

Construction of Control Charts for Some Circular Distributions

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Abstract

Quality is one of the most essential characteristics of any product. Monitoring and controlling quality is a continuous process. Statistical Process Control (SPC) is a quality control technique using statistical methods for monitoring and controlling the quality. Control charts are one of the most important tools of SPC. Laha and Gupta (2011) applied the concept of control charts to circular distributions such as Von mises, Wrapped Cauchy, Wrapped Normal and Cardioid distributions. Taking a cue from this work, an attempt has been made in this paper to extend this concept of control charts to two circular distribution developed by Srinivasa Subrahmanyam et al (2017), namely the Wrapped Exponential Inverted Weibull distribution (WEIWD) and Wrapped New Weibull Pareto Distribution(WNWPD). For these two distributions, the control charts at different sets of values of parameters are constructed, the theoretical values for Central Ray (CR), Anticlockwise Control Ray (ACR) and Clockwise Control Ray (CCR) angles are obtained, the acceptance region and rejection region at the theoretical values are identified and the expansion and contraction of the acceptance region with respect to change in parameter values are studied. The estimates for CR, ACR and CCR angles and their respective circular variances are computed and the impact of simulation size and sample size on the ACR and CCR angles and their circular variances are studied. Also, an effort has been made to figure out a reasonable simulation size as well as sample size for finding estimate values for CR, ACR and CCR angles in respect of these two circular distributions.

Keywords: Circular Statistics, Wrapping, Exponentiated Inverted Weibull, New Weibull Pareto, Quality control, Control Charts

I. INTRODUCTION

Quality is one of the most essential characteristics of any product. Monitoring and controlling quality is a continuous process. Statistical Process Control (SPC) is a quality control technique using statistical methods for monitoring and controlling the quality. Control charts are one of the most important tools of SPC. The control charts are constructed using the sample data on quality characteristic of a product, obtained from the production process. A control chart consists of a Centre Line (CL) representing the average of the quality characteristic. Another two lines horizontal to the CL, the Upper Control limit (UCL) above the CL and the other Lower Control Limit (LCL) below the CL are drawn on the control chart. These control lines are constructed in such a way that if a process is under control, then most of the observations in the sample fall within the region covered between UCL and LCL. If any point in the sample falls outside this region then it is an indication that the process is out of control. An example of a control chart in the linear case is shown hereunder.

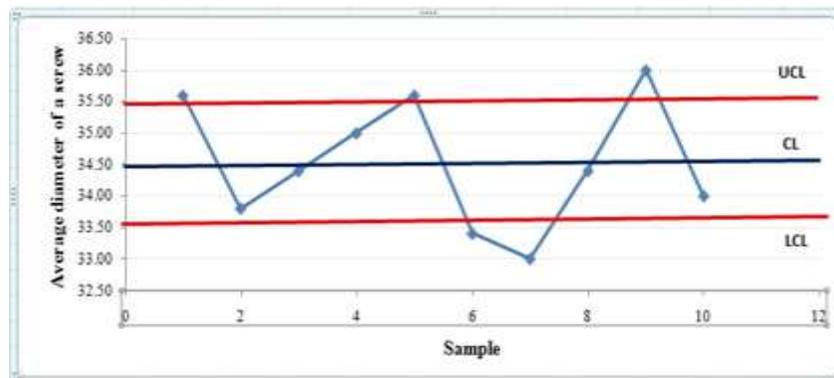


Fig. 1.1: Linear Control Chart

II. CONTROL CHARTS TO THE CIRCULAR DISTRIBUTIONS

Laha and Gupta (2011) applied the concept of control charts to circular distributions such as Von mises, Wrapped Cauchy, Wrapped Normal and Cardioid distributions and constructed Central Ray (CR), Anticlockwise Control Ray (ACR) and Clockwise Control

Ray (CCR) to the above mentioned circular distributions and studied the impact of simulation size on the circular variances of ACR and CCR angles of these distributions, the effect of variation of parameters of circular distributions on the variances of ACR and CCR angles and also studied the Average Run Length (ARL) and Median of Run Length (MRL) of circular control charts. The control chart in respect of Circular distributions as illustrated by Laha and Gupta (2011) is furnished below.

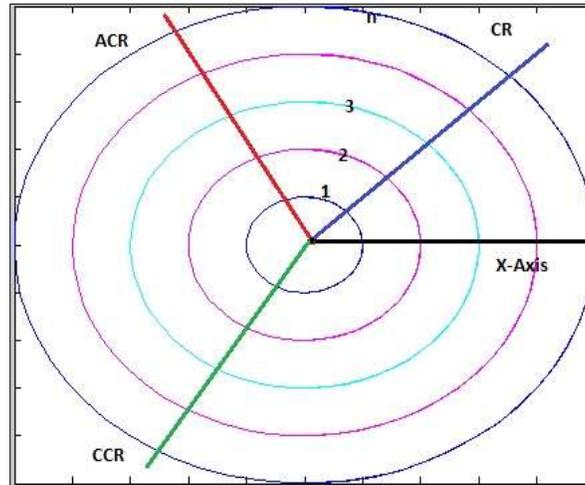


Fig. 2.1: Circular Control Chart for Mean Direction

The above figure represents the collection of concentric circles $1, 2, \dots, n$, with n^{th} circle having radius of n units. Central Ray represents the mean angle. The region bounded by CCR- CR-ACR in the anticlockwise direction is the acceptance region and the region bounded by ACR-CCR in the anticlockwise direction is the rejection region for the process for which the controls charts are constructed.

