

Section 1.5: Venn Diagrams – Shading

NOTE: The way I teach the material in the section 1.5 videos is different than the written explanations I wrote for this section.

I moved the written explanation part of this section to the end of this handout since it doesn't match up with my video explanation.

The written explanation part of this section explains an alternative way to solve the problems that most students didn't understand very well. **You may ignore the written explanation part of this section if you are comfortable with the method I use to solve the problems in the videos for this section.**

Homework #1-9: Sketch the region.

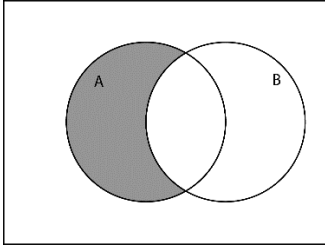
- | | |
|-------------------|-------------------|
| 1) $A \cap B'$ | 2) $A' \cap B'$ |
| 3) $A' \cup B$ | 4) $A \cup B'$ |
| 5) $(A \cap B)'$ | 6) $(A \cap B')'$ |
| 7) $(A' \cup B)'$ | 8) $(A \cup B')'$ |
| 9) $A' \cup B'$ | |

Homework #10-23: Sketch the shaded region.

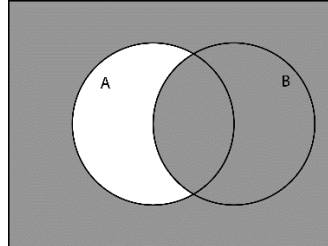
- | | |
|--------------------------|--------------------------|
| 10) $A \cap B' \cup C$ | 11) $A \cap B \cup C'$ |
| 12) $A \cap B' \cap C$ | 13) $A \cap B \cap C'$ |
| 14) $A \cup B \cup C'$ | 15) $A' \cup B \cup C$ |
| 16) $(B \cap C)' \cup A$ | 17) $(A \cap B)' \cup C$ |
| 18) $B \cup (A \cap C)$ | 19) $A \cup (B \cap C)$ |
| 20) $B \cap (A \cup C)$ | 21) $A \cap (B \cup C)$ |
| 22) $B \cap (A \cup C)'$ | 23) $A \cap (B \cup C)'$ |

Answers:

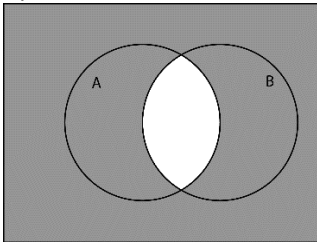
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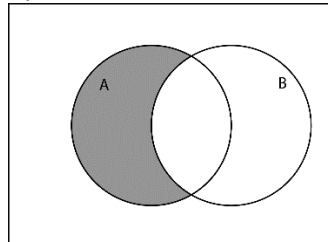
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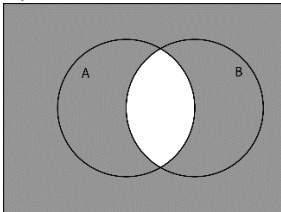
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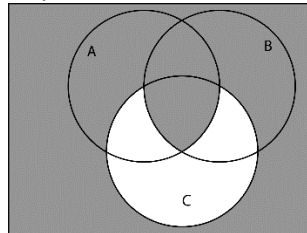
7)



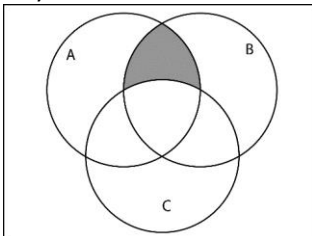
9)



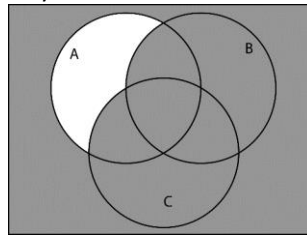
11)



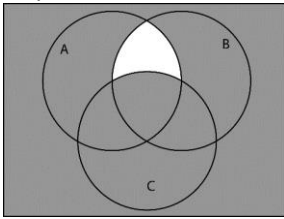
13)



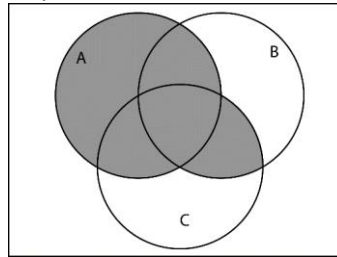
15)



