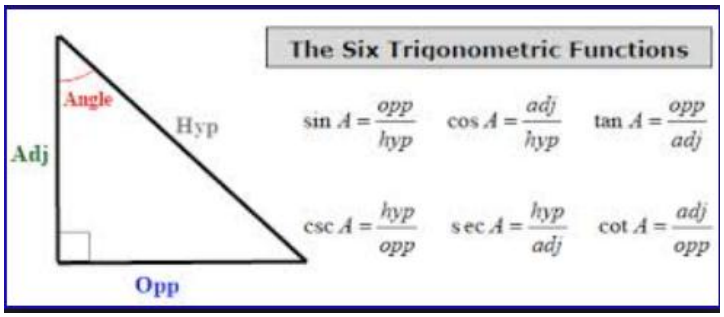
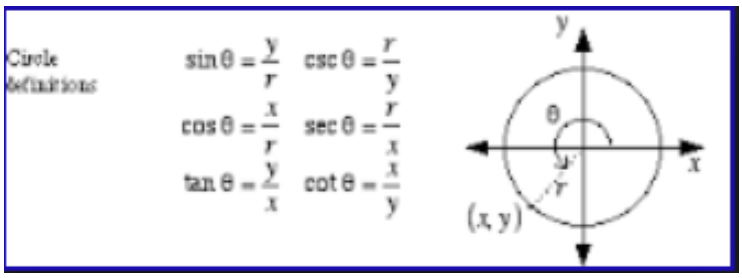
Pre Calculus	Trig Functions: The Unit Circle

Trigonometric Functions

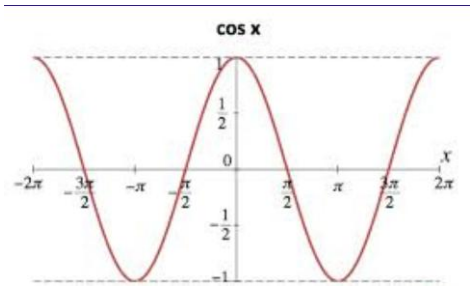
What are the six trigonometric functions?

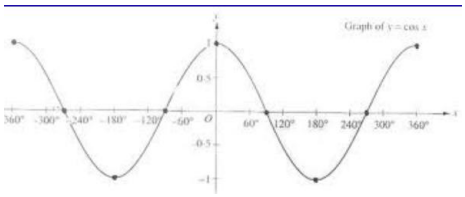




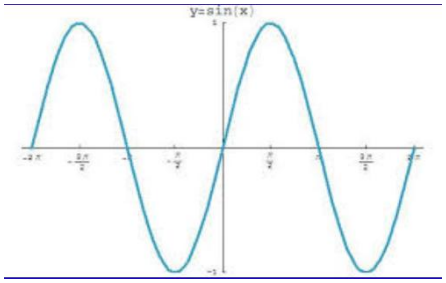
Rectangular Coordinate Plane

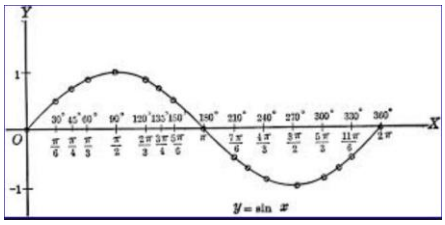
$f(x) = \cos x$





$f(x) = \sin x$





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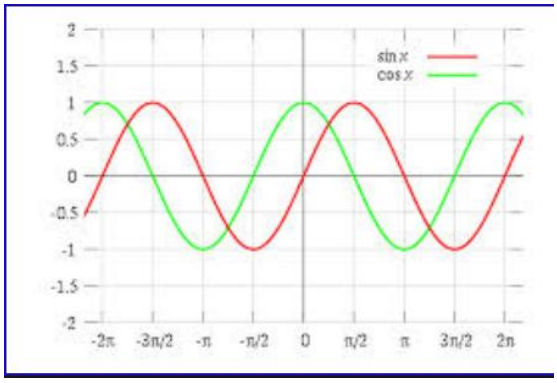

