M4.1 Probability and Venn diagrams

Before you start

You should be able to:

- draw and interpret Venn diagrams
- find the probability that an event will occur.

Why do this?

Venn diagrams can be used to help work out probabilities.

Objective

- You will be able to use set notation to describe events.
- You will be able to use Venn diagrams to find probabilities.

Get Ready

A bag contains 3 red, 2 blue and 6 green counters. A counter is taken at random. What is the probability that the counter is:

1 red **2** green **3** not green
4 blue or green **5** white

Key Points

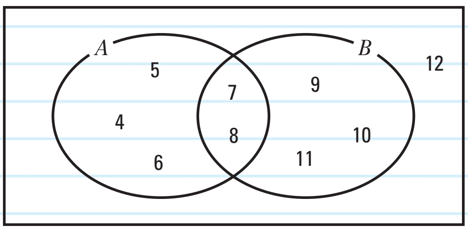
When working out probabilities from a Venn diagram:

- P(A) represents the probability that the item is in set A
- P(A') represents the probability that the item is *not* in set A
- $P(A') = 1 - P(A)$
- P($A \cap B$) represents the probability that the item is in both set A and set B
- P($A \cup B$) represents the probability that the item is in set A or in set B or in both sets.

Example 1

A03

The Venn diagram shows the integers from 4 to 12.



A number is taken at random from those shown on the Venn diagram.

Find: **a** P(A) **b** P(A') **c** P($A \cap B$).

a $P(A) = \frac{5}{9}$

There are 9 numbers in total in the Venn diagram so 9 goes on the bottom of the fraction.
There are 5 numbers altogether in set A so 5 goes on the top of the fraction.

b $P(A') = \frac{4}{9}$

There are 4 numbers that are not in set A.
Alternatively, work out $1 - \frac{5}{9}$

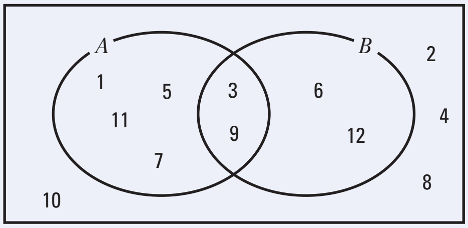
c $P(A \cap B) = \frac{2}{9}$

There are 2 numbers in both A and B.

Exercise 4A

D AO1

1 The Venn diagram shows the whole numbers from 1 to 12.

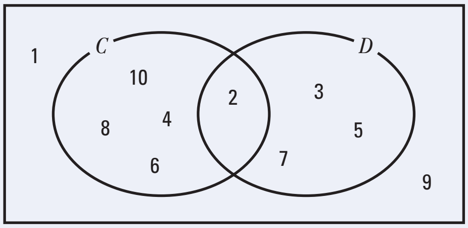


A number is chosen at random from those shown on the Venn diagram.

Find: **a** P(B) **b** P($A \cap B$) **c** P($A \cup B$)

AO1

2 The Venn diagram shows the whole numbers from 1 to 10.



A number is chosen at random from those shown on the Venn diagram.

Find: **a** P(D) **b** P(D') **c** P($C \cap D$) **d** P($C \cup D$)

C AO3

3 **a** On a Venn diagram show:
the whole numbers from 1 to 12
set E where $E = \{2, 4, 6, 8, 10, 12\}$
set F where $F = \{1, 2, 3, 4, 6, 12\}$

b A number is chosen at random from those in the Venn diagram. Find:
i P(E) **ii** P(F') **iii** P($E \cap F$) **iv** P($E \cup F$)

B AO3

4 **a** Draw a Venn diagram to show:
$\mathscr{E}$ = {integers from 10 to 20}
E = {even numbers}
M = {multiples of 5}

b A number is chosen at random from those in the Venn diagram. Find:
i P(M) **ii** P($E \cap M$) **iii** P($E \cup M$) **iv** P($E' \cap M$) **v** P($E \cap M$)$'$