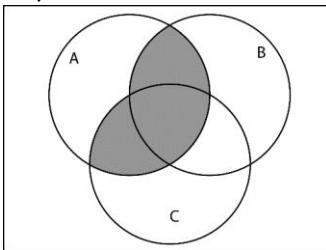
17)



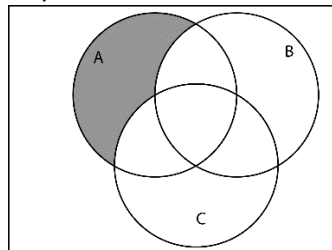
19)



21)



23)



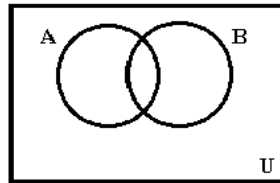
Here is the written explanation of an alternative way to solve the problems in section 1.5.

A **Venn diagram** is a way to graphically represent sets and set operations. Each Venn diagram begins with a rectangle representing the universal set. Then each set is represented by a circle inside the rectangle. We will only be studying with Venn diagrams that have 2 or 3 overlapping circles (sets). There are Venn diagrams that contain more than 3 circles (sets).

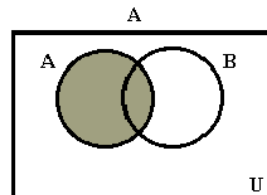
In this section we will learn how to graphically represent union, intersection and complement of sets using Venn diagrams. We will start with Venn diagrams that have two sets as they are easier.

Venn diagrams for two sets:

Here is a Venn diagram for two sets A and B. The universal set is depicted with a rectangle. The two sets “A” and “B” are depicted with overlapping circles inside the rectangle.



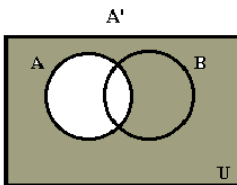
This is a Venn diagram associated with the set A. Notice we shade the set that the Venn diagram depicts.



Now for some set operations.

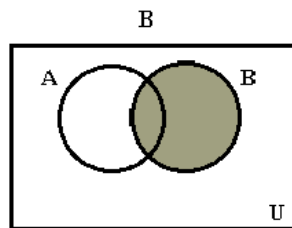
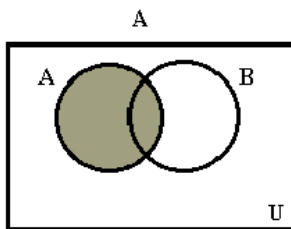
Example: Create a Venn diagram for A' . That is shade the area depicted by the set A' .

Recall that the complement of A (denoted A') is everything in the universal set that is not in set A. We shade everything except what's in the A circle. Here is the Venn diagram that depicts set A' .

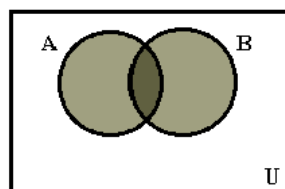


Example: Create a Venn diagram for $A \cup B$.

To do this: first create a shaded Venn diagram for each set described in the problem. I will create a Venn diagram for set A and another for set B.

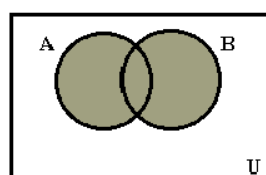


Now put them together. Notice that I darkened the area that is shaded in both diagrams.



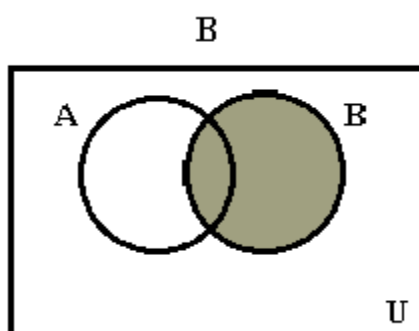
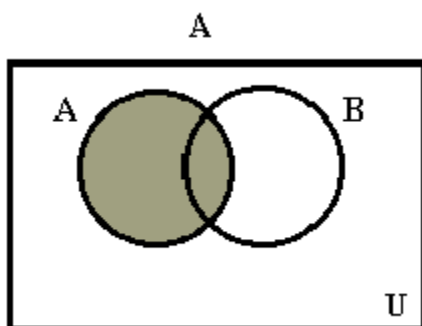
This is a union problem so anything that gets shaded at least once is in the union, so here's the Venn diagram that shows the answer.

$$A \cup B$$

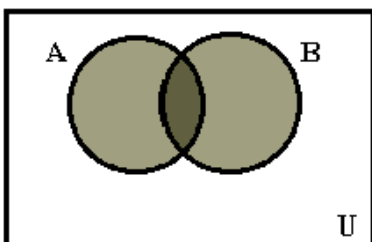


Example: Create a Venn diagram for $A \cap B$.