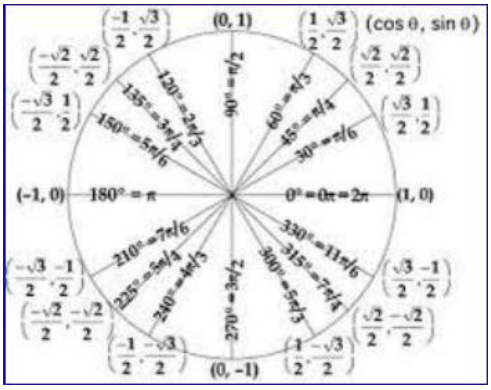

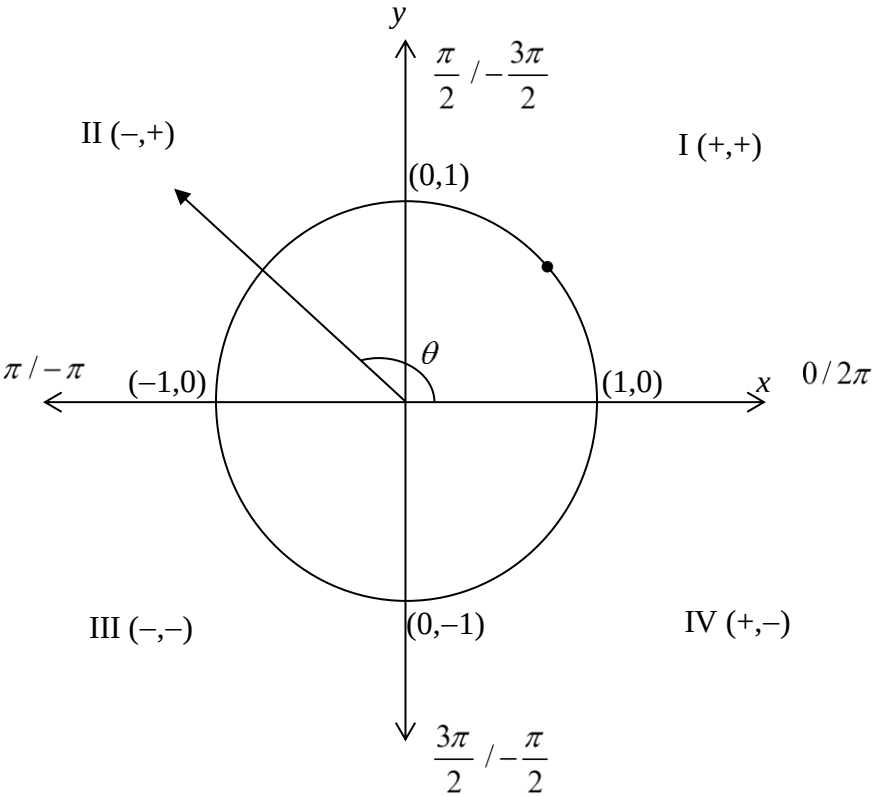

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Trigonometric Functions

What are the six trigonometric functions?

Sine, Cosine, Tangent, Secant, Cosecant and Cotangent

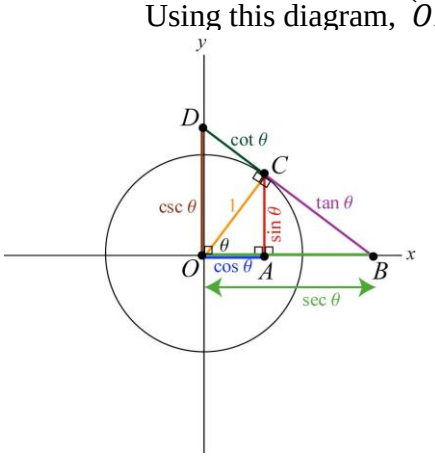
Can you complete the table below for these exact values in degrees? In radians?

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
x	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

The Unit Circle

How are the unit circle and right triangle trigonometry related?