Taking a cue from this work, an attempt has been made in this paper to extend this concept of control charts to two circular distributions developed by Srinivasa Subrahmanyam et al (2017), namely the Wrapped Exponential Inverted Weibull distribution (WEIWD) and Wrapped New Weibull Pareto Distribution(WNWPD).

In this Paper, for these two selected circular distributions, the control charts at different sets of values of parameters are constructed, the theoretical values for CR, ACR and CCR angles are obtained, the acceptance region and rejection region at the theoretical values are identified and the expansion and contraction of the acceptance region with respect to change in parameter values are studied. The estimates for CR, ACR and CCR angles and their respective circular variances are computed, the impact of simulation size and sample size on the ACR and CCR angles and their circular variances are studied. Also, an effort has been made to figure out a reasonable simulation size as well as sample size for finding the estimates for CR, ACR and CCR angles in respect of these two circular distributions. Here the reasonable simulation size or sample size is considered as the size at which the corresponding estimated angles of CR, ACR and CCR are close to their respective population values with minimum variance. Also, in the control charts constructed for the selected distributions, all angles are shown in degrees and for the graphical representation of CR, ACR and CCR angles, east as zero direction and anticlockwise as sense of rotation was considered.

A. Circular Distributions:

A circular random variable in a continuous circular distribution $g : [0, 2\pi) \rightarrow \mathbb{R}$ is said to be following a circular probability

density function of $g(\theta)$ if and only if g has the following basic properties

$$1) \quad g(\theta) \geq 0, \quad \forall \theta \quad \dots (1)$$

$$2) \quad \int_0^{2\pi} g(\theta) d\theta = 1 \quad \dots (2)$$

$$3) \quad g(\theta) = g(\theta + 2k\pi) \text{ } g \text{ is periodic, for any integer } k \text{ (Mardia,2000)} \quad \dots (3)$$

B. Circular Mean and Variance:

The circular mean and circular variance for a given set of angles can be computed as below.

1) Circular Mean:

Let p_i be a point on the circumference of the unit circle corresponding to the angle θ_i , $i = 1(1)n$ then the circular mean or the mean direction θ_0 of $\theta_1, \theta_2, \dots, \theta_n$ is defined to be the direction of the resultant of the unit vectors $\overline{OP_1}, \overline{OP_2}, \dots, \overline{OP_n}$. The Cartesian coordinates of P_i are $(\cos \theta_i, \sin \theta_i)$ so that the centre of gravity of these points is (C, S) where

$$C = \frac{1}{n} \sum \cos \theta_i \text{ and } S = \frac{1}{n} \sum \sin \theta_i$$

The value $R = \sqrt{C^2 + S^2}$ gives the length of the resultant vector. The direction θ_0 of the resultant vector is the mean direction. The direction θ_0 is given by

$$\theta_0 = \begin{cases} \tan^{-1}(S / C) & \text{if } S > 0, C > 0 \\ \tan^{-1}(S / C) + \pi & \text{if } C < 0 \\ \tan^{-1}(S / C) + 2\pi & \text{if } S < 0, C > 0 \end{cases}$$

2) Circular Variance:

A useful measure of the distance between two angles θ_1 and θ_2 is $1 - \cos(\theta_1 - \theta_2)$. Hence the dispersion of angles $\theta_1, \dots, \theta_n$ about its mean direction $\bar{\theta}$, i.e. circular variance V , is given by

$$V = D(\bar{\theta}) = \frac{1}{n} \sum_{i=1}^n [1 - \cos(\theta_i - \bar{\theta})]$$

III. METHODOLOGY

To carry out the intended study, random samples have been generated through simulation from the selected distributions. As the Cumulative Distribution Functions of Wrapped Exponential Inverted Weibull distribution and Wrapped New Weibull Pareto distribution are not in the closed form, the simulation was implemented using the Acceptance and Rejection method.

A. Finding CR, ACR AND CCR Angles for Different Simulation Sizes:

The Step wise procedure for finding CR, ACR and CCR for different Simulation sizes is detailed below.

- 1) For a selected simulation size say, N and for a fixed sample size k , random samples of k angles each between 0 to 2π degrees are generated.
- 2) The circular mean μ_i of the i^{th} sub sample consisting of k observations is computed.
- 3) Since N is the simulation size, steps 1 and 2 are repeated for N times. Running N simulations results in an array of circular means $\mu_1, \mu_2, \dots, \mu_N$.
- 4) Arrange this array of circular means in ascending order. The circular mean of all N circular means is computed and is the Central Ray (CR).
- 5) Taking the level of significance as 5% and by eliminating 0.025% circular means from both the ends of the sorted array of circular means, μ_u at the end of the array and μ_L at the beginning of the array can be obtained. Drawing a line at μ_u gives the Anticlockwise Control Ray (ACR) and drawing a line at μ_L gives the the Clockwise Control Ray(CCR).