First create a shaded Venn diagram for each set described in the problem.

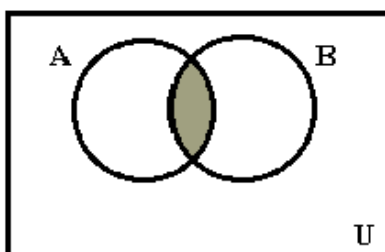


Now put them together.



This is an intersection problem. The final Venn diagram should be shaded only where the two sets cross. Here is the answer:

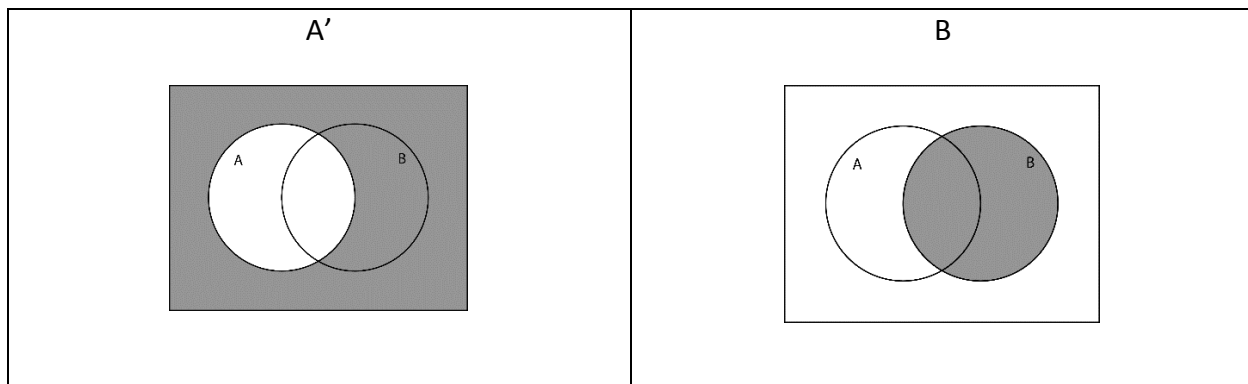
$$A \cap B$$



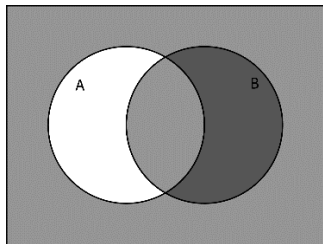
There are the basics, now let's try a couple that are more involved.

Example: Create a Venn diagram for $A' \cap B$

First create a Venn diagram for each set described in the problem.

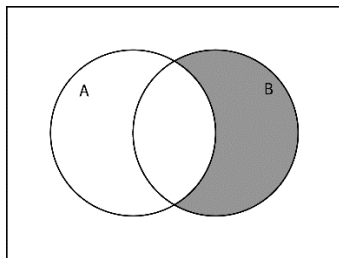


Next, put them together.



The intersection is the common region that is shaded. Here is my final answer.

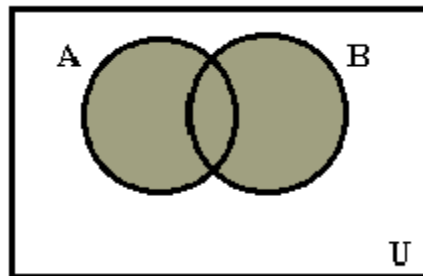
$A' \cap B$



Example: Find a Venn diagram for $(A \cup B)'$.

