

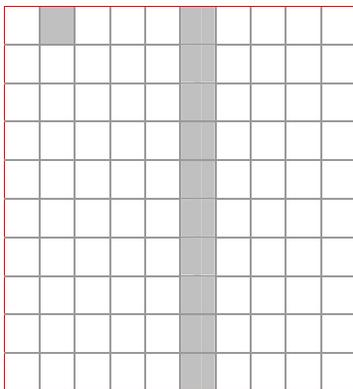
Activities with Graph Paper for Enhancing Learning of Mathematics

Harinder Mahajan

Graph paper is a very versatile and useful device for learning mathematics and is not used much in primary and middle schools. In this paper we suggest some activities with graph paper that can be used to enhance understanding of concepts and operations on numbers, geometry and measurement, communication and reasoning. Some opportunities are provided for exploration of ideas. We have restricted activities to 2005 syllabus of NCERT for classes V -VIII.

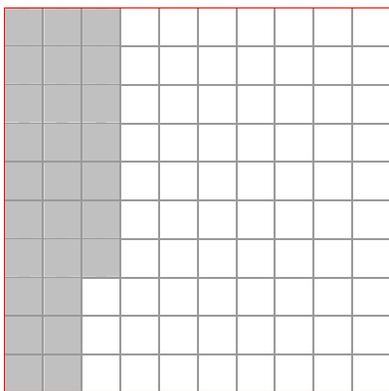
Numbers

Let a small square in represents one unit of area then the shaded long area represents a ten as it consists of ten squares.



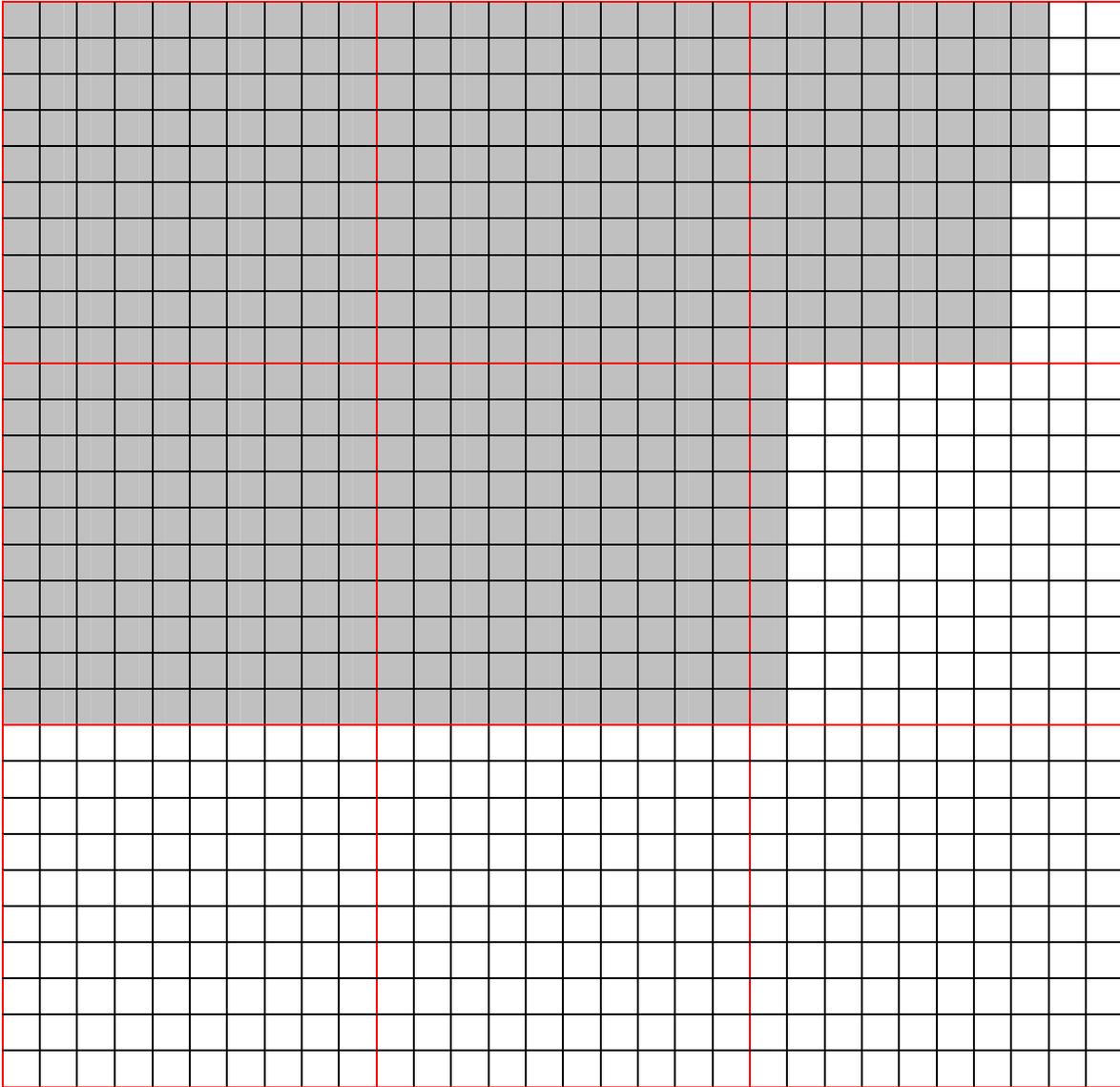
Short Long

We can represent a two-digit numbers by shading as many longs as tens and as many shorts as units in it. For example $27 = 2 \text{ Tens} + 7 \text{ Ones}$, we can represent it by shading 2 longs and 7 shorts as shown below:



Give more examples and ask them to represent some numbers using this method. Similarly we can represent a three digit number by use of a graph paper in which 10×10 squares are also marked in a different colour given in Appendix 1. A square with red border represents a hundred, a whole column or row within the square a long represents

tens and a small square represents a unit. We can represent the number by shading as many 10×10 squares as hundreds, as many longs as tens and as many shorts as units. Thus to represent a number say 475, we can shade 4 10×10 squares, 7 longs and 5 shorts as given below:



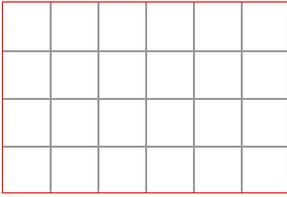
Give more examples and ask them to represent some numbers using this method.

Operation on numbers

The area model for multiplication and division extends to fractions and decimals and should be used. The length of a small square represents one unit of length and its area one unit of area.

Multiplication of one digit numbers

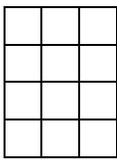
To find 4×6 mark a rectangle with 4 rows and 6 columns and count the number of squares which is 24, therefore $4 \times 6 = 24$.



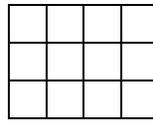
Properties of multiplication

Commutative law

Mark two rectangles 4×3 and 3×4 on the graph paper and count the number of squares. Both of them are 12. Therefore $4 \times 3 = 3 \times 4 = 12$.



$$4 \times 3$$



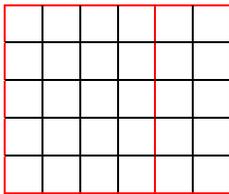
$$3 \times 4$$

Repeat with other numbers.

Generalise to $a \times b = b \times a$, where a , and b are any numbers.

Distributive law

The product of a number and sum of two numbers is equal to products of the number with sum of two numbers. For example $5 \times (4 + 2) = 5 \times 6$ as the figure given below shows.



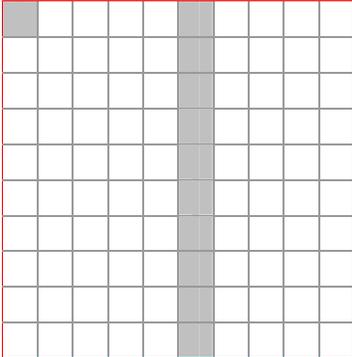
$$\leftarrow 5 \times 4 \rightarrow 5 \times 2$$

$$\leftarrow 5 \times 6 \rightarrow$$

Repeat with other numbers.

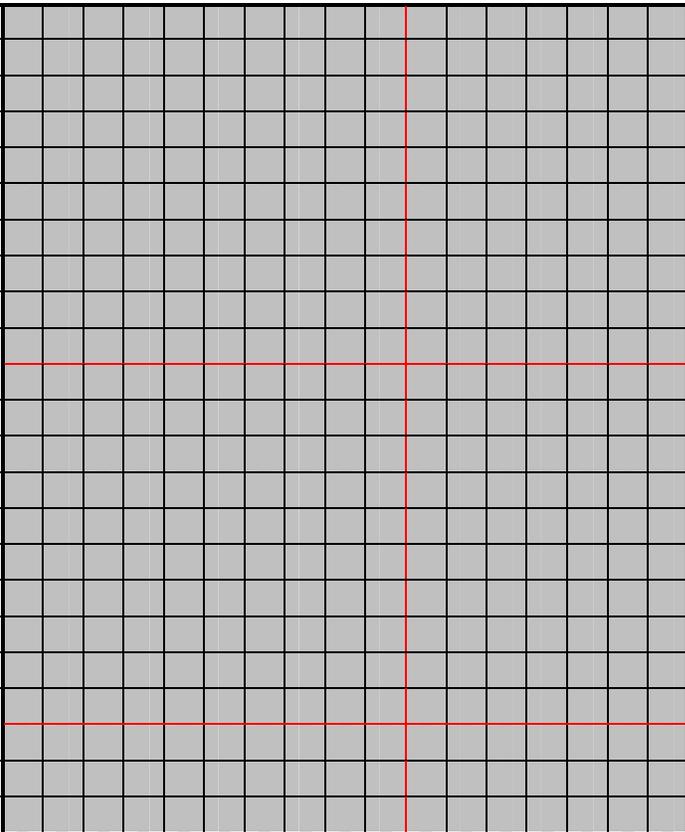
Generalise to $a \times b + a \times c = a \times (b + c)$, where a , b , and c are any numbers.

Multiplication of two-digit numbers



Short Long

Mark a rectangle with as many columns as one number and as many rows as the other number. Then count the number of 10×10 squares, longs and shorts and add them. For example to multiply 23×17 , Shade a 23×17 rectangle as given below

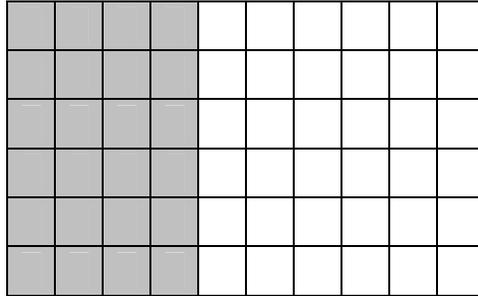


And count the number of small squares in it by counting 10×10 squares, longs and shorts in it. The number of 10×10 squares or hundreds is 2, the number of tens is $14 + 3 = 17$ and the number of ones is 21, therefore the $23 \times 17 = 200 + 170 + 21 = 391$.
Give more examples and ask them to do some exercises using this method.