B. Finding Estimates and Variances of CR, ACR AND CCR Angles for a Given Simulation Size:

After computing the CR, ACR and CCR angles for a selected simulation size, for finding estimates and circular variances for CR, ACR and CCR angles, the procedure detailed in section A above will be iterated for fifty times. After the completion of fifty iterations, arrays of CR, ACR and CCR angles, each consisting of fifty angles will be obtained, for a given simulation size. For obtaining an estimated value of ACR angle for a given simulation size, the circular mean of the array of ACR angles is computed. Similarly, circular mean of the array of CCR angles gives an estimated value of CCR angle and mean of the array of CR angles gives an estimated value of CR angle for a given simulation size. The circular variance of the arrays of CR ACR and CCR angles can also be computed. Changing the simulation size N from 250, 500, 1000, 2000 and up to 10,000, the corresponding estimated values for CR, ACR and CCR angles and their circular variances are computed. Applying this procedure on both the selected distributions WEIWD and WNWD at different sets of values to the parameters the estimates and variances of CR, ACR and CCR angles are obtained for each simulation size.

C. Finding CR, ACR AND CCR Angles for Different Sample Sizes:

For finding CR, ACR and CCR angles for a given sample size, the simulation size will be fixed at 1,000 and the sample size is varied from 5, 10 and up to 100 in steps of 10 in the procedure detailed in subsection A above in this section.

D. Finding Estimates and Variances of CR, ACRS AND CCR Angles for A Given Sample Size:

After computing the CR, ACR and CCR angles for a selected sample size as detailed in section C above, for finding estimates and circular variances of CR, ACR and CCR angles, this procedure is iterated for fifty times. After completion of fifty iterations, arrays of CR, ACR and CCR angles, each consisting of fifty angles for a given sample size are obtained. For obtaining an estimated value of ACR angle for a given sample size, the circular mean of the array of ACR angles is computed. Similarly, circular mean of the array of CCR angles gives an estimated value of CCR angle and mean of the array of CR angles gives an estimated value of CR angle for a given sample size. The circular variance of the arrays of CR ACR and CCR angles can also be computed. Changing the sample size k from 5, 10 and up to 100 in steps of 10, the corresponding estimated values for CR, ACR and CCR angles and their circular variances are computed. Applying this procedure on both the selected distributions WEIWD and WNWPD at different sets of values to the parameters the estimates and variances of CR, ACR and CCR angles are obtained for each sample size.

E. Finding Theoretical Values of CR, ACR AND CCR Angles:

The Population (or) theoretical value of the CR, ACR and CCR angles for the selected two distributions are computed by taking sufficiently large value for N , the simulation size. Theoretical values were computed for CR, ACR and CCR angles by considering simulation size as one million.

IV. CONSTRUCTION OF CONTROL CHARTS FOR WEIW DISTRIBUTION

The probability density function of Wrapped Exponentiated Inverted Weibull Distribution is

$$g(\theta) = \sum_{k=0}^{\infty} \lambda c(\theta + 2\pi k)^{-(c+1)} \left(e^{-(\theta + 2\pi k)^{-c}} \right)^{\lambda}, \text{ where } \theta \in (0, 2\pi), c > 0 \text{ and } \lambda > 0.$$

Five cases, each with different set of values to the parameters c and λ as detailed below are considered for the study.

Table - 1
Cases considered for WEIWD

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5
C	2	2	3	3	4
λ	2	3	2	3	3

A. Theoretical Values of CR, ACR and CCR Angles for Different Cases:

Using the procedure explained in subsection E of Section III, the theoretical values of parameters for WEIWD for the cases mentioned above are computed and tabulated. The tables along with corresponding graphs for case 1 is presented below.

Case 1: $C = 2$ and $\lambda = 2$

Table - 2
Theoretical Values for Case 1

C	λ	CR	ACR	CCR
2	2	96.86	169.92	50.26

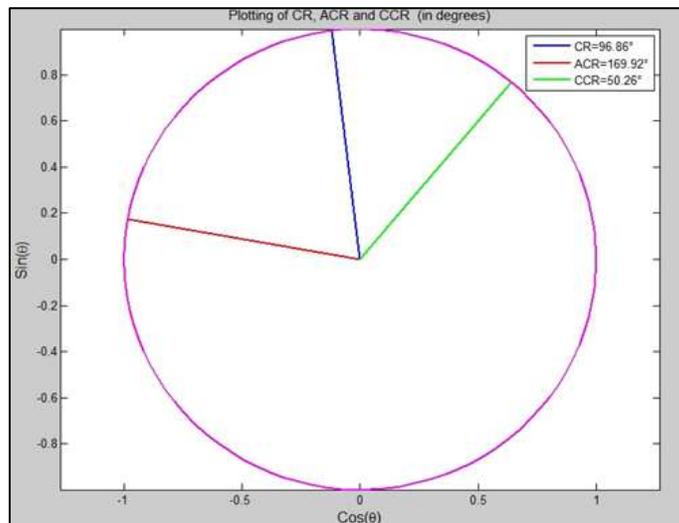


Fig. 3: Graph of CR, ACR and CCR for Case 1

The acceptance region in the figure 3 is the region bounded by $CCR - CR - ACR$ i.e. from 50.26^0 to 169.92^0 in the anticlockwise direction taking east as zero direction. The Theoretical Values obtained for the rest of the cases are tabulated hereunder.