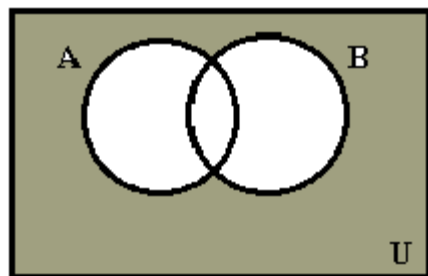
First shade what's in the parenthesis, that is $A \cup B$. (I took this shading from an earlier example.)

$$A \cup B$$



Now take the complement of that set. The complement is the region that is not already shaded. This is my answer.

$$(A \cup B)'$$

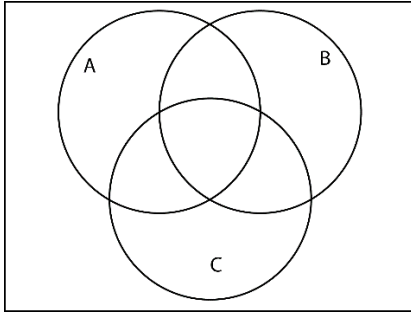


Homework #1-9: Sketch the region.

- | | |
|-------------------|-------------------|
| 1) $A \cap B'$ | 2) $A' \cap B'$ |
| 3) $A' \cup B$ | 4) $A \cup B'$ |
| 5) $(A \cap B)'$ | 6) $(A \cap B')'$ |
| 7) $(A' \cup B)'$ | 8) $(A \cup B')'$ |
| 9) $A' \cup B'$ | |

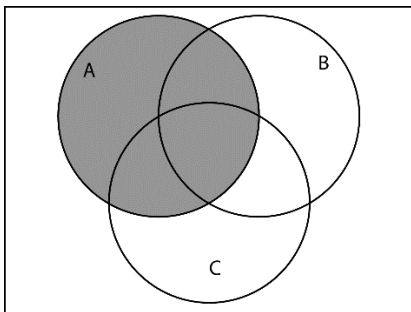
Venn Diagrams for 3 Sets:

Here's the beginning Venn diagram associated with 3 sets.



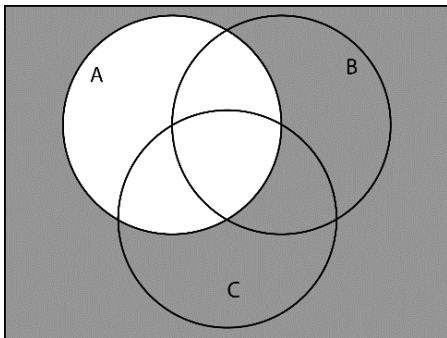
Complements, unions and intersections are handled in the same way as they were with 2 set Venn diagrams.

Here is the Venn diagram associated with the set A. Notice we shade the inside of the A circle.



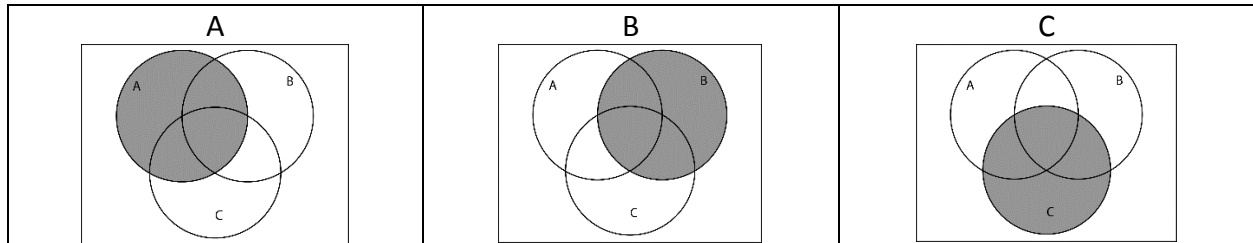
Example: Create a Venn diagram for A' .

Recall that the complement of A is everything that is in the universal set but not in the set A, so we shade everything except what's in the circle A.

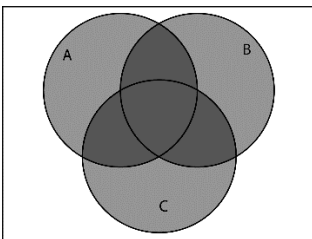


Example: Create a Venn diagram that represents $A \cup B \cup C$.

To do a Venn diagram for a union of three sets, first shade each set.

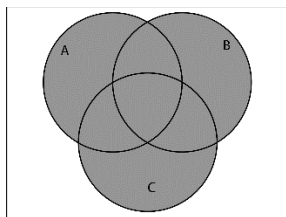


Now put them together



Anything that gets shaded at least once is in the union, so here's the answer.

