0.00	820.56	0.00	2864.65	1967.42	0	1	0	1	1
I15	3	0.00	2020.99	6041.32	0.00	3052.53	0	1	1	0	1
I16	3	0.00	1101.99	1578.21	1400.16	0.00	0	1	1	1	0
I17	3	0.00	839.29	1201.99	1066.38	0.00	0	1	1	1	0
I18	3	0.00	752.05	1077.05	955.54	0.00	0	1	1	1	0
I19	3	957.54	1246.55	989.79	1450.29	0.00	1	2	1	2	0
I20	3	0.00	1191.08	1292.97	1147.11	0.00	0	1	1	1	0
I21	3	0.00	974.80	774.01	1134.13	0.00	0	1	1	1	0
I22	3	1461.34	890.09	851.16	1134.43	0.00	1	1	1	1	0
I23	4	2543.33	2064.69	7460.51	9371.43	9384.96	1	1	2	2	2
I24	4	2978.13	2417.67	8735.96	10973.57	10989.42	1	1	2	2	2
I25	4	3138.18	2547.60	9205.43	11563.30	11580.00	1	1	2	3	3
I26	4	0.00	0.00	516.63	0.00	0.00	0	0	1	0	0
I27	4	0.00	0.00	1275.63	0.00	0.00	0	0	1	0	0
I28	4	0.00	0.00	842.00	0.00	0.00	0	0	1	1	0
I29	4	0.00	790.22	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 11- 90% Initial Inventory											
		Production Quantity of Finished Goods					Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	15048.00	27000.00	27000.00	27000.00	27000.00	1	2	2	2	2
E1	2	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	3	1862.92	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	697.20	1952.80	0.00	2650.00	1820.00	1	1	0	1	1
E4	5	0.00	1202.46	4577.76	0.00	2415.57	0	2	1	0	1
E5	6	1474.15	674.14	787.71	1040.00	0.00	1	1	1	1	0
E6	7	0.00	0.00	0.00	0.00	791.00	0	0	0	0	1
E7	8	2710.00	2200.00	0.00	5641.03	3907.16	1	1	0	1	1
E7	9	0.00	0.00	8200.00	4108.40	6078.42	0	0	1	1	1
E8	10	151.46	660.60	1100.00	1000.00	1000.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
		Production Quantity of Intermediate Products					Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	26026.61	52452.26	52452.26	52452.26	0	1	2	2	2
I2	1	0.00	27512.27	55446.37	55446.37	55446.37	0	2	3	3	3
I3	2	0.00	0.00	46911.52	64501.43	64501.43	0	0	1	2	2
I4	2	0.00	19790.68	46571.43	46571.43	46571.43	0	2	4	4	4
I5	2	0.00	24935.21	70240.50	70240.50	70240.50	0	1	2	2	2
I6	3	0.00	37025.39	63222.77	63222.77	63222.77	0	1	2	2	2
I7	3	451.26	22188.55	22356.00	22356.00	22356.00	1	2	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	659.59	2991.73	0	0	0	1	1
I13	3	0.00	0.00	0.00	0.00	2136.89	0	0	0	0	1
I14	3	0.00	565.05	0.00	2864.65	1967.42	1	1	0	1	1
I15	3	0.00	1389.55	6019.75	0.00	3176.47	0	1	1	0	1
I16	3	0.00	761.81	1394.90	1585.36	0.00	0	1	1	1	0
I17	3	0.00	580.21	1062.38	1207.43	0.00	0	1	1	1	0
I18	3	0.00	519.90	951.95	1081.93	0.00	0	1	1	1	0
I19	3	1077.24	1010.82	991.34	1450.29	0.00	1	1	1	2	0
I20	3	0.00	948.41	1142.80	1298.83	0.00	0	1	1	1	0
I21	3	0.00	790.46	775.23	1134.13	0.00	0	1	1	1	0
I22	3	1608.00	735.35	859.23	1134.43	0.00	1	1	1	1	0
I23	4	2543.33	2064.69	8502.34	8343.13	9371.43	1	1	2	2	2
I24	4	2978.13	2417.67	9011.32	10714.05	10973.57	1	1	2	2	2