Table - 3
Theoretical values Obtained for other Cases

Case	C	λ	CR	ACR	CCR
2	2	3	113.32	205.44	56.29
3	3	2	87.65	126.91	60.53
4	3	3	98.88	142.93	68.17
5	4	3	88.61	118.57	67.83

B. Analysis of Theoretical Values:

Theoretical values for CR, ACR and CCR angles for the Wrapped Exponentiated Inverted Weibull distribution at different values for parameters c and λ are computed to study the expansion and contraction of the acceptance region in response to the changes in the value of parameters. The results are furnished below.

Table - 4
Theoretical Values of CR, ACR and CCR of WEIWD

Sl. No	C	λ	CR	ACR	CCR	ACR - CCR	Change
(1)	(2)	(3)	(4)	(5)	(6)	(7) = (5)-(6)	(8)
1	2	2	96.86	169.92	50.26	119.66	
2	2	3	113.32	205.44	56.29	149.15	29.49
3	3	3	98.88	142.93	68.17	74.76	-74.39
4	3	4	107.58	155.55	74.02	81.52	6.77
5	4	4	94.85	126.69	72.69	54.00	-27.53
6	4	5	99.93	133.16	76.56	56.60	2.60

In the table 4, the column (7) indicates the size of the acceptance region whereas column (8) indicates the change occurred in the size of the acceptance region, when the value of one of the two parameters is increased by a one unit. A positive value in column (8) indicates the expansion of acceptance region and a negative value indicates the contraction of acceptance region. It can be observed that whenever the value for the parameter c is increasing, the acceptance region is decreasing. On the other hand, with the increase in value for the parameter λ the acceptance region is expanding. But the magnitude of change happening in both directions gradually decreasing with increasing values for the parameters c and λ .

C. Finding CR, ACR AND CCR Angles and Variances for Different Simulation Sizes:

For WEIW Distribution, the CR, ACR, and CCR angles and their respective variance for Case 1 at parameter values $c = 2$ and $\lambda = 2$, for different simulation sizes ranging from 250 to 10,000 are obtained as tabulated below.

Table - 5
CR, ACR and CCR Angles with Circular Variance

Simulation Size	CR	CVCR ¹	ACR	CVACR ²	CCR	CVCCR ³
250	96.99	0.02	173.51	1.55	51.08	0.17
500	96.96	0.01	170.15	0.65	51.69	0.10
1000	96.80	0.01	168.94	0.28	50.52	0.05
2000	96.98	0.00	170.03	0.20	50.47	0.03
3000	96.90	0.00	170.02	0.08	50.31	0.02
4000	96.79	0.00	169.82	0.10	50.37	0.01
5000	96.83	0.00	169.98	0.08	50.22	0.01
6000	96.86	0.00	169.31	0.04	50.20	0.01
7000	96.82	0.00	169.52	0.04	50.01	0.01
8000	96.92	0.00	169.76	0.03	50.52	0.01
9000	96.89	0.00	170.18	0.04	50.31	0.01
10000	96.88	0.00	169.97	0.04	50.19	0.01

1. Circular Variance of Central Ray, 2. Circular Variance of Anticlockwise Control Ray,
3. Circular Variance of Clockwise Control Ray

The graph of circular variances for ACR angles for different simulation sizes is obtained as below.

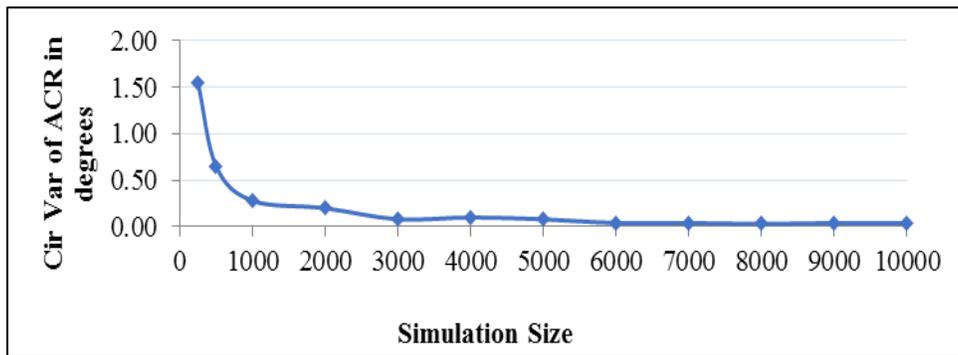


Fig. 4: Circular Variance of ACR Angles

The graph of circular variances for CCR angles for different simulation sizes is obtained as below.