Division

Division facts

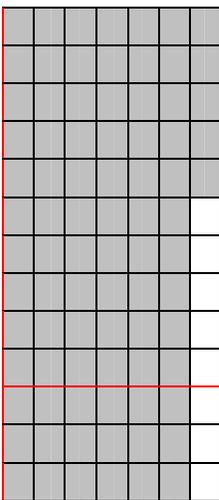
For example to find $28 \div 7$, shade area using as many rows as the divisor and count the number of columns required to shade as much area as the dividend. The number of columns gives the quotient. As it requires 4 columns to shade 28 units of area using 7 rows. $28 \div 7 = 4$. The number of columns in this case 4 gives the quotient.



Give more examples and ask them to find some division facts using this method.

Division of two-digit numbers by one or two digit number

Use a graph paper in which ten \times ten squares are also marked in a different colour or thicker lines. Shade an area corresponding to the dividend using as many rows as the divisor. The number of whole columns that can be shaded gives the quotient and the difference between the dividend and the squares shaded gives the remainder. For example to divide 83 by 13, using 13 rows when we shade 1 column 1 long and 3 short or 13 squares are shaded, when we shade 2 columns 2 long and 6 shorts that is 26 squares are shaded, we go on shading columns until the difference between dividend and squares shaded is less than the divisor. After we have shaded 6 columns, we have 6 longs and 18 shorts that is 78 squares shaded and the difference between dividend and the squares shaded is 5 which is less than 13 and is the remainder. The number of columns fully shaded is 6 which is the quotient.



Give more examples and ask them to do some exercises using this method.

Fractions

If the whole rectangle represents a unit how much of the total area is shaded in the following figures?

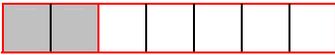
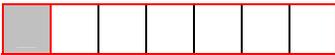


What is common in these fractions? (They all have 1 as numerator.)

Such fractions are called **unit fractions**.

Compare these fractions. What do you notice? (The unit fractions with larger denominator are smaller.)

Mark a rectangle on a graph paper that is 1 unit wide and 7 units long.



What does the shaded region taking the whole rectangle as a unit represent? ($1/7$, $2/7$, $3/7$).

What is common in these fractions? (They all have the same denominator.) Such fractions are called **like fractions**.

Repeat with other numbers.

Compare these fractions. What do you notice? (The fractions with larger numerator are larger.)

Equivalent fractions



Mark three 6×1 rectangles on the graph paper and take that as unit.

Divide the first rectangle into 2, second in 3 and third in 6 equal parts by drawing red border around each part.

What does one part in (a) represent? ($1/2$)

What does two parts in (a) represent? ($2/2$)

What does a part in (b) represent? ($1/3$)

What do two parts in (b) represent? ($2/3$)

What does one part in (c) represent? ($1/6$)

What do two parts in (c) represent? ($2/6$)

What do three parts in (c) represent? ($3/6$)

What do four parts in (c) represent?

What do five parts in (c) represent?

What do six parts in (c) represent?

Is the region covered by $1/2$ the same as that covered by $3/6$?

Is the region covered by $1/3$ the same as that covered by $2/6$?

The fractions that cover equal regions of a figure are called **equivalent fractions**.

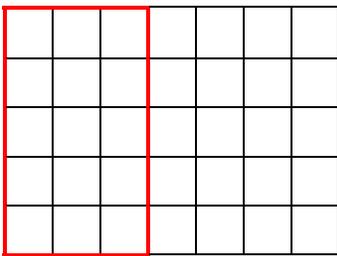
Are there any other equivalent fractions in the above figures?

Repeat with other examples.

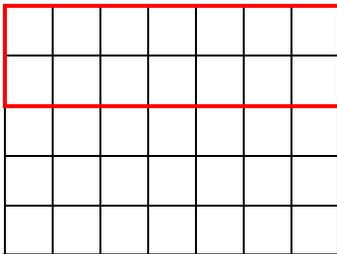
Give rules for forming equivalent fractions.

Comparison of unlike fractions

Mark two rectangles on the graph paper whose length and breadth are given by the denominators of the unlike fractions. Take the rectangle as a unit and mark or shade regions corresponding to fractions and count the number of small squares in each. The fraction that has larger number of squares or larger area is larger. For example to compare $3/7$ and $2/5$, mark a rectangle whose length is 7 units and breadth 5 units. Taking this rectangle as a unit, mark $3/7^{\text{th}}$ of the rectangle by drawing a border around 3 of 7 columns, and count the number of small squares in it which is 15. Mark another 7×5 rectangle and draw a border around 2 rows to show $2/5$ of the rectangle and count the number of small squares in it which is 14. Therefore $3/7 > 2/5$.



$$\rightarrow \frac{3}{7}$$



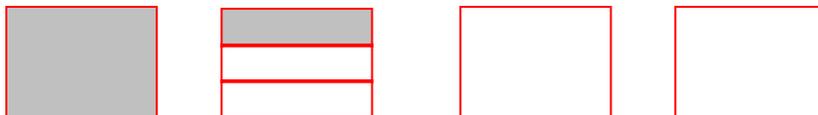
$$\rightarrow \frac{2}{5}$$

Or find the LCM of their denominators. Change the fractions to equivalent fractions so that their denominator is the same as LCM. Now these are like fractions and we can compare them like those.