Using this diagram, $\overleftrightarrow{OD}\ ll\ \overleftrightarrow{AC}$ so therefore $< COA \cong < CDO \cong < ACB \cong \theta$



$\sin\theta = \frac{AC}{OC} = AC$ $\cos\theta = \frac{OA}{OC} = OA$

$\csc\theta = \frac{OD}{OC} = OD$ $\sec\theta = \frac{OB}{OC} = OB$

$\tan\theta = \frac{BC}{OC} = BC$ $\cot\theta = \frac{CD}{OC} = CD$

sin

What are the domain and range of $\sin\theta$ and $\cos\theta$? Why?

$$D\colon\ (-\infty, +\infty)\ \ and\ R\colon [-1, 1]$$

over

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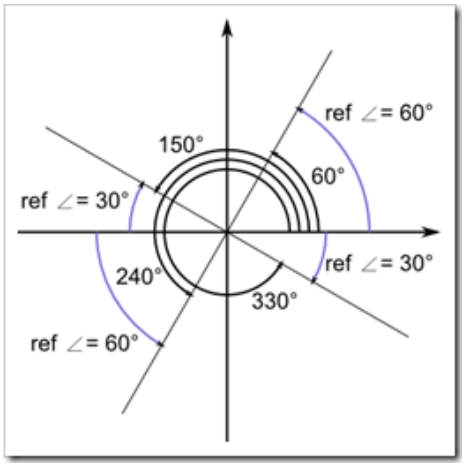
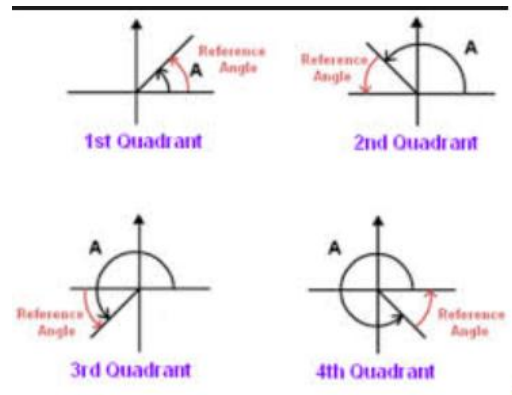
The Reference Angle

Let θ be an angle in standard position. Its reference angle is the acute angle between the terminal side of θ and the *x-axis*.

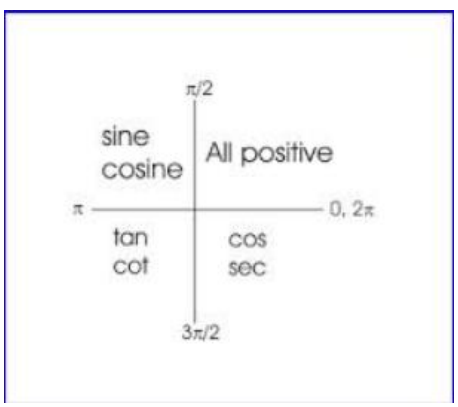




Example:

(a) Find the reference angle for $t = \frac{7}{4}\pi$.

$$\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$$



Examples: Note: $t = \theta$.

(a) Evaluate the six trigonometric functions at $t = \frac{3}{4}\pi$.

This angle is in the II quadrant

$$\sin t = \sin\frac{3\pi}{4} = \sin\left(\pi - \frac{3\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos t = \cos\frac{3\pi}{4} = \cos\left(\pi - \frac{3\pi}{4}\right) = -\cos\frac{\pi}{4} = \frac{-\sqrt{2}}{2}$$

$$\tan t = \tan\frac{3\pi}{4} = \tan\left(\pi - \frac{3\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$$

over

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$$\csc t = \csc\frac{3\pi}{4} = \csc\left(\pi - \frac{3\pi}{4}\right) = \csc\frac{\pi}{4} = \sqrt{2}$$
$$\sec t = \sec\frac{3\pi}{4} = \sec\left(\pi - \frac{3\pi}{4}\right) = -\sec\frac{\pi}{4} = -\sqrt{2}$$

$$\cot t = \cot\frac{3\pi}{4} = \cot\left(\pi - \frac{3\pi}{4}\right) = -\cot\frac{\pi}{4} = -1$$

(a) Evaluate the six trigonometric functions at $t = -\frac{5}{6}\pi$.