I25	4	3138.18	2547.60	9495.60	11289.83	11563.30	1	1	2	3	3
I26	4	0.00	0.00	0.00	485.34	0.00	0	0	0	1	0
I27	4	0.00	0.00	0.00	1198.37	0.00	0	0	0	1	0
I28	4	0.00	0.00	0.00	791.00	0.00	0	0	0	1	1
I29	4	0.00	682.40	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 12- 110% Initial Inventory											
		Production Quantity of Finished Goods					Number of Setups				
			Time Period					Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	12392.00	27000.00	27000.00	27000.00	27000.00	1	2	2	2	2
E1	2	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	3	721.34	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	275.92	3093.60	0.00	1930.48	1820.00	1	1	0	1	1
E4	5	0.00	1986.12	2937.40	1089.38	1410.62	0	1	1	1	1
E5	6	1474.15	0.00	1528.26	973.59	0.00	1	0	1	1	0
E6	7	0.00	0.00	0.00	0.00	689.00	0	0	0	1	1
E7	8	2710.00	2200.00	0.00	1801.37	3666.07	1	1	0	1	1
E7	9	0.00	0.00	8200.00	7948.05	6319.51	0	0	1	1	1
E8	10	185.12	451.85	1100.00	1000.00	1000.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
		Production Quantity of Intermediate Products					Number of Setups				
			Time Period					Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	8498.18	52452.26	52452.26	52452.26	0	1	2	2	2
I2	1	0.00	8983.28	55446.37	55446.37	55446.37	0	1	3	3	3
I3	2	0.00	0.00	0.00	52573.39	64501.43	0	0	0	2	2
I4	2	0.00	0.00	31098.89	46571.43	46571.43	0	0	3	4	4
I5	2	0.00	0.00	69498.86	70240.50	70240.50	0	0	2	2	2
I6	3	0.00	17154.25	63222.77	63222.77	63222.77	0	1	2	2	2
I7	3	447.23	17287.65	22356.00	22356.00	22356.00	1	2	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	0.00	768.14	0	0	0	0	1
I13	3	0.00	0.00	0.00	0.00	646.53	0	0	0	0	1
I14	3	0.00	831.83	0.00	2086.85	1967.42	0	1	0	1	1
I15	3	0.00	2377.47	3862.68	1432.54	1854.96	0	1	1	1	1
I16	3	0.00	0.00	1403.69	1661.82	0.00	0	1	1	1	0
I17	3	0.00	0.00	1069.07	1265.66	0.00	0	0	1	1	0
I18	3	0.00	0.00	957.95	1134.11	0.00	0	1	1	1	0
I19	3	1150.00	639.24	1150.00	1361.47	0.00	1	1	1	2	0
I20	3	0.00	396.34	1150.00	1361.47	0.00	0	1	1	1	0
I21	3	0.00	560.74	889.72	883.09	0.00	0	1	1	1	0
I22	3	1608.00	0.00	1667.03	1061.99	0.00	1	0	1	1	0
I23	4	2543.33	2064.69	8369.74	8475.73	9371.43	1	1	2	2	2
I24	4	2978.13	2417.67	9011.32	10714.05	10973.57	1	1	2	2	2
I25	4	3138.18	2547.60	9495.60	11289.83	11563.30	1	1	2	3	3
I26	4	0.00	0.00	0.00	422.75	0.00	0	0	0	1	1
I27	4	0.00	0.00	0.00	1043.84	0.00	0	0	0	1	1
I28	4	0.00	0.00	0.00	689.00	0.00	0	0	0	1	1
I29	4	0.00	466.76	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 13- 120% Initial Inventory											
		Production Quantity of Finished Goods					Number of Setups				
		Time Period					Time Period				
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	11064.00	27000.00	27000.00	27000.00	27000.00	1	2	2	2	2
