Look into
Test stuff

Math 1314
Lesson 8
Business Applications: Break Even Analysis, Equilibrium Quantity/Price

Three functions of importance in business are cost functions, revenue functions and profit functions.

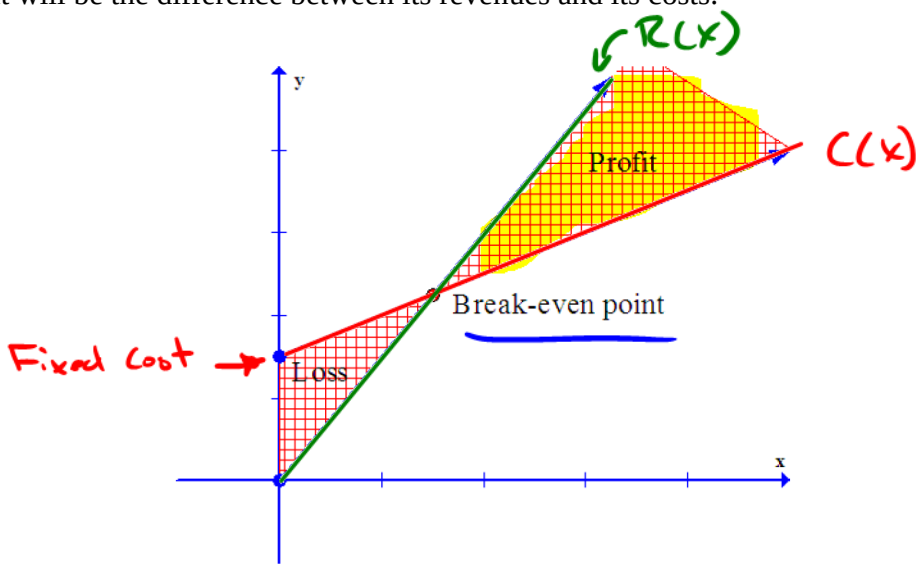
Cost functions model the cost of producing goods or providing services. These are often expressed as linear functions, or functions in the form $C(x) = mx + b$. A linear cost function is made up of two parts, fixed costs such as rent, utilities, insurance, salaries and benefits, etc., and variable costs, or the cost of the materials needed to produce each item. For example, fixed monthly costs might be $125,000 and material costs per unit produced might be $15.88, so the cost function could be expressed as $C(x) = 15.88x + 125,000$.

Revenue functions model the income received by a company when it sells its goods or services. These functions are of the form $R(x) = xp$, where x is the number of items sold and p is the price per item. Price however is not always static. For this reason, the unit price is often given in terms of a demand function. This function gives the price of the item in terms of the number demanded over a given period of time (week, month or year, for example). No matter what form you find the demand function, to find the revenue function, you'll multiply the demand function by x. So if demand is given by $p = 1000 - 0.01x$, the revenue function is given by $R(x) = x(1000 - 0.01x) = 1000x - 0.01x^2$.

Profit functions model the profits made by manufacturing and selling goods or by providing services. Profits represent the amount of money left over after goods or services are sold and costs are met. We represent the profit as $P(x) = R(x) - C(x)$.

Break-Even Analysis

The break-even point in business is the point at which a company is making neither a profit nor incurring a loss. At the break-even point, the company has met all of its expenses associated with manufacturing the good or providing the service. The *x* coordinate of the break-even point gives the number of units that must be sold to break even. The *y* coordinate gives the revenues at that production and sales level. After the break-even point, the company will make a profit, and that profit will be the difference between its revenues and its costs.



We can find the break-even point algebraically or graphically.

Example 1: Find the break-even point, given this information: Suppose a manufacturer has monthly fixed costs of $100,000 and production costs of $12 for each item produced. The item sells for $20. Assume all functions are linear.

$C(x) = 12x + 100{,}000$

$R(x) = 20x$

$20x = 12x + 100{,}000$

$8x = 100000$

$x = 12{,}500$

Break Even Quantity

$R(BEQ) = 20(12{,}500)$

$= 250{,}000$ (y-value)

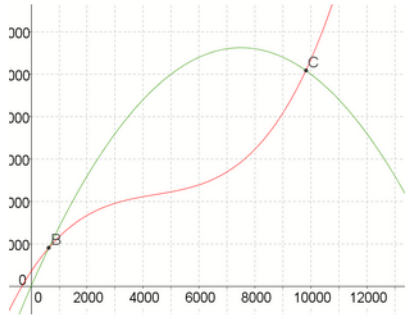
Break Even Revenue

$\boxed{(12500,\ 250000)}$

(BEQ, BER)

Change Decimal pt option

Example 2: Suppose a company can model its costs according to the function $C(x) = 0.000003x^3 - 0.04x^2 + 200x + 70,000$ where $C(x)$ is given in dollars and demand can be modeled by $p = -0.02x + 300$.



a. Find the revenue function.