A AO3

5 $\mathscr{E}$ = {integers from 1 to 20}
M = {multiples of 4}
F = {factors of 20}

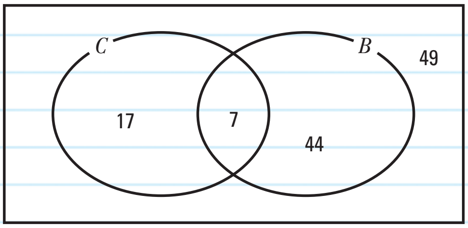
A number is chosen at random from the universal set, $\mathscr{E}$.
Work out:

a P(M) **b** P(F') **c** P($M \cap F$) **d** P($M \cup F$) **e** P($M' \cap F$)

Example 2 The Venn diagram shows information about the students in Year 12.

B = {students who take Biology}

C = {students who take Chemistry}



A03

If a student is chosen at random work out:

a $P(B)$ **b** $P(B \cap C)$ **c** $P(C \cap B')$ **d** $P(B \cup C)$

a $P(B) = \frac{51}{117}$

49 + 44 + 17 + 7 = 117 there are 117 students in Year 12, so the bottom number of each fraction will be 117.
44 + 7 = 51 so 51 students study Biology.

b $P(B \cap C) = \frac{7}{117}$

7 is in the intersection. This shows that 7 students study Biology and Chemistry.

c $P(C \cap B') = \frac{17}{117}$

There are 17 students who study Chemistry and not Biology.

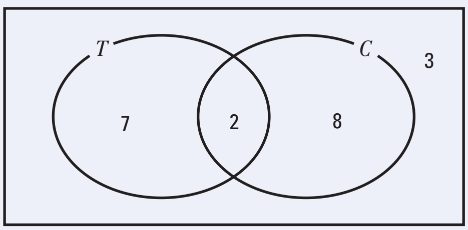
d $P(B \cup C) = \frac{68}{117}$

17 + 7 + 44 = 68 so 68 students study Biology or Chemistry (or both).

Exercise 4B

1 Some students were asked if they played tennis or cricket.

D A01



The Venn diagram shows information about their answers.
A student is chosen at random. Work out:

a $P(T)$ **b** $P(C)$ **c** $P(T \cap C)$

C A01

2 In a group of 42 people, 13 belong to a badminton club, 19 belong to a tennis club and 7 belong to both a badminton and a tennis club.

a Draw a Venn diagram to represent this information.
A person is chosen at random from this group.

b Find the probability that this person:

i does not belong to a badminton club

ii does not belong to either a badminton or a tennis club

iii belongs to a tennis club but not a badminton club.

B A03

3 There are 26 students in a tutor group. Of these students 11 study History, 17 study PE and 6 students study both History and PE. A student is chosen at random. Work out the probability that this student studies:

a History **b** PE **c** History but not PE **d** neither History nor PE.

A03

4 There are 37 cars parked in a car park. 12 of the cars are red, 22 of the cars are Fords and 8 of the cars are red Fords. One of the cars in the car park is chosen at random. What is the probability that it is:

a not red

b a red car that is not a Ford

c neither red nor a Ford?

A A03

5 There are 29 students in a music class.

13 can play the guitar,
8 can play the piano,
10 cannot play the guitar and cannot play the piano.

One of the 29 students is chosen at random.
Work out the probability that this student can play the guitar but not the piano.

B A03

6 There are 120 people watching a film.

68 have popcorn,
29 have popcorn and a drink,
35 have neither popcorn nor a drink.

One of these people is chosen at random. Work out the probability that this person has a drink but does not have any popcorn.

A03

7 In a group of 35 girls 6 wear glasses, 17 have brown hair and 2 girls have brown hair and wear glasses. One of these girls is chosen at random. Work out the probability that she:

a has brown hair but does not wear glasses

b does not have brown hair and does not wear glasses.

M4.2 Compound events

Before you start

You should be able to:

- add and multiply fractions.