E1	2	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	3	150.56	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	228.24	2608.14	0.00	2463.62	1820.00	1	1	0	1	1
E4	5	0.00	2727.35	2957.85	254.89	1097.28	0	1	1	1	1
E5	6	1474.15	0.00	1519.03	982.82	0.00	1	0	1	1	0
E6	7	0.00	0.00	0.00	0.00	638.00	0	0	0	1	1
E7	8	2710.00	2200.00	0.00	1801.37	3545.52	1	1	0	1	1
E7	9	0.00	0.00	8200.00	7948.05	6440.05	0	0	1	1	1
E8	10	201.94	347.47	1100.00	1000.00	1000.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
		Production Quantity of Intermediate Products					Number of Setups				
		Time Period					Time Period				
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	0.00	52186.23	52452.26	52452.26	0	0	2	2	2
I2	1	0.00	0.00	55165.15	55446.37	55446.37	0	0	3	3	3
I3	2	0.00	0.00	0.00	23153.62	64501.43	0	0	0	1	2
I4	2	0.00	0.00	13467.28	46571.43	46571.43	0	0	2	4	4
I5	2	0.00	0.00	56660.44	70240.50	70240.50	0	0	2	2	2
I6	3	0.00	7218.68	63222.77	63222.77	63222.77	0	1	2	2	2
I7	3	431.37	14851.05	22356.00	22356.00	22356.00	1	1	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I13	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I14	3	0.00	0.00	0.00	2663.17	1967.42	0	0	0	1	1
I15	3	0.00	3330.89	3889.57	335.19	1442.93	0	1	1	1	1
I16	3	0.00	0.00	1049.65	1677.58	0.00	0	0	1	1	0
I17	3	0.00	0.00	799.43	1277.67	0.00	0	0	1	1	0
I18	3	0.00	0.00	716.33	1144.86	0.00	0	0	1	1	0
I19	3	1436.32	230.94	1144.58	1374.38	0.00	2	1	1	2	0
I20	3	0.00	147.74	1144.58	1374.38	0.00	0	1	1	1	0
I21	3	0.00	180.60	895.06	1074.77	0.00	0	1	1	1	0
I22	3	1608.00	0.00	1656.96	1072.06	0.00	1	0	1	1	0
I23	4	2543.33	2064.69	8303.44	8542.03	9371.43	1	1	2	2	2
I24	4	2978.13	2417.67	9011.32	10714.05	10973.57	1	1	2	2	2
I25	4	3138.18	2547.60	9495.60	11289.83	11563.30	1	1	2	3	3
I26	4	0.00	0.00	0.00	391.46	0.00	0	0	0	1	1
I27	4	0.00	0.00	0.00	966.57	0.00	0	0	0	1	1
I28	4	0.00	0.00	0.00	638.00	0.00	0	0	0	1	1
I29	4	0.00	358.93	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 14- Setup to Inventory Cost Ratio- 25											
			Production Quantity of Finished Goods				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	11064.00	27000.00	27000.00	27000.00	27000.00	1	2	2	2	2
E1	2	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	3	150.56	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	2650.00	0.00	0.00	2650.00	1820.00	1	0	0	1	1
E4	5	0.00	1362.34	4577.76	0.00	1097.28	0	1	1	0	1
E5	6	0.00	2148.29	787.71	1040.00	0.00	0	1	1	1	0
E6	7	0.00	0.00	0.00	0.00	638.00	0	0	0	0	1
E7	8	2710.00	2200.00	0.00	5641.03	3559.95	1	1	0	1	1
E7	9	0.00	0.00	8200.00	4093.97	6440.05	0	0	1	1	1
E8	10	0.00	549.41	1100.00	1000.00	1000.00	0	1	1	1	1

Production Quantity of Intermediate Goods											