Improper fractions and mixed numbers

A fraction in which denominator is greater than numerator is called a **proper fraction** e.g. $\frac{4}{6}$, $\frac{7}{8}$, $\frac{2}{9}$.

A fraction in which numerator is greater than denominator is called an **improper fraction** e.g. $\frac{3}{2}$, $\frac{7}{4}$, $\frac{8}{3}$. These represent a number of equal parts into which a number of wholes are divided. For example, to find $\frac{4}{3}$ Mark 4 (numerator) rectangles with the same length and width 3 (the denominator) units and shade $\frac{1}{3}$ of the rectangles.



The shaded part is $\frac{3}{3} + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}$

We can also express it as $1 + \frac{1}{3} = 1\frac{1}{3}$. Such numbers that are a combination of a whole number and a fraction are called **mixed numbers**.

Repeat with more examples.

Fractions in lowest terms

Fractions that do not have a common factor other than 1 are called fractions in lowest terms. A fraction whose numerators and denominators have common factors can be reduced to fractions with lowest terms by dividing its numerator and denominator by the HCF of the numerator and denominator. For example, in $\frac{4}{6}$ HCF of 4 and 6 is 2; we can reduce it to its lowest terms, by dividing its numerator and denominator by 2. Now 2 and 3 do not have any common factor other than 1, therefore it is a fraction in lowest terms.

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

Operations on fractions

Addition of like fractions

We can add like fractions e.g. $\frac{1}{5} + \frac{2}{5}$, by marking a 5×1 rectangle on Graph paper and first shading $\frac{1}{5}$ of the rectangle and then shading $\frac{2}{5}$ of the rectangle next to it.



How much of the rectangle is shaded now? ($\frac{3}{5}$) Therefore $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.

Repeat with more examples. What do you notice?

Thus sum of two like fractions is a fraction whose numerator is the sum of numerators and denominator is the same as denominators of like fractions.

Addition of unlike fractions

Find the LCM of their denominators

Change the fractions to equivalent fractions so that their denominator is the same as LCM. Now these are like fractions and we can add them like those.

Subtraction of like fractions

To subtract like fractions e.g. $\frac{4}{7} - \frac{2}{7}$, make a 7×1 rectangle on Graph paper and first

shade $\frac{4}{7}$ of the rectangle and then cross $\frac{2}{7}$ of the rectangle from that. How much of the

uncrossed shaded rectangle is left now? ($\frac{2}{7}$)



Therefore $\frac{4}{7} - \frac{2}{7} = \frac{2}{7}$

Thus difference of two like fractions is a fraction whose numerator is the difference of numerators and denominator is the same as denominators of like fractions.

Addition and subtraction of unlike fractions

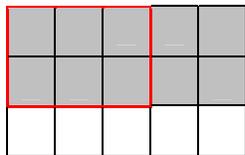
Find the LCM of their denominators. Change the fractions to equivalent fractions so that their denominator is the same as LCM. Now these are like fractions and we can add and subtract them like those.

Multiplication of two fractions

To multiply two fractions, say $\frac{2}{3}$ and $\frac{3}{5}$ mark a rectangle whose length is 5 linear units and breadth is 3 linear units (same as that of denominator of two fractions). Taking the rectangle as a unit, shade $\frac{2}{3}$ rd of the area and then mark $\frac{3}{5}$ th of the area shaded earlier by

a red border. The area within red border represents $\frac{3}{5}$ th of $\frac{2}{3}$ or $\frac{6}{15}$ th of the area of

rectangle. Therefore $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$



Repeat with more examples. What do you notice?

The product of two fractions is a fraction whose numerator is the product of numerators of two fractions and whose denominator is the product of denominators of two fractions.

Reciprocal of a fraction If the product of two fractions is 1 each is called the reciprocal of the other e.g. 4 and 1/4, 3/5 and 5/3.

Division of two fractions

The denominator of a fraction is a factor of the denominator of the other fraction

To divide a fractions, say $\frac{1}{2}$ by $\frac{1}{6}$ draw a rectangle whose length is 6 linear units (same as that of denominator of smaller fraction) and breadth is 1 linear unit and take it as a unit.

What does a square represent? ($\frac{1}{6}$)

What do 3 squares represent? ($\frac{3}{6}$ or $\frac{1}{2}$)

Shade $\frac{1}{2}$ of the area and then find the number of $\frac{1}{6}$ s in it. How many $\frac{1}{6}$ s are there in $\frac{1}{2}$? (3)



Therefore $\frac{1}{2} \div \frac{1}{6} = 3$.

Repeat with more examples. What do you notice?

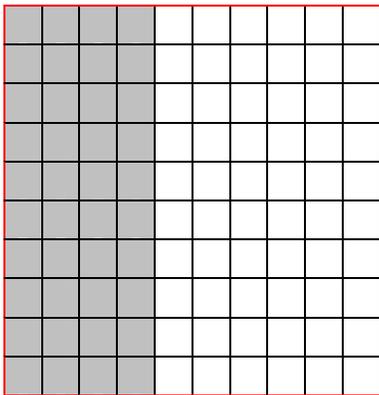
The quotient of two fractions can be obtained by multiplication of the dividend by reciprocal of divisor. This holds for all fractions.