Why do this?

Set notation can be used to describe the probability of two events occurring at the same time.

Objectives

- You will be able to use set notation to describe compound events.

Get Ready

1. A fair dice is thrown. Work out the probability of throwing:
   a a 1 or a 2 **b** either an even number or a prime number.
2. Two fair dice are thrown together. The scores are added together. Work out the probability of throwing:
   a a total of 3 **b** a total of 7.

Key Points

- Two events are mutually exclusive when they cannot occur at the same time.
  For mutually exclusive events A and B:
  $P(A \cup B) = P(A) + P(B)$
- Two events are independent if one event does not affect the other event.
  For two independent events A and B:
  $P(A \cap B) = P(A) \times P(B)$

Example 3

A03

M and N are mutually exclusive events.
$P(M) = \frac{4}{9}$ $P(N) = \frac{1}{3}$
Work out $P(M \cup N)$.

$P(M \cup N) = \frac{4}{9} + \frac{1}{3}$

$= \frac{4}{9} + \frac{3}{9}$

$= \frac{7}{9}$

M and N are mutually exclusive events. So use $P(M \cup N) = P(M) + P(N)$

Example 4

A03

A dice and a coin are thrown.
Event F is getting a 5 on the dice. Event H is getting a head on the coin.
Work out:
a $P(F)$ **b** $P(H)$ **c** $P(F \cap H)$.

a $P(F) = \frac{1}{6}$

b $P(H) = \frac{1}{2}$

c $P(F \cap H) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

Throwing a dice and throwing a coin are independent events, since the outcome of one event does not affect the outcome of the other event. So use $P(A \cap B) = P(A) \times P(B)$

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Exercise 4C

C

A01

1 A bag contains 5 red, 3 green and 4 yellow counters.

Event R is getting a red counter.

Event G is getting a green counter.

Event Y is getting a yellow counter.

A counter is taken at random from the bag. Work out:

a P(R) **b** P(G) **c** P(Y)

d P($R \cup Y$) **e** P($G \cup Y$)

A01

2 A bag contains 3 red and 4 blue counters.

A box contains 2 red and 5 blue counters.

Event A is getting a red counter from the bag.

Event B is getting a red counter from the box.

One counter is taken at random from the bag and another counter is taken at random from the box. Work out:

a P(A) **b** P(B) **c** P($A \cap B$)

B

A03

3 The events A and B are mutually exclusive.

Given that P(A) $= \frac{1}{3}$ and P(B) $= \frac{5}{8}$ work out:

a P(A') **b** P($A \cup B$)

A03

4 The events A and B are independent.

Given that P(A) $= \frac{2}{5}$ and P(B) $= \frac{1}{4}$ work out:

a P(B') **b** P($A \cup B$)

A03

5 The events D and E are mutually exclusive.

Given that P(D) $= \frac{2}{5}$ and P($D \cup E$) $= \frac{3}{4}$ work out:

a P(D') **b** P(E)

A

A03

6 P(C) $= \frac{1}{4}$, P(D) $= \frac{2}{5}$, P($C \cap D$) $= \frac{1}{10}$

Are events C and D independent?

You must give a reason for your answer.

A03

7 P(E) $= \frac{1}{4}$, P(F) $= \frac{2}{5}$, P($E \cup F$) $= \frac{7}{10}$

Are events E and F mutually exclusive?

You must give a reason for your ans∪∩wer.

A03

8 The event X and Y are independent.

Given that P(X) $= \frac{3}{8}$ and P($X \cap Y$) $= \frac{9}{44}$ work out P(Y').

Review

- P(A) represents the probability that the item is in set A.
- P(A') represents the probability that the item is *not* in set A.
- P(A') $= 1 - P(A)$
- P($A \cap B$) represents the probability that the item is in both set A and set B.
- P($A \cup B$) represents the probability that the item is in set A or in set B or in both sets.
- Two events are mutually exclusive when they cannot occur at the same time.