This angle would be in the III quadrant

$$\sin t = \sin\frac{-5\pi}{6} = \sin\left(\pi - \frac{5\pi}{6}\right) = -\sin\frac{\pi}{6} = \frac{-1}{2}$$

$$\cos t = \cos\frac{-5\pi}{6} = \cos\left(\pi - \frac{5\pi}{6}\right) = -\cos\frac{\pi}{6} = \frac{-\sqrt{3}}{2}$$

$$\tan t = \tan\frac{-5\pi}{6} = \tan\left(\pi - \frac{5\pi}{6}\right) = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\csc t = \csc\frac{-5\pi}{6} = \csc\left(\pi - \frac{5\pi}{6}\right) = -\csc\frac{\pi}{6} = -2$$
$$\sec t = \sec\frac{-5\pi}{6} = \sec\left(\pi - \frac{5\pi}{6}\right) = -\sec\frac{\pi}{6} = \frac{-2\sqrt{3}}{3}$$

$$\cot t = \cot\frac{-5\pi}{6} = \cot\left(\pi - \frac{-5\pi}{6}\right) = \cot\frac{\pi}{6} = \sqrt{3}$$

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Definition of a Periodic Function

A function *f* is periodic if there exists a positive real number *c* such that $f(t+c) = f(t)$ for all *t* in the domain of *f*. The smallest number of *c* for which *f* is periodic is called the period of *f*.

Even and Odd Trigonometric Functions

What does it mean for a function to be even?

For a function to be even, it reflects over the y-axis. Algebraically if $f(x) = f(-x)\ is\ true$, then a function is even.

What does it mean for a function to be odd?

For a function to be odd, it rotates about the origin. Algebraically if $f(-x) = -f(x)\ \ is\ true$, then a function is odd.

Determine whether each of the six trigonometric functions are even, odd, or neither.

$f(x) = \sin(-x) = -\sin(x)\quad this\ means\ \ f(-x) = -f(x)\quad therefore\ \sin x\ \ is\ \ odd$

$f(x) = \cos(-x) = \cos(x)\quad this\ means\ \ f(-x) = f(x)\quad therefore\ \cos x\ \ is\ \ even$

$f(x) = \tan(-x) = -\tan(x)\quad this\ means\ \ f(-x) = -f(x)\quad therefore\ \tan x\ \ is\ \ odd$

$f(x) = \csc(-x) = -\csc(x)\quad this\ means\ \ f(-x) = -f(x)\quad therefore\ \csc x\ \ is\ \ odd$

$f(x) = \sec(-x) = \sec(x)\quad this\ means\ \ f(-x) = f(x)\quad therefore\ \sec x\ \ is\ \ even$

$f(x) = \cot(-x) = -\cot(x)\quad this\ means\ \ f(-x) = -f(x)\quad therefore\ \cot x\ \ is\ \ odd$

ODD Trig functions	EVEN Trig functions
Sine	Cosine
Tangent	Secant
Cosecant	
Cotangent	

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Evaluating Trigonometric Functions with a Calculator

Read *Class Sheet Trig Functions w/ Ti – 83*, then complete the following problems.

Examples:

(a) Evaluate in radians: $\cot 28$

$$\cot 28 = \frac{1}{\tan 28} = -3.553286448$$

(b) Evaluate in radians: $\csc 0.98$

$$\csc 0.98 = \frac{1}{0.98} = 1.204097732$$

(c) Evaluate in radians: $\sec\left(-\frac{\pi}{3}\right) =$

$$\sec\left(-\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{-\pi}{3}\right)} = \frac{1}{cos\left(\frac{\pi}{3}\right)} = \frac{1}{\frac{1}{2}} = 2$$

Just a thought:

$$\cot 90° = \frac{1}{\tan 90°} = \frac{1}{undefined}$$

But this is not true!

$$\tan 90° = \frac{1}{0} = undefined \;\; therefore$$

$$\cot 90° = \frac{0}{1} = 0$$

BE CAREFUL!!

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