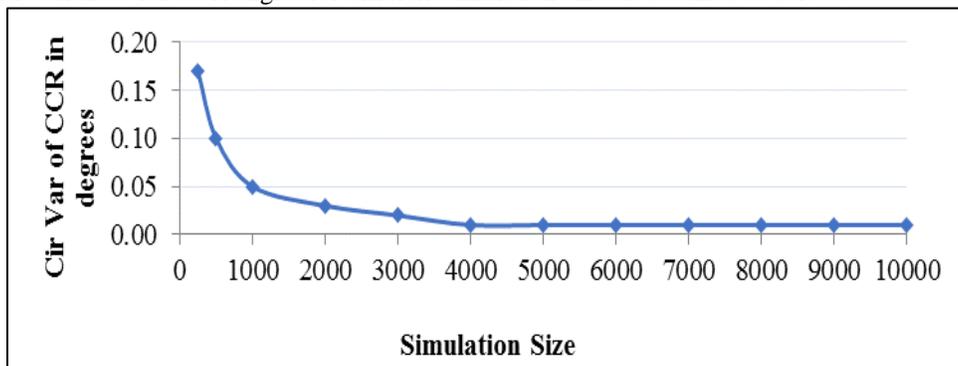


Fig. 5: Circular Variance of CCR Angles

Comparing with the theoretical values of CR, ACR and CCR angles for WEIWD, at $c = 2$ and $\lambda = 2$ tabulated at table 2, with values obtained for CR, ACR and CCR angles for different simulation sizes at table 5, It can be observed that with the increasing simulation size, the CR, ACR and CCR angles are close to their respective theoretical values. Also, from figures 4 and 5, the circular variances of ACR and CCR are moderate when the simulation size is around 250 to 500 but with the increasing simulation size, especially after 1000, a sharp reduction in the circular variances can be noticed finally reaching to zero for higher simulation sizes. Also, the results for Cases 2, 3, 4 and 5 are obtained and all of these outcomes confirm the tendencies of CR, ACR and CCR angles observed in the first case.

D. Finding CR, ACR AND CCR Angles and Variances for Different Sample Sizes:

For WEIW Distribution, the CR, ACR, and CCR angles and their respective variance for Case 1 at parameter values $c = 2$ and $\lambda = 2$, for different sample sizes ranging from 5 to 100 are computed and are tabulated below.

Table – 6
CR, ACR and CCR Angles with Circular Variances

Sample Size	CR	CVCR ¹	ACR	CVACR ²	CCR	CVCCR ³
5	96.70	0.01	170.46	0.31	50.81	0.06
10	96.34	0.00	138.72	0.07	64.04	0.01
20	95.75	0.00	122.81	0.02	73.25	0.01
30	95.56	0.00	116.98	0.01	77.17	0.00
40	95.41	0.00	113.71	0.00	79.24	0.00
50	95.31	0.00	111.54	0.00	80.98	0.00
60	95.36	0.00	109.99	0.00	82.29	0.00
70	95.33	0.00	108.91	0.00	83.15	0.00
80	95.28	0.00	107.87	0.00	83.75	0.00
90	95.25	0.00	106.96	0.00	84.50	0.00
100	95.30	0.00	106.45	0.00	85.11	0.00

1. Circular Variance of Central Ray, 2. Circular Variance of Anticlockwise Control Ray,
3. Circular Variance of Clockwise Control Ray

The graph of CR, ACR and CCR angle for different sample sizes is furnished below.

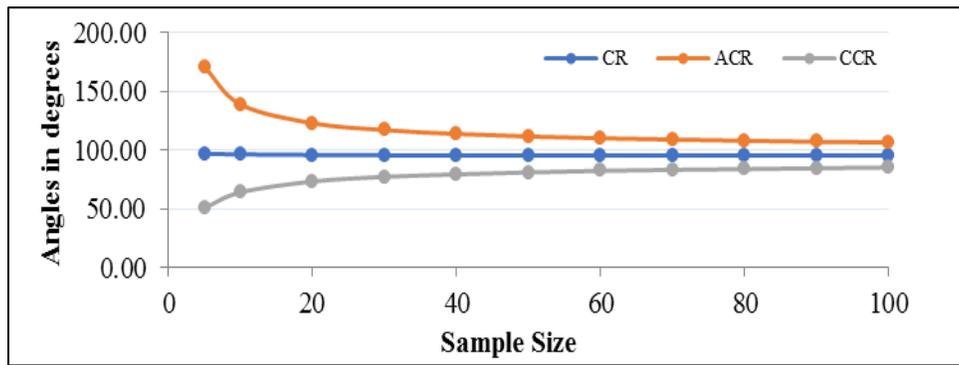


Fig. 6: CR, ACR and CCR angles of WEIWD for different Sample Sizes

The graph of circular variances for ACR angles for different sample sizes is obtained as below.

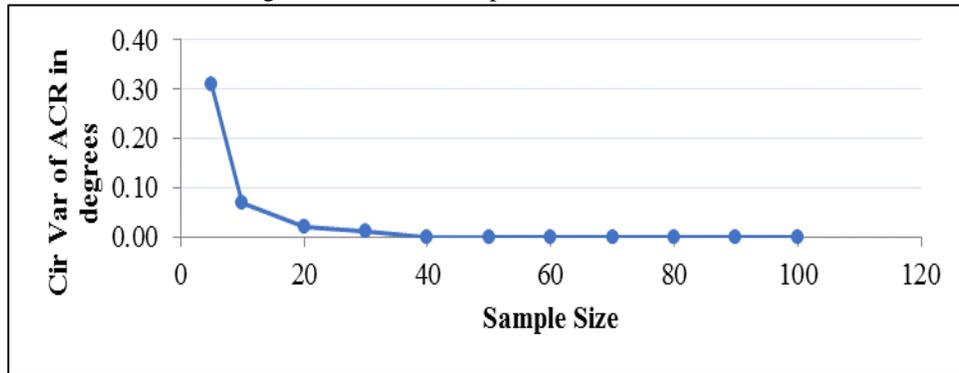


Fig. 7: Circular Variance of ACR Angles

The graph of circular variances for CCR angles for different sample sizes is obtained as below.