			Production Quantity of Intermediate Products				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	0.00	52186.23	52452.26	52452.26	0	0	2	2	2
I2	1	0.00	0.00	55165.15	55446.37	55446.37	0	0	3	3	3
I3	2	0.00	0.00	0.00	23659.73	63995.31	0	0	0	1	2
I4	2	0.00	0.00	23054.14	37350.00	46206.00	0	0	2	3	4
I5	2	0.00	0.00	95141.43	51000.00	51000.00	0	0	2	1	1
I6	3	0.00	7218.68	63222.77	63222.77	63222.77	0	1	2	2	2
I7	3	510.93	14771.49	22356.00	22356.00	22356.00	1	1	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I13	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I14	3	0.00	0.00	0.00	2663.17	1967.42	0	0	0	1	1
I15	3	1535.90	0.00	6019.75	0.00	1442.93	1	0	1	0	1
I16	3	0.00	0.00	2727.23	0.00	0.00	0	0	1	0	0
I17	3	0.00	0.00	2077.10	0.00	0.00	0	0	1	0	0
I18	3	0.00	0.00	1861.20	0.00	0.00	0	0	1	0	0
I19	3	0.00	1744.60	991.34	1450.29	0.00	0	2	1	2	0
I20	3	0.00	225.07	2441.63	0.00	0.00	0	1	1	0	0
I21	3	0.00	241.07	775.23	1134.13	0.00	0	1	1	1	0
I22	3	0.00	2343.35	859.23	1134.43	0.00	0	2	1	1	0
I23	4	2543.33	2064.69	8289.91	8542.03	9384.96	1	1	2	2	2
I24	4	2978.13	2417.67	9011.32	10698.20	10989.42	1	1	2	2	2
I25	4	3138.18	2547.60	9495.60	11273.13	11580.00	1	1	2	3	3
I26	4	0.00	0.00	0.00	391.46	0.00	0	0	0	1	1
I27	4	0.00	0.00	0.00	966.57	0.00	0	0	0	1	1
I28	4	0.00	0.00	0.00	638.00	0.00	0	0	0	1	0
I29	4	0.00	358.93	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 15- Setup to Inventory Cost Ratio- 50											
			Production Quantity of Finished Goods					Number of Setups			
				Time Period					Time Period		
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	11064.00	27000.00	27000.00	27000.00	27000.00	1	2	2	2	2
E1	2	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	3	150.56	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	2650.00	0.00	0.00	2650.00	1820.00	1	0	0	1	1
E4	5	0.00	2319.50	3620.60	0.00	1097.28	0	1	1	0	1
E5	6	0.00	1716.17	1219.83	1040.00	0.00	0	1	1	1	0
E6	7	0.00	0.00	0.00	0.00	638.00	0	0	0	0	1
E7	8	0.00	0.00	0.00	1800.00	3559.95	0	0	0	1	1
E7	9	4910.00	0.00	7935.00	8200.00	6440.05	1	0	1	1	1
E8	10	0.00	549.41	1100.00	1000.00	1000.00	0	1	1	1	1
Production Quantity of Intermediate Goods											
			Production Quantity of Intermediate Products					Number of Setups			
				Time Period					Time Period		
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	19810.76	34320.00	34320.00	34320.00	34320.00	1	1	1	1	1
I2	1	0.00	0.00	55165.15	55446.37	55446.37	0	0	3	3	3
I3	2	0.00	0.00	0.00	23659.73	63995.31	0	0	0	1	2
I4	2	0.00	0.00	23054.14	37350.00	46206.00	0	0	2	3	4
I5	2	0.00	0.00	95141.43	51000.00	51000.00	0	0	2	1	1
I6	3	0.00	7218.68	63222.77	63222.77	63222.77	0	1	2	2	2
