

Section 1.2: Break-Even Analysis and Market Equilibrium

A **mathematical model** is a mathematical description (often by means of a function or an equation) of a real-world phenomenon. The purpose of the model is to understand the phenomenon and perhaps to make predictions about future behavior.

Linear Demand and Supply Curves

Example 1: For a particular commodity, it is found that 20 units will be supplied at a unit price of \$9 whereas 10 units will be supplied at a unit price of \$7. For this commodity, it is found that the consumers are willing to consume 10 units at a unit price of \$23 but are only willing to consume 2 units at a unit price of \$24.60. Assuming that both supply and demand functions are linear, find the

a) supply equation

b) demand equation

Market Equilibrium: The point at which the consumer and supplier agree upon (i.e. the point of intersection of the supply and demand curves)

Example 2: Find and interpret the market equilibrium for the commodity in Example 1.

Cost, Revenue, and Profit Functions

Cost Function (cost of manufacturing x units of a product)

where c is the cost per unit and F is the fixed cost.

Revenue Function (revenue realized from selling x units of the product)

Profit Function (profit realized from making and selling x units of the product)

Example 3: A manufacturer of garbage disposals, has a monthly fixed cost of \$14,000 and a production cost of \$5 for each garbage disposal manufactured. The units sell for \$75 each.

- a) What is the cost function?
- b) What is the revenue function?
- c) What is the profit function?

Break-Even Point: The point at which the company suffers neither a loss or gain. (i.e. the point of intersection of the revenue and cost functions: $R(x) = C(x)$ or $P(x) = 0$)

Example 4: Find (and interpret) the break-even point for the garbage disposals in Example 3.

Quadratic Revenue and Profit Functions

Example 5: It is found that the consumers of a particular toaster will demand 64 toaster ovens when the unit price is \$35 whereas they will demand 448 toaster ovens when the unit price is \$5. Assuming that the demand function is linear and the selling price is determined by the demand function,

- a) Find the demand equation.

- b) Find the revenue function.

- c) Find the number of items sold that will give the maximum revenue. What is the maximum revenue?

- d) If the company has a fixed cost of \$1,000 and a variable cost of \$15 per toaster, find the company's linear cost function.

- e) What is the company's maximum profit?

- f) How many toasters should be sold for the company to break even?