Price

$R(x) = p \cdot x = (-0.02x + 300) \cdot x$

$= -0.02x^2 + 300x$

b. Find the break even point.

Command: intersect[<object>, <object>]

Answer:

$x \geq 0$

intersect[R, C]

B (624.4552, 140637.4357)

C (9919.3321, 1017413.9749)

c. Find the smallest positive quantity for which all costs are covered.

624.4552 Next Number Up

Command:

Answer:

$R(624) - C(624) < 0$ Loss

$R(629) - C(629) > 0$ Gain / Profit

$x = 629$

Popper 06

Question 4: A manufacturer has a monthly fixed cost of \$80,000.00 and a production cost of \$22 for each unit produced. The product sells for \$33 per unit. Find the break-even point.

a. $(0, 3200)$
b. $\left(24000, \frac{80000}{11}\right)$
c. $\left(\frac{80000}{33}, 240000\right)$
d. $\left(\frac{80000}{11}, 240000\right)$

You may be given raw data concerning costs and revenues. In that case, you'll need to start by finding functions to represent cost and revenue.

Example 3: Suppose you are given the cost data and demand data shown in the tables below.

x-values
y-values

quantity produced	50	100	200	500	1000	1200	1500	2000
total cost	100400	140100	182000	217400	229300	232600	239300	245500

> list1

x-values
y-values

quantity demanded	50	100	200	500	1000	1200	1500	2000
price in dollars	295	285	280	270	250	248	249	248

> list2

a. Find a cubic regression equation that models costs, and a quadratic regression equation that models demand. *Begin by entering the data in GGB. Then create lists.*

Cubic Cost Model:

Command:

fitpoly[list1, 3]

Answer:

$f(x) = 0.0001x^3 - 0.3321x^2 + 384.1395x + 99320764$

Quadratic Demand Model:

Command:

fitpoly[list2, 2]

Answer: Round to 5 Decimals

$g(x) = 0.00002x^2 - 0.60883x + 293.79593$ "p"

b. State the revenue function.

Command:

$g \cdot x$

Answer:

$h(x) = (\text{green stuff}) \cdot x$ = "Revenue"

c. Find the break-even point.

Command: f - cost h - Revenue

intersect[f, h]

Answer: (949.28, #)

d. Find the smallest positive quantity for which all costs are covered.

$x = 950$

Market Equilibrium

The price of goods or services usually settles at a price that is dictated by the condition that the demand for an item will be equal to the supply of the item. If the price is too high, consumers will tend to refrain from buying the item. If the price is too low, manufacturers have no incentive to produce the item, as their profits will be very low.

Market equilibrium occurs when the quantity produced equals the quantity demanded. The quantity produced at market equilibrium is called the equilibrium quantity and the corresponding price is called the equilibrium price.

Mathematically speaking, market equilibrium occurs at the point where the graph of the supply function and the graph of the demand function intersect. We can solve problems of this type either algebraically or graphically.

Example 4: Suppose that a company has determined that the demand equation for its product is $5x+3p-30=0$ where p is the price of the product in dollars when x of the product are demanded (x is given in thousands). The supply equation is given by $52x-30p+45=0$, where x is the number of units that the company will make available in the marketplace at p dollars per unit. Find the equilibrium quantity and price.

Demand function

$5x+3p-30=0$

$(5x+3p=30)\cdot 10$

Supply function

$52x-30p+45=0$

$52x-30p=-45$

$$\begin{array}{r} 50x+30p=300 \\ +\ 52x-30p=-45 \\ \hline \end{array}$$

$$\frac{102x}{102}=\frac{255}{102}$$

$x=2.5$

Solve for p, substitute into something

$p=5.83$

Price \$ 5.83

$2.5\times 1000 \rightarrow$ Quantity of 2500

Example 5: The quantity demanded of a certain electronic device is 8000 units when the price is $260. At a unit price of $200, demand increases to 10,000 units. The manufacturer will not market any of the device at a price of $100 or less. However for each $50 increase in price above $100, the manufacturer will market an additional 1000 units. Assume that both the supply equation and the demand equation are linear. Find the supply equation, the demand equation and the equilibrium quantity and price.

Demanded

(8000, 260)

(10000, 200)

$$m = \frac{200 - 260}{10000 - 8000}$$

$$= \frac{-6\cancel{0}}{200\cancel{0}} = \frac{-3}{100}$$

$$= -0.03$$

$$y = mx + b$$

$$p = mx + b$$

$$200 = -0.03(10000) + b$$

$$200 = -300 + b$$

$$500 = b$$

$$p = -0.03x + 500$$

Supply

(0, 100) ← y-int (b)

(1000, 150)

$$m = \frac{150 - 100}{1000}$$

$$= \frac{50}{1000} = 0.05$$

$$p = mx + b$$

$$p = 0.05x + 100$$

$$-0.03x + 500 = 0.05x + 100$$

$$400 = 0.08x$$

$$5000 = x$$

$$p = 350$$

Equilibrium Quantity = 5000

" Price = 350

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