I7	3	0.00	15282.42	22356.00	22356.00	22356.00	0	1	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I13	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I14	3	0.00	0.00	0.00	4630.59	0.00	0	0	0	1	0
I15	3	0.00	2794.57	4761.08	0.00	1442.93	0	1	1	0	1
I16	3	0.00	2727.23	0.00	0.00	0.00	0	1	0	0	0
I17	3	0.00	2077.10	0.00	0.00	0.00	0	1	0	0	0
I18	3	0.00	1861.20	0.00	0.00	0.00	0	1	0	0	0
I19	3	1150.00	504.05	1081.89	1450.29	0.00	1	1	1	2	0
I20	3	0.00	414.45	2252.26	0.00	0.00	0	1	1	0	0
I21	3	0.00	170.27	846.04	1134.13	0.00	0	1	1	1	0
I22	3	0.00	1872.00	1330.59	1134.43	0.00	0	1	1	1	0
I23	4	4608.02	0.00	7446.97	9384.96	9384.96	1	0	2	2	2
I24	4	5395.81	0.00	8720.10	10989.42	10989.42	1	0	2	2	2
I25	4	5685.78	0.00	9188.73	11580.00	11580.00	2	0	3	3	3
I26	4	0.00	0.00	391.46	0.00	0.00	0	0	1	0	0
I27	4	0.00	0.00	966.57	0.00	0.00	0	0	1	0	0
I28	4	0.00	0.00	638.00	0.00	0.00	0	0	1	0	0
I29	4	0.00	358.93	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 16- Setup to Inventory Cost Ratio- 0.1											
			Production Quantity of Finished Goods				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	11064.00	27000.00	27000.00	27000.00	27000.00	1	2	2	2	2
E1	2	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	3	150.56	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	566.27	2909.21	0.00	1824.52	1820.00	1	1	0	1	1
E4	5	0.00	876.21	3661.17	1402.72	1097.28	0	1	1	1	1
E5	6	1474.15	395.17	1201.51	905.17	0.00	1	1	1	1	0
E6	7	0.00	0.00	0.00	0.00	638.00	0	0	0	0	1
E7	8	2710.00	2200.00	0.00	5641.03	3545.52	1	1	0	1	1
E7	9	0.00	0.00	7963.85	4344.55	6440.05	0	0	1	1	1
E8	10	201.94	347.47	1100.00	1000.00	1000.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
			Production Quantity of Intermediate Products				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	0.00	52186.23	52452.26	52452.26	0	0	2	2	2
I2	1	0.00	0.00	55165.15	55446.37	55446.37	0	0	3	3	3
I3	2	0.00	0.00	0.00	23153.62	64501.43	0	0	0	1	2
I4	2	0.00	0.00	13467.28	46571.43	46571.43	0	0	2	4	4
I5	2	0.00	0.00	56660.44	70240.50	70240.50	0	0	2	2	2
I6	3	0.00	7218.68	63222.77	63222.77	63222.77	0	1	2	2	2
I7	3	431.37	14851.05	22356.00	22356.00	22356.00	1	1	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I13	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I14	3	0.00	690.86	0.00	1972.30	1967.42	0	1	0	1	1
I15	3	0.00	896.64	4814.44	1844.57	1442.93	0	1	1	1	1
I16	3	0.00	0.00	1182.19	1545.04	0.00	0	0	1	1	0
I17	3	0.00	0.00	900.37	1176.72	0.00	0	0	1	1	0
I18	3	0.00	0.00	806.79	1054.41	0.00	0	0	1	1	0
I19	3	1436.32	406.06	1078.05	1265.80	0.00	2	1	1	2	0
I20	3	0.00	322.86	1078.05	1265.80	0.00	0	1	1	1	0
I21	3	0.00	317.54	843.03	989.86	0.00	0	1	1	1	0
I22	3	1608.00	431.05	1310.61	987.36	0.00	1	1	1	1	0
I23	4	2543.33	2064.69	7474.04	9371.43	9371.43	1	1	2	2	2
I24	4	2978.13	2417.67	8751.81	10973.57	10973.57	1	1	2	2	2
