Optimal Production and Sales Plans for a New Product in the Presence of Scarcity Effects

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Abstract
We develop integrated production and sales planning model for new product under supply constraint. We consider product scarcity situations and its consequences namely hype and retarding effects. We identify situations where strategic sales delay and myopic sales policies can be optimal. Under retarding effect, we show that the length of build-up period depends upon magnitude of retarding effect.

Key words: marketing-manufacturing interface, control theory.

Introduction
Several new products such as game consoles, tablet PCs face capacity problems (Balachander et al. 2009). To model a new product diffusion process under a supply constraint, researchers have suggested modifications to the original Bass model (Ho et al. 2002), which assumed that only the customers who had successfully purchased the product and not the customers who had demanded the product controlled the future product growth. Their models thus have assumed that waiting customers did not directly influence the subsequent product growth.

However, there are many situations where the demand for the new product indeed depends upon the behavior of waiting customers. Sometimes, the response of the waiting customers can accelerate the product growth. Such customer behaviors are referred as hype effects where the scarcity of the product encourages customers to buy sooner.Thesehype effects have also been found in automobile and fashion industries (Balachander et al. 2009).

On some other occasions, the unavailability of the product can slow down the product growth. For example, Jain et al. (1991) studied the diffusion of new telephones in Israel and found that the waiting customers tend to communicate negative information about the product which subsequently resulted in the slower growth. We call this as a retarding effect where the scarcity of the product discourages immediate purchases. Our research is motivated by these contrasting customer behaviors in the presence of the product backlogs.

Such new product diffusion patterns can influenceboth production and sales planning decisions .Sony delayed roll out of its Play Station 2 until it had 1 million units of inventory and ID Four Ltd increased initial inventory of its new software in anticipation of rapid demand growth (Kumar and Swaminathan 2002). Given these interactions, we believe that there is a need to develop integrated sales and production plans for the new products. How should a marketing manager design a sales strategy over the entire life cycle in the presence of the hype and retarding effects?

In this paper, we characterize optimal production and sales plan for the new product under a supply constraint by explicitly including the response of waiting customers. We use a supply restricted new product diffusion model (Jain et al. 1991) to represent the scarcity effects. We assume that when the unmet demand is completely backordered, it is served on FCFS basis whenever sale occurs. We solve the problem via discrimination to study the effects of production characteristics such as inventory holding and backorder costs on the optimal profits, sales policies and build-up periods. We identify situations where the myopic and strategic sales delay policies can be optimal.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. The mathematical model formulation is presented in the Section 3. Section 4 discusses the behavior of optimal sales, productionand capacity plans in the presence of scarcity effects. The paper ends with conclusions and the directions for future research.

Motivation and Literature Review

Several models have been suggested to represent an innovation diffusion process .Bass (1969) is a seminar paper which has proposed a parsimonious mathematical diffusion model using the theory of communication where the diffusion process can be represented using the following differential equation. $z(t)=\left\{p+\frac{q_1}{m}S(t)\right\}\left[m-S(t)\right]$ where $z(t)$ is instantaneous demand at time *t*, *p* and q_1 are constants $\in$ (0,1), which represent the effects of mass media and adopters on potential adopters respectively. *S(t)* is the cumulative sales up to time *t* while *m* is the market potential. Mahajan et al. (2000) provide an excellent review on this literature.

Despite of the valuable contributions, from an operations management perspective, the fundamental limitation of the Bass model is its inability to model supply restrictions. A research stream in operationshas attempted to analytically derive the optimal dynamic sales and production plans in the presence of supply constraints by using a modified Bass model. Ho et al. (2002) analyzed a supply constrained diffusion process by including inventory holding and lost sales costs. They showed that the myopic policy is always optimal under supply constraints. Cantamessa and Valentini (2000) developed a production planning model for the new product considering the diffusion process and capacity constraints. Kumar and Swaminathan (2003) included backorder costs in addition to inventory and lost sales costs and showed that the build-up policy is optimal in the lost sales setting. In an extensive numerical study, they found that the build-up plan is dominant in contrast to Ho et al. (2002) results which favored the myopic plan. Amini and Li (2011) found that lower supply chain cost can be achieved by using the myopic policy.

We believe that none of the above research papershave considered the broader implications of supply constrained diffusion process while developing the production and sales plans because their models did not consider the influence of waiting customers on the diffusion process.Jain et al. (1991) is the firstpaper which has proposed and validated a modification to the Bass diffusion model to represent thesescarcity effects.However, this empirical stream of research has not investigated the impact of the scarcity effects on the optimal production and sales plans.

In this paper, we attempt to bridge the gap between these two research streamsby presenting an enhancedmathematical model to characterize the optimal sales and production plans for the new productby incorporating the **scarcity effect**.

Mathematical Model Formulation

Consider a firm that wants to decide production and sales plans for its new product. We assume that the firm can start the production of the product at a known date, which we define as t =0.