We can divide any fractions by using this rule.

Decimals

Decimals are special fractions that have 10, 100, 1000 and so on as a denominator

Concept of decimals-Tenths

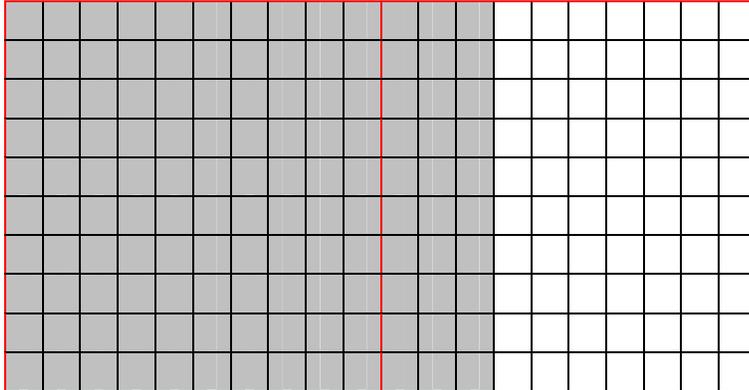


Take the square with red border as a unit.

Each long divides it into 10 equal parts or is $\frac{1}{10}$ of the whole.

In the above grid 4 longs are shaded or $\frac{4}{10}$ or 0.4 of the grid is shaded.

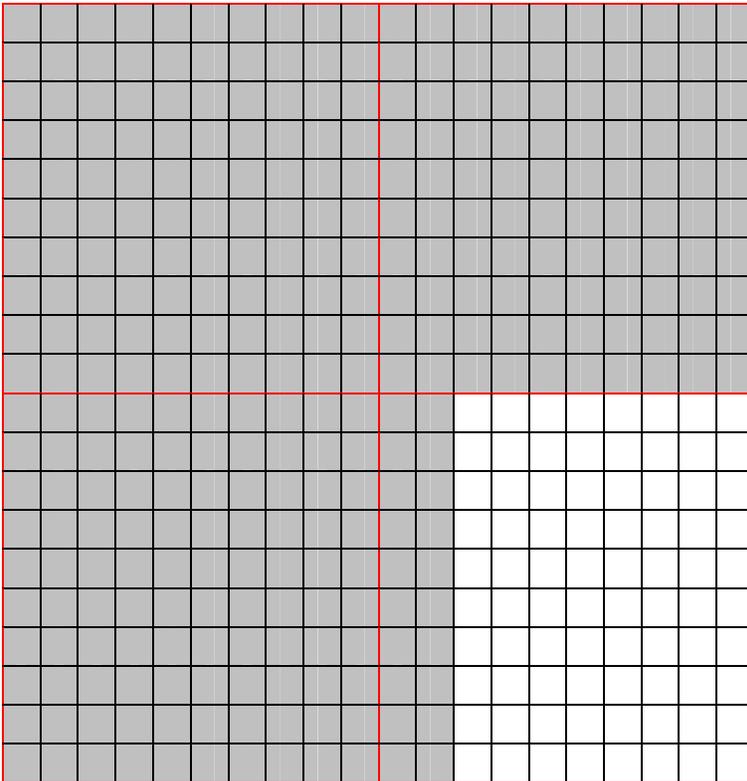
Decimals larger than 1



If our unit is big square, we have one unit and three tenths shaded. We may write it as

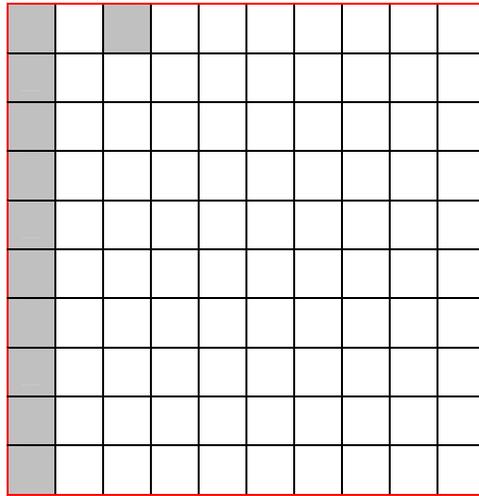
$$1\frac{3}{10} \text{ or } 1.3.$$

We can show 3.2 shading 3 whole squares and 2 longs in the fourth square

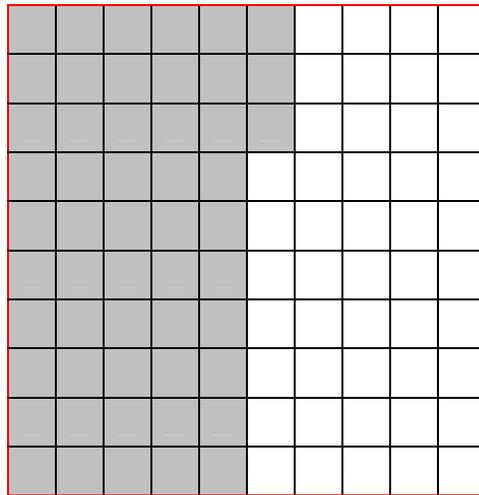


Concept of decimals-Hundredths

Think of the 10×10 square in your graph paper notebook as one unit.