I25	4	3138.18	2547.60	9222.13	11563.30	11563.30	1	1	2	3	3
I26	4	0.00	0.00	391.46	0.00	0.00	0	0	1	0	0
I27	4	0.00	0.00	966.57	0.00	0.00	0	0	1	0	0
I28	4	0.00	0.00	638.00	0.00	0.00	0	0	1	1	1
I29	4	0.00	358.93	1136.30	1033.00	1033.00	0	1	1	1	1

Appendix 3: Sensitivity Analysis on Production Planning and Scheduling Results

Case 17- Setup to Inventory Cost Ratio- 0.5											
			Production Quantity of Finished Goods				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
E1	1	11064.00	27000.00	27000.00	27000.00	27000.00	1	2	2	2	2
E1	2	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
E2	3	150.56	7000.00	7000.00	7000.00	7000.00	1	1	1	1	1
E3	4	566.27	3104.80	0.00	1628.93	1820.00	1	1	0	1	1
E4	5	0.00	1452.88	3084.50	1402.72	1097.28	0	1	1	1	1
E5	6	1474.15	0.00	1461.85	1040.00	0.00	1	1	1	1	0
E6	7	0.00	0.00	0.00	0.00	638.00	0	0	0	1	1
E7	8	2710.00	2200.00	0.00	1801.37	3545.52	1	1	0	1	1
E7	9	0.00	0.00	8200.00	7948.05	6440.05	0	0	1	1	1
E8	10	201.94	347.47	1100.00	1000.00	1000.00	1	1	1	1	1
Production Quantity of Intermediate Goods											
			Production Quantity of Intermediate Products				Number of Setups				
				Time Period				Time Period			
Product	Plant	1	2	3	4	5	1	2	3	4	5
I1	1	0.00	0.00	52186.23	52452.26	52452.26	0	0	2	2	2
I2	1	0.00	0.00	55165.15	55446.37	55446.37	0	0	3	3	3
I3	2	0.00	0.00	0.00	23153.62	64501.43	0	0	0	1	2
I4	2	0.00	0.00	13467.28	46571.43	46571.43	0	0	2	4	4
I5	2	0.00	0.00	56660.44	70240.50	70240.50	0	0	2	2	2
I6	3	0.00	7218.68	63222.77	63222.77	63222.77	0	1	2	2	2
I7	3	431.37	14851.05	22356.00	22356.00	22356.00	1	1	2	2	2
I8	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I9	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I10	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I11	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I12	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I13	3	0.00	0.00	0.00	0.00	0.00	0	0	0	0	0
I14	3	0.00	902.30	0.00	1760.87	1967.42	0	1	0	1	1
I15	3	0.00	1654.96	4056.12	1844.57	1442.93	0	1	1	1	1
I16	3	0.00	0.00	1182.19	1545.04	0.00	0	0	1	1	0
I17	3	0.00	0.00	900.37	1176.72	0.00	0	0	1	1	0
I18	3	0.00	0.00	806.79	1054.41	0.00	0	0	1	1	0
I19	3	1436.32	351.51	1132.60	1265.80	0.00	2	1	1	2	0
I20	3	0.00	268.30	1132.60	1265.80	0.00	0	1	1	1	0
I21	3	0.00	274.88	885.69	989.86	0.00	0	1	1	1	0
I22	3	1608.00	0.00	1594.59	1134.43	0.00	1	0	1	1	0
I23	4	2543.33	2064.69	8303.44	8542.03	9371.43	1	1	2	2	2
I24	4	2978.13	2417.67	9011.32	10714.05	10973.57	1	1	2	2	2
I25	4	3138.18	2547.60	9495.60	11289.83	11563.30	1	1	2	3	3
I26	4	0.00	0.00	0.00	391.46	0.00	0	0	0	1	1
I27	4	0.00	0.00	0.00	966.57	0.00	0	0	0	1	1
I28	4	0.00	0.00	0.00	638.00	0.00	0	0	0	1	1
I29	4	0.00	358.93	1136.30	1033.00	1033.00	0	1	1	1	1