Let,

q_2 = influence of waiting applicants on potential adopters

s(t)= instantaneous sales of the product at time *t*

a(t) = instantaneous backorders at time *t*

Z(t) = cumulative demand for the product up to time *t*

A(t) = cumulative number of backorders up to time *t*

$Z(t) = S(t) + A(t)$

$z(t) = s(t) + a(t)$

Under a supply constraint, the diffusion process can be described as follows. At time *t*, potential adopters who have not yet purchased the product may place an order. The demand is fulfilled if the product is available at time *t*; otherwise the customer waits (backordering). Jain et al. (1991) propose a modification to the Bass model (Bass 1969) to include this supply constrained situation. The model is parsimonious, intuitive, and more importantly, empirically validated. The empirical validation is important because the lack of it can question the generalizability of the implications of the results (Krishnan and Jain 2006). Also, the model reduces to the Bass model when $q_1 = q_2$.

$$z(t) = \left\{ p + \frac{q_1}{m} S(t) + \frac{q_2}{m} A(t) \right\} [m - Z(t)] \tag{1}$$

q_2 is unrestricted in sign. Figure 1 shows how this diffusion model represents both hype and retarding effects. When $q_2 > q_1$, we observe the hype effects and when $q_2 < q_1$ we have retarding effect. Both Kumar and Swaminathan (2003) and Ho et al. (2002) diffusion models can be considered as a special case of this model when word of mouth effects of waiting customers is zero (q_2 =0). Because of these strengths, we have used this diffusion model (equation 1) in our mathematical formulation.

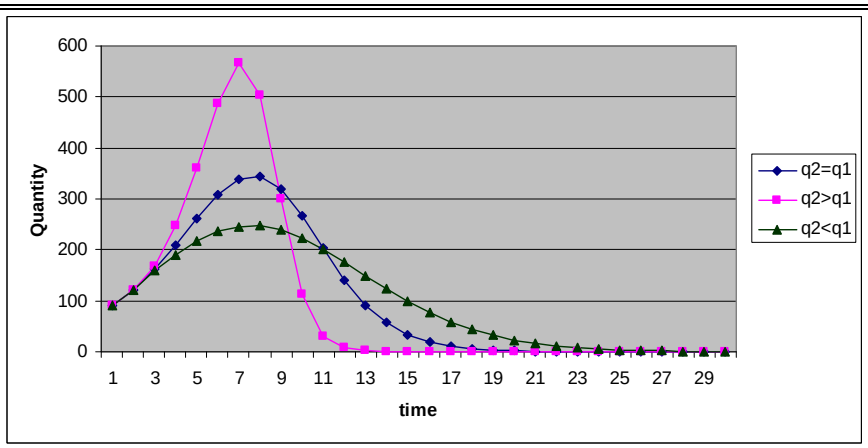

Figure 1- Hype and retarding effects in the supply constrained diffusion process.

Let,
α = unit production cost,
π = selling price per unit;
w = back order cost per unit backlogged per unit time,
h = inventory holding cost per unit inventoried per unit time,
γ = discount rate
$i(t)$ = inventory at time t
$x(t)$ = production at time t
c = production capacity

The problem can be formulated as (P1) where the objective is to maximize the discounted profit of the firm over the life cycle of the product by subtracting discounted inventory, backorder and production costs from discounted revenue (equation 2). We assume that the unmet demandis completely backordered. The firm manages the diffusion process using equations (3-5). Instantaneous inventory and backorders follow equations (6-7).

$$(P1) \max_{s(t),x(t),o\le t\le T} \phi = \int_o^T e^{-\gamma t}\{\pi s(t) - \alpha x(t) - wA(t) - hi(t)\}dt \tag{2}$$

subject to

$$\dot{Z}(t)=z(t) \tag{3}$$

$$\dot{S}(t)=s(t) \tag{4}$$

$$\dot{z}(t)=-pz(t)+\frac{q_1}{m}\{-S(t)z(t)+(m-Z(t))s(t)\}+\frac{q_2}{m}\{-A(t)z(t)+(m-Z(t))a(t)\} \tag{5}$$

$$\dot{i}(t) = x(t) - s(t) \tag{6}$$

$$\dot{A}(t) = a(t) = z(t) - s(t) \tag{7}$$

$$0 \le x(t) \le c \tag{8}$$

$$s(t) \ge 0, z(t) \ge 0 \tag{9}$$

$$A(t) \ge 0, i(t) \ge 0,\ A(0) = 0, i(0) = 0\ \textit{for all}\ t \ge 0 \tag{10}$$

At optimality, the firm sells as much as possible or sells nothing at any time t. We define a strategic sales delay policy (SSD) as a policy that delays the sales over finite time interval on finite number of occasions over the life cycle with the objective of maximizing profits. Build-up policy is a special case of SSD where the sales delay occurs only at the start of the product life cycle and subsequently the firm sells as much possible. Thus, under build-up policy, the production department produces products and builds enough inventoryto never lose sale once the sale starts. The length of build-up period decides the amount of inventory the firm holds.On the other hand, myopic policy (M) sells as much as possible at each time$t$and produces units to meet the sales plan.