1. The square with red border represents one unit
2. The shaded long part within the red square represents one tenth.
3. The shaded small square -a short represents one hundredth of the unit.
4. There are ten shorts in one long.
5. Hundredths are written to the right of the tenths column when writing numbers in figures. That is we write units, decimal point, tenths and hundredths from left to right.



In the above graph 0 units, 5 tenths and 3 hundredths or 53 hundredths are shaded. We read it as zero (units) point five (tenths) three (hundredths) (NOT zero point fifty three). We write it as 0.53.

Give more examples.

Activities

1. Taking the square with red border as a unit show by shading with different colours
 - (a) one tenth
 - (b) one hundredth
2. In the graph paper find the numbers that are represented by shaded parts. (Include numbers with different digit as given in the next activity.)

3. In the graph paper, shade the parts that represent following decimal numbers:
- | | | |
|----------|----------|----------|
| (a) 0.4 | (b) 2.7 | (c) 1.0 |
| (d) 1.42 | (e) 0.57 | (f) 2.70 |
| (g) 2.05 | (h) 0.60 | (i) 0.05 |

Operations on Decimal numbers

In all the operations on numbers we would consider 10×10 grid as a unit.

Addition

Shade the region corresponding to one of the numbers and then shade a region adjacent to that the region corresponding to the other number. Find the total region that is shaded which gives the sum of two numbers.

Give examples and exercises on these.

Subtraction

Shade the region corresponding to the minuend and cross out the region corresponding to the subtrahend. The left over shaded region gives the difference.

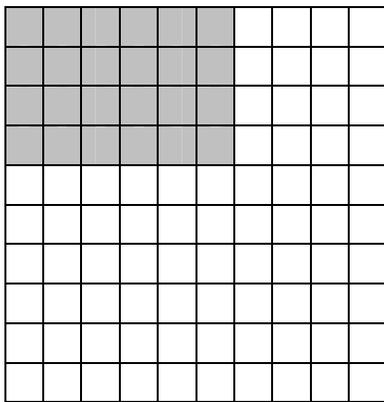
Give examples and exercises on these.

Multiplication

Multiplication of tenths

The whole square has a length of 1; therefore length of each square is one tenth or 0.1, length of 2 squares is two tenths or 0.2, length of 3 squares is three tenths or 0.3 and so on.

To multiply 0.6 by 0.4, shade a rectangle whose length is 0.6 or 6 small squares and breadth is 0.4 or 4 small squares. As the area shaded is 24 hundredths, $0.4 \times 0.6 = 0.24$.



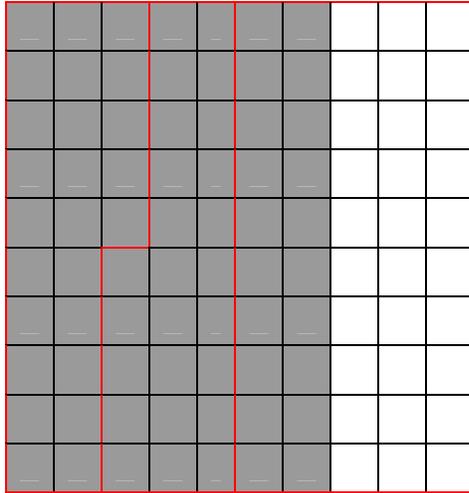
Give more examples and exercises on these and let them discover the rule that it is as many hundredths as the product of two numbers ignoring the decimals.

Similarly we can extend it to multiplication of a whole number and a tenth and a number greater than one and a tenth.

Division

Shade graph that represents the dividend and then enclose the number of squares represented by the divisor. Continue doing that till the number left over is less than the divisor. The quotient is given by the number of enclosures and remainder by the left over squares. Express remainder by divisor as a decimal and add that to the quotient. For example, to divide 0.7 by 0.25, we first shade 7 longs and then enclose 0.25 that is 2 long and 5 shorts in it twice (enclosed by red borders). After we have done that only 2 longs

are left and $.20/.25 = 4/5 = 8/10 = 0.8$ adding that to the quotient we have 2.8, therefore $0.7 \div 0.25 = 2.8$



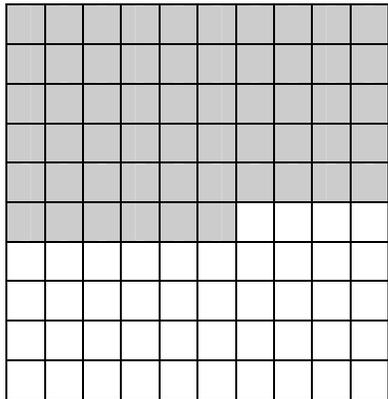
Concept of percent

Percent means per centum or out of 100.

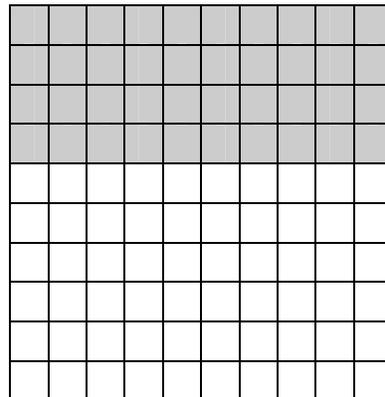
We can represent a percent by using a 10×10 grid and shading it.

This square is divided into 100 equal parts, 56 of out of 100 are shaded. Another way of saying it is 56 percent of the small squares are shaded. We write it as 56%.