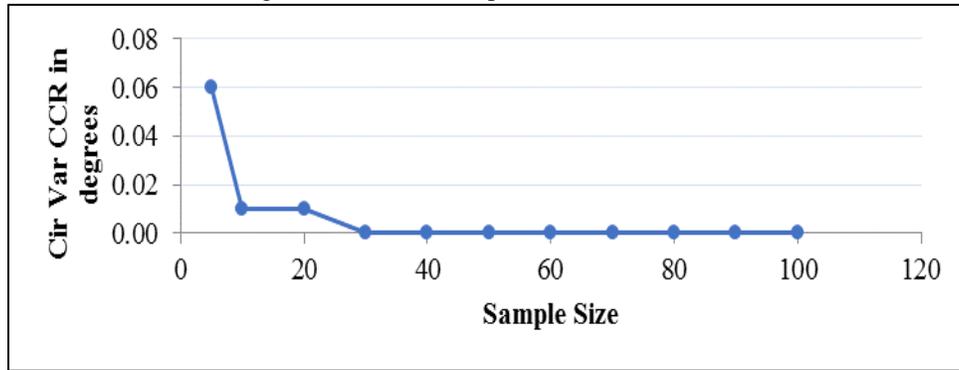


Fig. 8: Circular Variance of CCR Angles

From the figure 6, it is clear that with increasing sample size the ACR and CCR angles are coming close to CR angle. Also, from figures 7 and 8 the circular variances of ACR and CCR are reaching zero with increasing sample size after a sharp reduction in circular variance at sample size 10. Also, the results for Cases 2, 3, 4 and 5 are also obtained and all of these outcomes confirm the tendencies of CR, ACR and CCR angles observed in the first case.

V. CONSTRUCTION OF CONTROL CHARTS FOR WNWP DISTRIBUTION

The probability density function of Wrapped New Weibull Pareto Distribution (WNWPD) is

$$g(\theta) = \sum_{k=0}^{\infty} \frac{c \delta}{\lambda} \left(\frac{\theta + 2k\pi}{\lambda} \right)^{c-1} e^{-\delta \left(\frac{\theta + 2k\pi}{\lambda} \right)^c}, \text{ where } \theta \in (0, 2\pi), c > 0, \delta > 0 \text{ and } \lambda > 0.$$

Five cases, each with different set of values to the parameters c , δ and λ are considered for the study.

Table – 7
Cases considered for WNWPD

Parameters	Case 1	Case 2	Case 3	Case 4	Case 5
C	2	2	3	3	3
δ	2	3	2	3	3
λ	2	2	2	2	3

A. Theoretical Values of CR, ACR AND CCR Angles for Different Cases:

Using the procedure explained in subsection E of Section III, the theoretical values of parameters for WNWPD for the cases mentioned above are computed and tabulated. The tables along with corresponding graphs for case 1 is presented below.

Case 1: $C = 2$, $\delta = 2$ and $\lambda = 2$

Table – 8
Theoretical Values for Case 1

C	δ	λ	CR	ACR	CCR
2	2	2	70.68	106.42	40.49

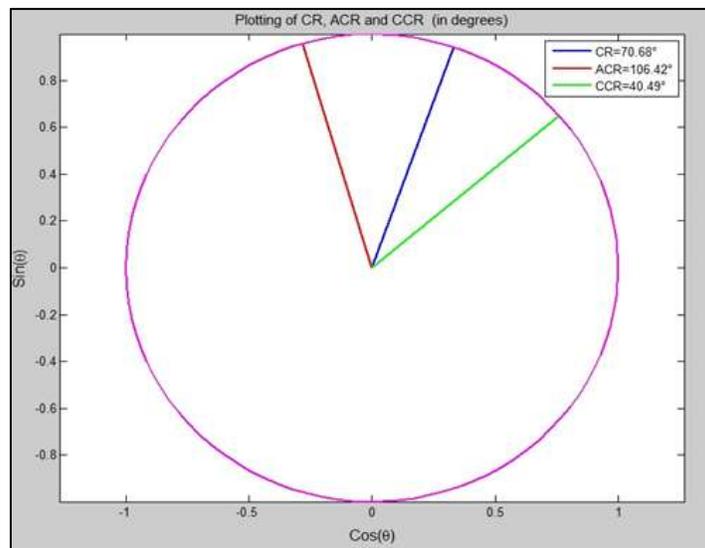


Fig. 9: Graph of CR, ACR and CCR for Case 1

From the figure 9, the acceptance region is from 40.49° to 106.42° in the anticlockwise direction and east as zero direction. The region excluding this in the circle is the rejection region. The Theoretical Values obtained for the rest of the cases are tabulated hereunder.