Implications of Scarcity Effects

We conduct a numerical study to demonstrate the importance of including the scarcity effects while determining the optimal production and sales strategies. First, we show how optimal sales policy structure changes with q_2. Subsequently, we show that profits increase when the firm considers the q_2 effects. Finally, we analyze the impact of q_2 on the capacity planning decisions. We discretize the problem (P1) as shown below to numerically compute the optimal strategies using non linear programming.Interval branch and bound algorithm with a multi start option embodied in Frontline Solver'sengines was used to solve our non linear program. The algorithm gave solutions which in probability converged to global optimums. We have also used the results of theorem 1 to verify the optimality of the solutions obtained by using the solver.

In this formulation, we set N such that the market potential is almost entirely exhausted. We also convert γ to $\beta \in (0,1)$ for the discrete case. It is important to set parameters of our discrete model at appropriate values. Based on the analysis of 213 data sets drawn from different industries, Sultan (1990) reported that the mean values for parameters p and q_1 were 0.03 and 0.38 respectively. We use p=0.03 and q=0.4 in our experiments. We set m =3000, T= 36(sufficient time period to exhaust the market demand) and C = 125. If each period is of two weeks, then the corresponding product life cycle is around 1.5 years. We set per unit production cost at 1 unit and selling price per unit at 1.3 units.

To investigate the effect of inventory holding and backorder costs, we set 'h' at two levels (low: 0.001, high: 0.01) and 'w' at two levels (low: 0.001, high: 0.01). Discounting parameter β was set at 0.995. We consider lost sales and complete backlogging situations. For the lost sales case, we add the constraint $0 \le s(t) \le z(t)$ to P2 and set w =0. This resulted in 16 scenarios. For each scenario, we consider the following levels for q_2 (0.5, 0, and -0.5). Table 1shows how the optimal sales plan changes when the firm includes waiting customer's effects. In the Table 1, M=Myopic, BU=Build Up, and number in the bracket denotes the build-up period.

Table 1 -Behavior of optimal sales strategies under different situations.

	h	w	β =0.995		
			q2=0.5	q2=0	q2=-0.5
Lost Sales	low		M	BU(5)	SSD
	high		M	BU(5)	M
Complete Back Order	low	low	SSD	M	M
	high	low	M	M	M
	low	High	M	BU(8)	M
	high	High	M	M	M

Although all the problem characteristics (except q_2) are the same, the Table 1 demonstrates how the optimal sales structure changes from myopic-build-up- SSD policies with q_2. Experiments suggest that in the presence of the hype effect, the myopic policy becomesthe most preferred optionunder high backorder cost and lost salessituations. Consistent with this finding, after facing the supply shortage and the resulting hype effect, both Sony and Nintendo

have used costly options such as airfreight to keep the retailer shelves full with their game consoles (Business Week 2007). Also, Sony's and ID Four Ltd's strategy of building initial inventory before their product launchesmay be interpreted as a mechanism to implement the myopic sales strategy. In contrast, when backordercost is low and time discounting is high, then the SSD plan is optimal for the hype effect. For an automobile industry, backorder cost is low as customers are willing to wait for some time to get their delivery. Also, time discounting influence is relatively higher as the product life cycle is higher compared to products such as game consoles. This may be the reason why several automobile firms may have used strategic sales delay strategy during the initial periods to bring forward the demand (Wall Street Journal 1999). Thus, the optimal sales and production policy can take different shapes depending upon the dynamics of the diffusion process in the presence of capacity constraint.

Next, we analyze the behavior of the build-up policy. We assume q_1=0.6, m =3000, T= 30, C=125, h=0.001, w=0.01 and β=1. We assumed a linear demand between the periods to compute the build-up period in a continuous time. When the retarding effect is low (q_2 = 0), and p = 0.001, the length of BU period is approximately 6. However, When q_2 = -1.2, the length becomes approximately 1. This is because the firm cannot afford to delay the sales for a long time because such strategy can erode the profits due to slow product diffusion. Clearly, the firm should carefully anticipate the reaction of waiting applicants while deciding the build-up inventory.Ourresults show that the optimal build-up period (and thus opening inventory) could be significantly different than the build-up period suggested by Kumar and Swaminathan (2003) model which assumed q_2=0.

Managerial Implications and Conclusion

In this paper, we provided an analysis of integrated sales and operations planning decisions for a supply constrained new product diffusion process. The diffusion model incorporated word of mouth effects of waiting customers in addition to adopters and innovators when the firm is supply constrained. Analytical results showed that the presence of the scarcity effects arising from the product shortages affected the optimal sales plan, and hence optimal profits. Also, we found that operations characteristics such as inventory holding and backorder costs influenced the sales planning decision.In our numerical experiments, we used q_2=0 as the base case for comparing our results with the past research. We showed that the length of the build-up period increased (decreased) as the magnitude of the retarding effect decreased (increased) with reference to the base case. Hence, our results indicate that managers should anticipate the scarcity effects and include them while building sales and production plans.

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