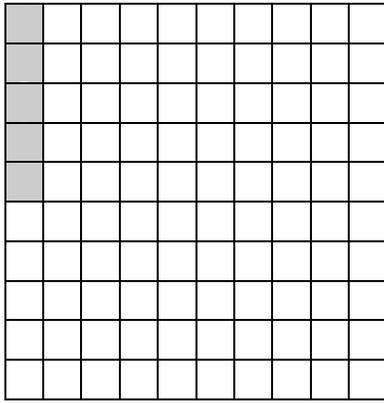
The percent of areas shaded in other figures are given below them. (The last two grids are parts of one figure)



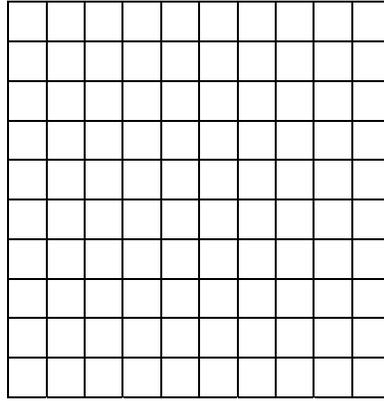
56 %



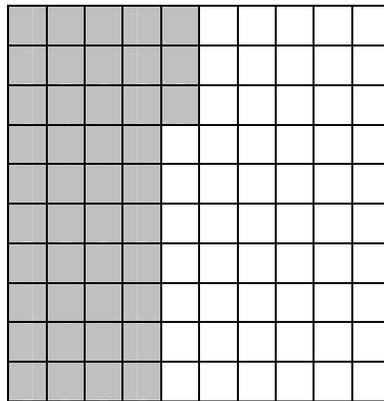
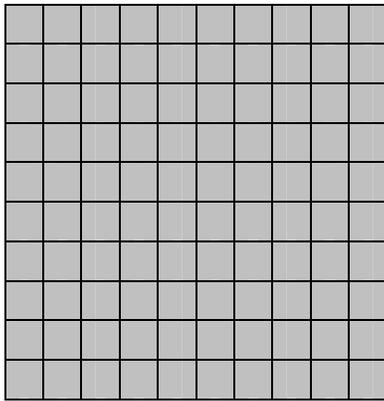
40 %



5%



0%



← 143 % →

1. Give them figures in which less than 10, between 10 and 100 and more than 100 squares are shaded using more than one grid and ask them to find out what percent of the figures are shaded.
2. Shade the following percents in a graph paper where a 10×10 square represents 100.
 (a) 65% (b) 30% (c) 7% (d) 137% (e) 0.25%
3. Let the whole square represent a certain amount say 500 rupees, then what does one square represent?(Divide the total amount by 100).
 What would 20 squares represent?
 What would half a square represent?
4. If a small square represents 8 persons, shade part of the square will represent 240 persons?

Grids can be used to solve many types of percent problems. For more on these see the lesson on Grid and Percent It in illuminations.nctm.org/LessonDetail.aspx?id=L249

Geometry and Measurement

Use triangle graph paper given in Appendix 2 for all the activities given below:

Angle

Draw an acute and obtuse angle.

Draw an angle of 60° , 90° , 120° , 180° , 240° , 270° and 300° .

Triangle

If the side of the small triangle is taken as a linear unit of measurement, find the perimeters of a given equilateral triangle.

Draw equilateral triangles with specific perimeters e.g. 3, 9 and 15.

If the area of the small triangle is taken as a unit of area, find the area of a given equilateral triangle.

Draw equilateral triangles with specific areas e.g. 4, 9 and 25.

Draw equilateral triangles similar to given triangles with specific similarity ratios.

Find the relation between the similarity ratio and the ratio of perimeters of equilateral triangles.

Find the relation between the similarity ratio and the ratio of areas of equilateral triangles.

Polygons

Draw a rhombus.

If the side of the small triangle is taken as a linear unit of measurement, find the perimeter of the rhombus.

If the area of the small triangle is taken as a unit of area, find the area of rhombus.

Draw rhombuses with specific perimeters.

Draw rhombuses with specific areas.

Draw a rhombus similar to a given rhombus with a specific similarity ratio.

Repeat all of these for a parallelogram, trapezium and a hexagon.

Repeat for a square and a rectangle using Dot or square graph paper.

Graphs

The activities given under graph by the author in

<http://www.vidyaonline.org/writeup/dotpaper.pdf> can also be done using graph paper.

Bar graphs

Collection and tallying of real-world data

Make a table as the one given below on the Blackboard. Ask students one by one about the type of TV programme they like to watch most. Write the name of the TV programme and mark a tally-| for each students that names it in a table as the one given below.

TV programme	Number of students who would like to have it (A tally represents one student)
Cartoons	
Serials	
Cricket or other matches	
Others	

Call on a volunteer to count number of tallies in each row and record the number at the end of the row. Tell them it would make the counting easier if we record the fifth entry by making a diagonal mark across the first four lines (||||).]. Call on a volunteer to redraw the tallies using the grouping notation in another table and then ask

1. Did it make easier to count the tallies in the tally graph?
2. Why did that notation make it easier?
3. What heading would you like to give to the table that will describe the data in it?
4. How did we show what we found out?