Table – 9
Theoretical Values obtained for other cases

Case	C	δ	λ	CR	ACR	CCR
2	2	3	2	58.09	86.85	33.48
3	3	2	2	81.08	107.60	55.57
4	3	3	2	70.86	93.89	48.59
5	3	3	3	106.11	141.40	72.18

B. Analysis of Theoretical Values:

Theoretical values for CR, ACR and CCR angles for the Wrapped New Weibull Pareto distribution at different values of parameters c , δ and λ are computed to study the expansion and contraction of the acceptance region in response to the changes in the value of parameters. The results are furnished below.

Table – 10
Theoretical Values of CR, ACR and CCR of WNWPD

Sl. No	C	δ	λ	CR	ACR	CCR	ACR - CCR	Change
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8) = (6) - (7)	(9)
1	2	2	2	70.68	106.42	40.49	65.92	
2	2	3	2	58.09	86.85	33.48	53.37	-12.55

3	3	3	2	70.86	93.89	48.59	45.30	-8.07
4	3	3	3	106.11	141.40	72.18	69.22	23.92
5	3	4	3	96.45	128.31	65.89	62.41	-6.81
6	4	4	3	110.27	137.25	82.58	54.67	-7.74
7	4	4	4	147.09	183.66	109.49	74.17	19.50
8	4	5	4	139.08	173.51	103.60	69.91	-4.26

From the table 10 above, it can be observed that whenever the value for the parameter c and δ are increasing the acceptance region is shrinking. Also, relatively more contraction of acceptance region is happening when the parameter c is increased than δ . On the other hand, whenever the value for the parameter λ is increased the acceptance region is expanding.

C. Finding CR, ACR AND CCR Angles and Variances for Different Simulation Sizes:

Case 1: WNWPD at $C = 2$, $\delta = 2$ and $\lambda = 2$

For WNWP Distribution, the CR, ACR, and CCR angles and their respective variance for Case 1, i.e. at parameter values $C = 2$, $\delta = 2$ and $\lambda = 2$, for different simulation sizes ranging from 250 to 10,000 are obtained as tabulated below.

Table – 11
CR, ACR and CCR Angles with Circular Variance for Case 1

Simulation Size	CR	CVCR ¹	ACR	CVACR ²	CCR	CVCCR ³
250	70.81	0.01	106.02	0.08	41.02	0.04
500	70.58	0.01	105.68	0.06	41.01	0.02
1000	70.62	0.00	106.50	0.02	40.60	0.01
2000	70.71	0.00	106.08	0.02	40.81	0.01
3000	70.70	0.00	106.08	0.01	40.62	0.00
4000	70.61	0.00	106.27	0.00	40.52	0.00
5000	70.71	0.00	106.39	0.01	40.55	0.00
6000	70.74	0.00	106.48	0.00	40.61	0.00
7000	70.70	0.00	106.51	0.00	40.48	0.00
8000	70.66	0.00	106.32	0.00	40.43	0.00
9000	70.67	0.00	106.10	0.00	40.55	0.00
10000	70.68	0.00	106.30	0.00	40.50	0.00

1. Circular Variance of Central Ray, 2. Circular Variance of Anticlockwise Control Ray,
3. Circular Variance of Clockwise Control Ray

The graph of circular variances for ACR angles for different simulation sizes is obtained as below.

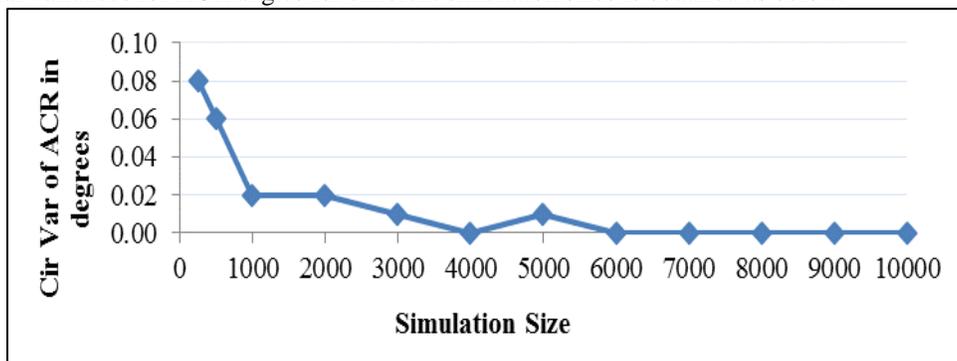


Fig. 10: Circular Variance of ACR Angles for Case 1

The graph of circular variances for CCR angles for different simulation sizes is obtained as below.