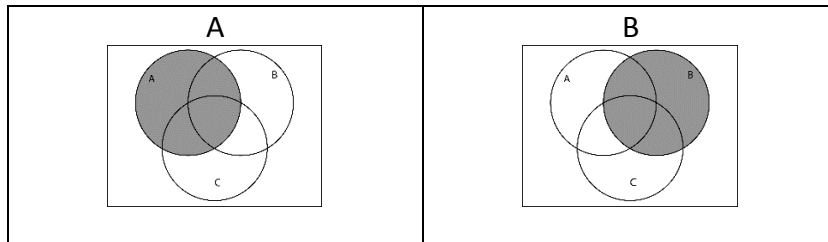
$$A \cup B \cup C$$



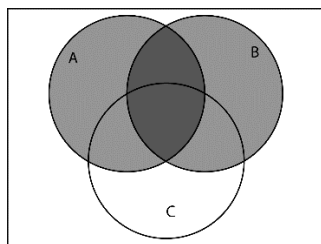
Example: Create a Venn diagram for $A \cap B \cup C$.

We need to work from left to right. First we will find $A \cap B$

First shade each set.

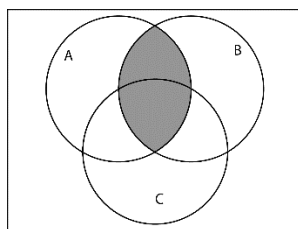


Now put them together

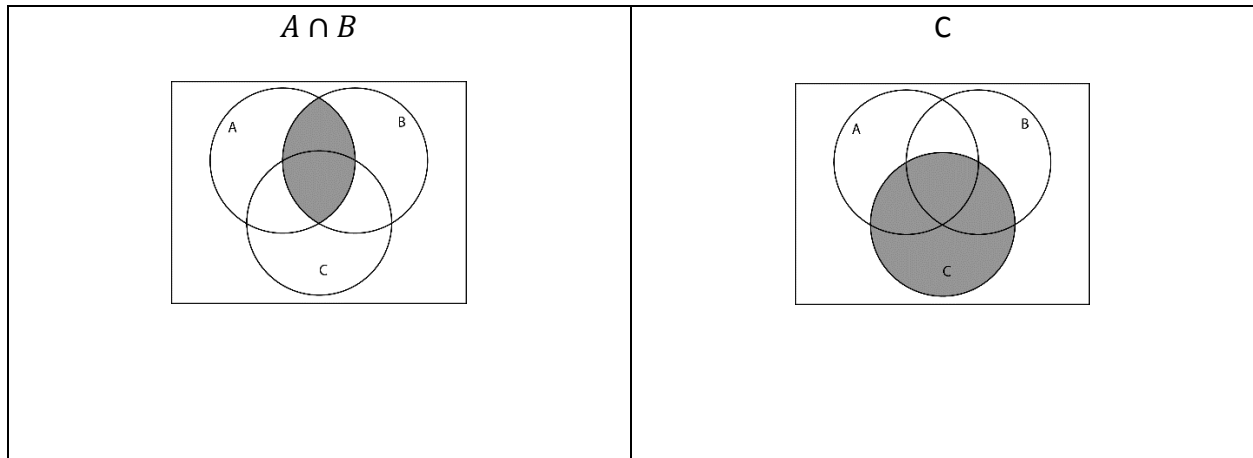


$A \cap B$ is the region where the two graphs overlap.

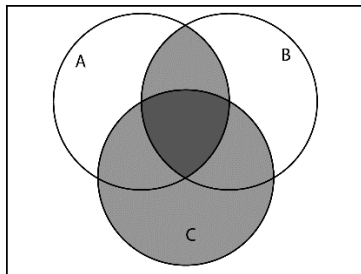
$A \cap B$



Next put the graphs of $A \cap B$ and of C next to each other.

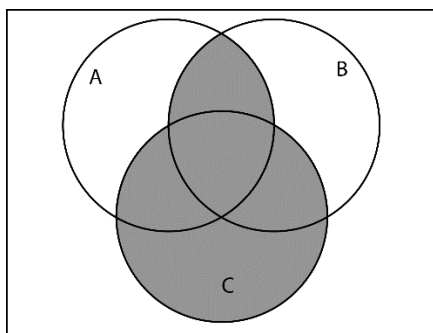


Put the graphs together



The union consists of any region that is shaded. This is my answer.