We can present a summary of this information in a similar table by writing the number against TV programmes instead of tallies

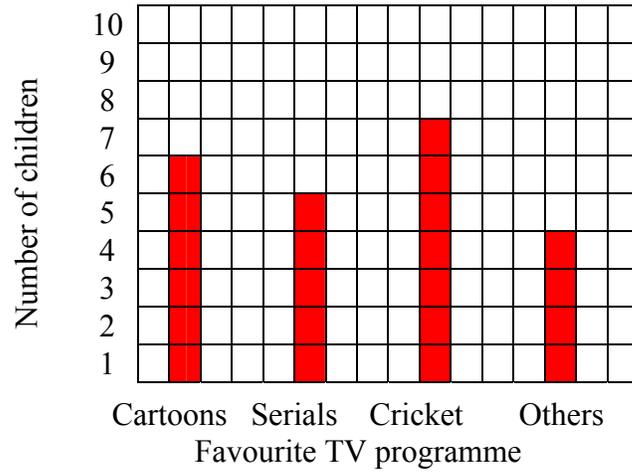
We can present the data in a table by a **bar graph**.

To create a bar graph,

1. Draw two lines at right angles to each other.
2. Write the name of categories along the horizontal line (or vertical) and choose a scale for length of the bar along the vertical (or horizontal) scale.
3. Erect bars of uniform width vertically (or horizontally) for each category with length equal to the number in a category using a square as a unit. The bars should have equal spacing between them.

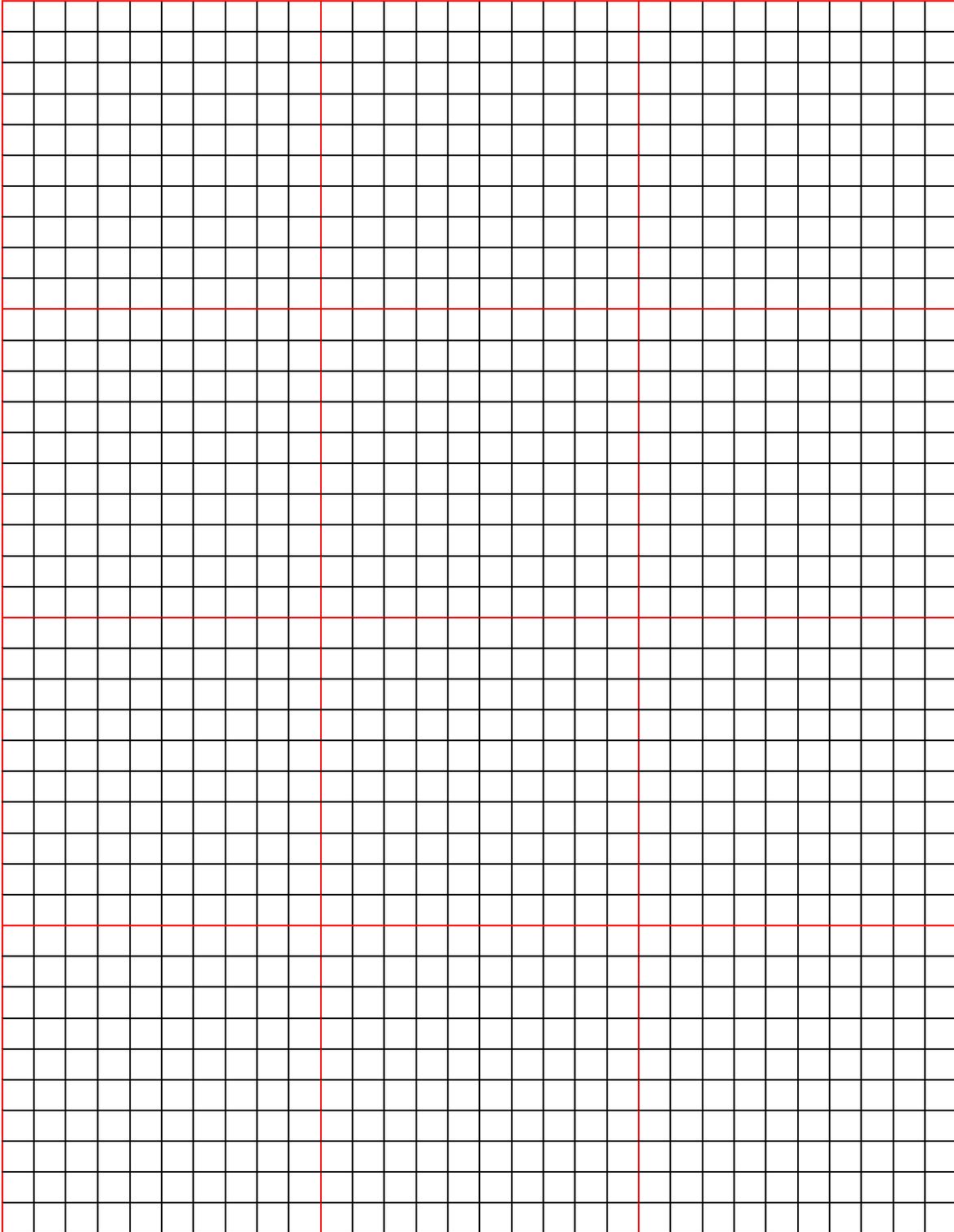
For example, we can represent the data on favourite TV programme of a class in a bar graph as follows:

Favourite TV Programmes of a Class



1. How many children prefer to watch cartoons?
2. How many children prefer to watch cricket matches?
3. Which kind of programme is most popular?
4. Which kind of programme is least popular?
5. How many more children prefer to watch cricket matches than serials?
6. What heading would you like to give to the table?

Appendix 1



Appendix 2

Triangle Graph Paper