  For mutually exclusive events A and B:
  $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
- Two events are independent if one event does not affect the other event.

  For two independent events A and B:
  $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$

Answers

Chapter 4

M4.1 Get Ready answers

1 $\frac{3}{11}$

2 $\frac{6}{11}$

3 $\frac{5}{11}$

4 $\frac{8}{11}$

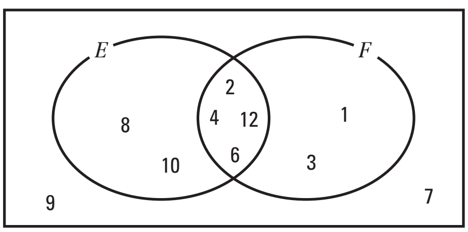
5 0

Exercise 4A

1 **a** $\frac{4}{12}$ **b** $\frac{2}{12}$ **c** $\frac{8}{12}$

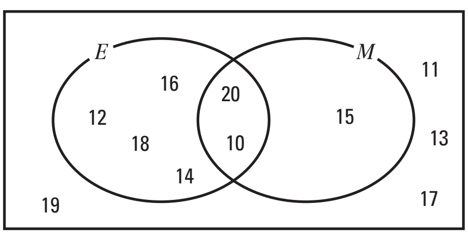
2 **a** $\frac{4}{10}$ **b** $\frac{6}{10}$ **c** $\frac{1}{10}$ **d** $\frac{8}{10}$

3 **a**



b **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{3}$ **iv** $\frac{2}{3}$

4 **a**



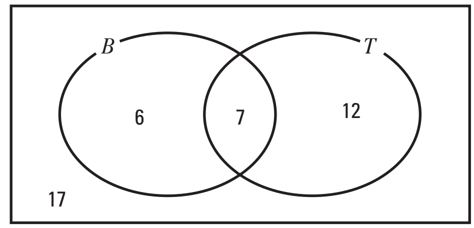
b **i** $\frac{3}{11}$ **ii** $\frac{2}{11}$ **iii** $\frac{7}{11}$ **iv** $\frac{1}{11}$ **v** $\frac{9}{11}$

5 **a** $\frac{1}{4}$ **b** $\frac{14}{20}$ **c** $\frac{1}{10}$
d $\frac{9}{20}$ **e** $\frac{1}{5}$

Exercise 4B

1 **a** $\frac{9}{20}$ **b** $\frac{10}{20}$ **c** $\frac{2}{20}$

2 **a**



b **i** $\frac{29}{42}$ **ii** $\frac{17}{42}$ **iii** $\frac{12}{42}$

3 **a** $\frac{11}{26}$ **b** $\frac{17}{26}$ **c** $\frac{5}{26}$ **d** $\frac{4}{26}$

4 **a** $\frac{23}{37}$ **b** $\frac{4}{37}$ **c** $\frac{9}{37}$

5 $\frac{11}{29}$

6 $\frac{17}{120}$

7 **a** $\frac{15}{35}$ **b** $\frac{14}{35}$

M4.2 Get Ready answers

1 **a** $\frac{1}{3}$ **b** 1

2 **a** $\frac{1}{18}$ **b** $\frac{1}{6}$

Exercise 4C

1 **a** $\frac{5}{12}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$
d $\frac{3}{4}$ **e** $\frac{7}{12}$

2 **a** $\frac{3}{7}$ **b** $\frac{2}{7}$ **c** $\frac{6}{49}$

3 **a** $\frac{2}{3}$ **b** $\frac{23}{24}$

4 **a** $\frac{3}{4}$ **b** $\frac{1}{10}$

5 **a** $\frac{3}{5}$ **b** $\frac{7}{20}$

6 Yes as $\frac{1}{4} \times \frac{2}{5} = \frac{1}{10}$

7 No as $\frac{1}{4} + \frac{2}{5} = \frac{13}{20}$

8 $P(Y') = \frac{5}{11}$