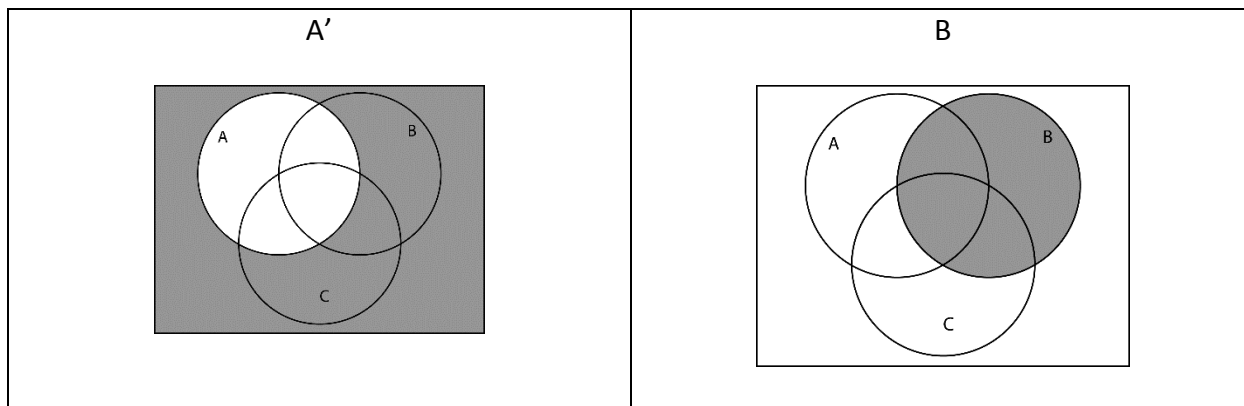
$$A \cap B \cup C$$



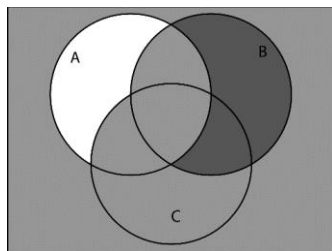
Example: Create a Venn diagram for $A' \cap B \cup C$.

Let's work from left to right. Find $A' \cap B$.

Create a diagram for each set

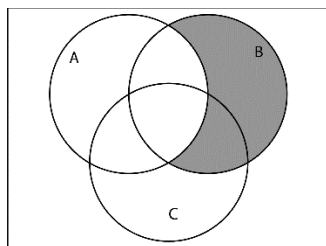


Put the sets together.

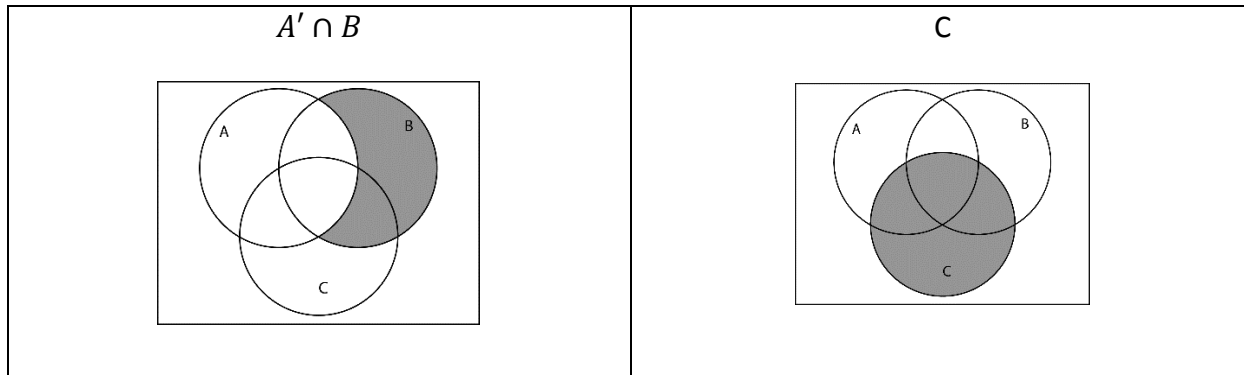


$A' \cap B$ is the region where the two graphs overlap.

$A' \cap B$

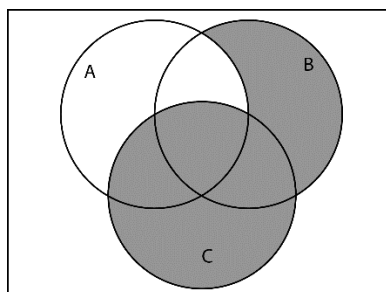


Next we put the graphs for $A' \cap B$ and C next to each other.



Now put them together. We are finding the union of the two sets. Anything that gets shaded is part of my answer. Here is the answer.

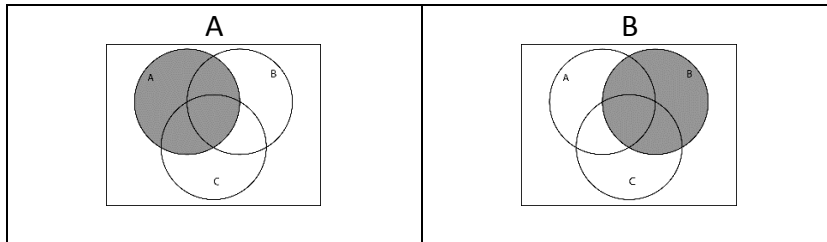
$$A' \cap B \cup C$$



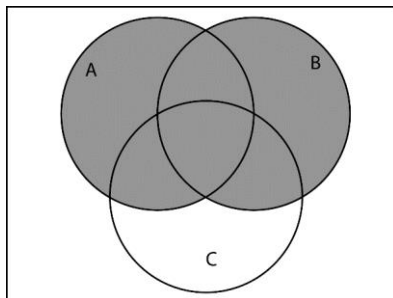
Example: Sketch $(A \cup B)' \cap C$

We need to work on the inside of the parenthesis of $(A \cup B)'$ first.
We will first graph $A \cup B$

First shade each set

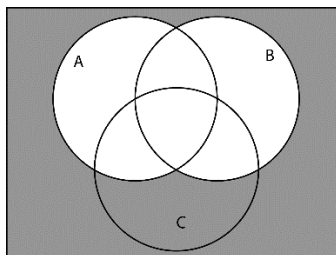


Now put them together. We are finding the union of the two sets. Anything that gets shaded is part of my answer. This is a diagram for $A \cup B$



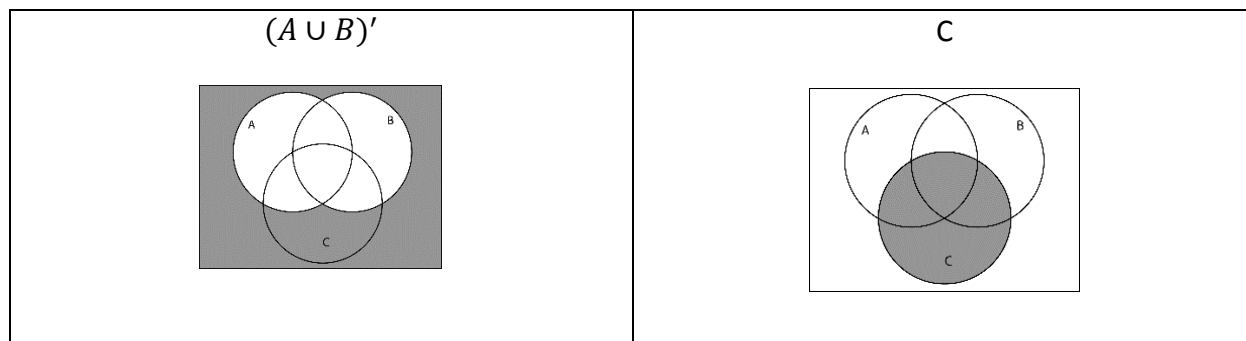
Now we can find $(A \cup B)'$, which is just the complement of the set $A \cup B$, which is anything not shaded in our last drawing.

$(A \cup B)'$

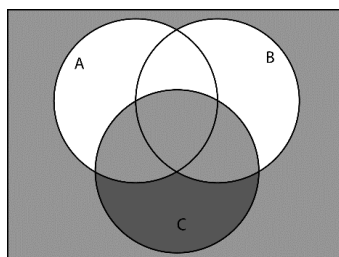


Now we can get back to the problem: $(A \cup B)' \cap C$

Put the sets $(A \cup B)'$ and C next to each other.

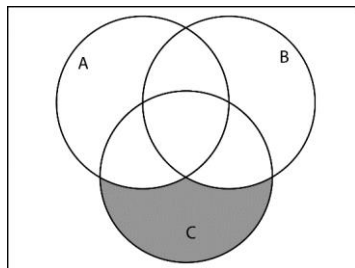


Now put the graphs together



Remember the intersection only includes the region where the shadings overlap. Here is the answer.

$$(A \cup B)' \cap C$$



Homework #10-23: Sketch the shaded region.

10) $A \cap B' \cup C$

11) $A \cap B \cup C'$

12) $A \cap B' \cap C$

13) $A \cap B \cap C'$

14) $A \cup B \cup C'$

15) $A' \cup B \cup C$

16) $(B \cap C)' \cup A$

17) $(A \cap B)' \cup C$

18) $B \cup (A \cap C)$

19) $A \cup (B \cap C)$

20) $B \cap (A \cup C)$

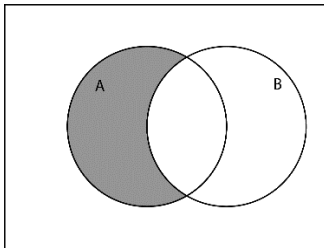
21) $A \cap (B \cup C)$

22) $B \cap (A \cup C)'$

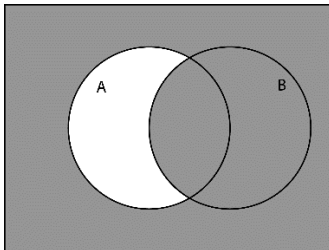
23) $A \cap (B \cup C)'$

Answers:

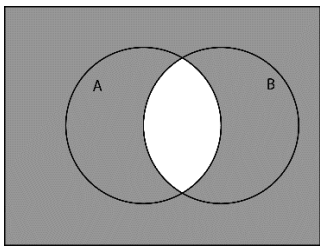
1)



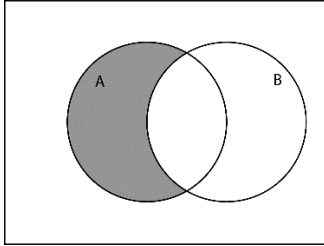
3)



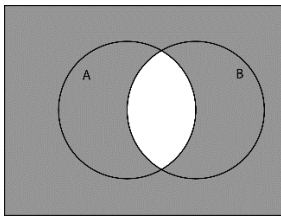
5)



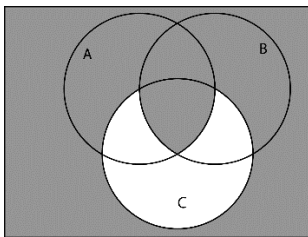
7)



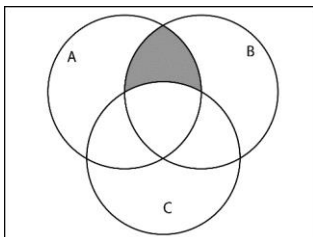
9)



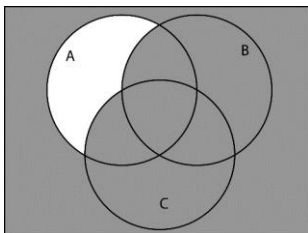
11)



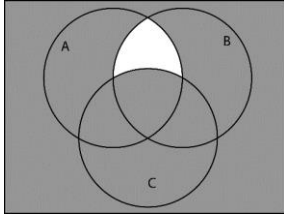
13)



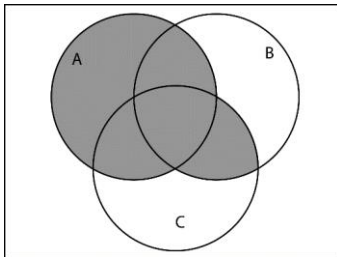
15)



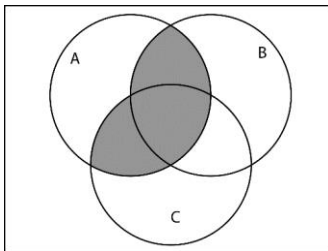
17)



19)



21)



23)