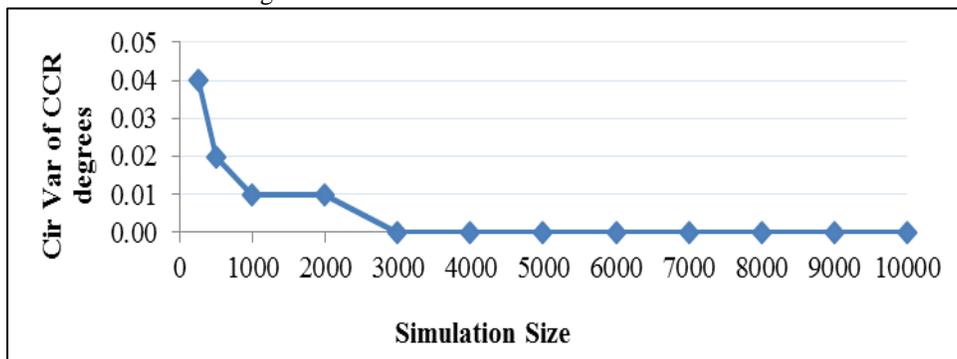


Fig. 11: Circular Variance of CCR Angles for Case 1

Comparing with the theoretical values tabulated at table 8 for CR, ACR and CCR for WNWPD at $C = 2$, $\delta = 2$ and $\lambda = 2$ with values obtained for CR, ACR and CCR for different simulation sizes furnished above at table 11, It can be noticed that with the increasing simulation size, the CR, ACR and CCR angles are approaching to their respective theoretical values. Also, from figures 10 and 11 the circular variances of ACR and CCR are more when the simulation size is around 250 and 500 but with the increasing simulation size, especially after 1000, it is clearly evident that after a sharp reduction the circular variances are attaining zero for both ACR and CCR angles. Also, the results obtained for Cases 2, 3, 4 and 5 confirms the tendencies of CR, ACR and CCR angles, observed in the case 1.

D. Finding CR, ACR and CCR Angles and Variances for Different Sample Sizes:

Case 1: WNWPD at $C = 2$, $\delta = 2$ and $\lambda = 2$

For WNWP Distribution, the CR, ACR, and CCR angles and their respective variance for Case 1, i.e. at parameter values of $C = 2$, $\delta = 2$ and $\lambda = 2$ for different sample sizes ranging from 5 to 100 are computed and are tabulated below.

Table – 12
CR, ACR and CCR Angles with Circular Variance for Case 1

Sample Size	CR	CVCR ¹	ACR	CVACR ²	CCR	CVCCR ³
5	70.65	0.00	106.26	0.02	40.63	0.01
10	70.46	0.00	95.21	0.01	48.74	0.01
20	70.29	0.00	87.45	0.01	54.44	0.00
30	70.19	0.00	84.17	0.00	57.19	0.00
40	70.18	0.00	82.15	0.00	58.86	0.00
50	70.14	0.00	80.78	0.00	60.12	0.00
60	70.16	0.00	79.98	0.00	60.87	0.00
70	70.16	0.00	79.15	0.00	61.51	0.00
80	70.15	0.00	78.58	0.00	62.10	0.00
90	70.15	0.00	78.09	0.00	62.53	0.00
100	70.16	0.00	77.65	0.00	62.88	0.00

1. Circular Variance of Central Ray, 2. Circular Variance of Anticlockwise Control Ray,
3. Circular Variance of Clockwise Control Ray

The graph of CR, ACR and CCR angle for different sample sizes is furnished below.

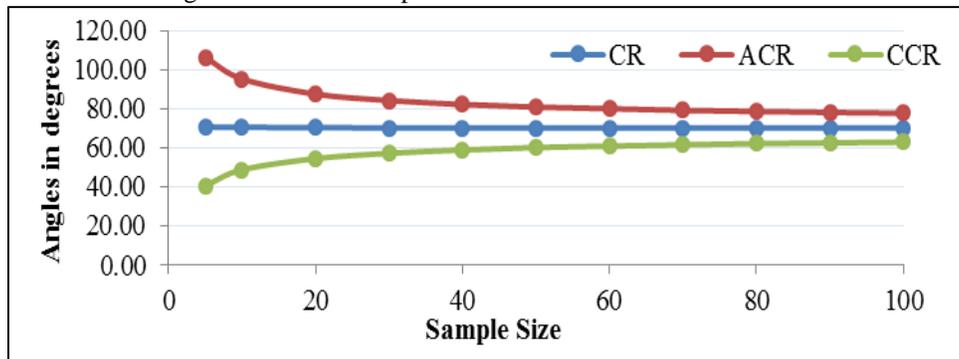


Fig. 12: CR, ACR and CCR angles for Case 1

The graph of circular variances for ACR angles for different sample sizes is obtained as below.

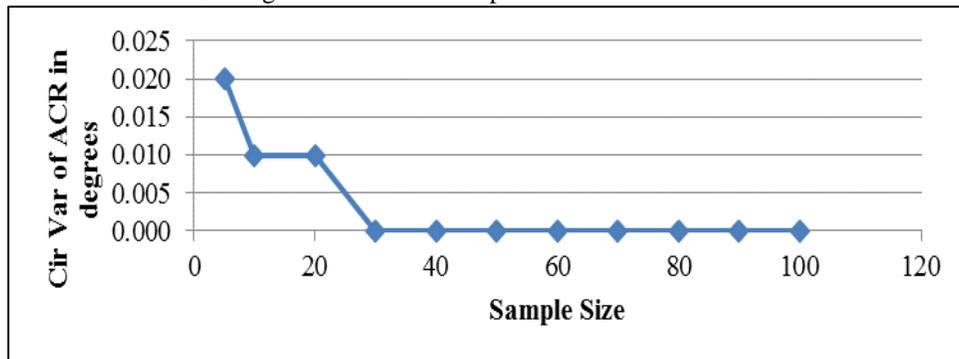


Fig. 13: Circular Variance of ACR Angles for Case 1

The graph of circular variances for CCR angles for different sample sizes is obtained as below.

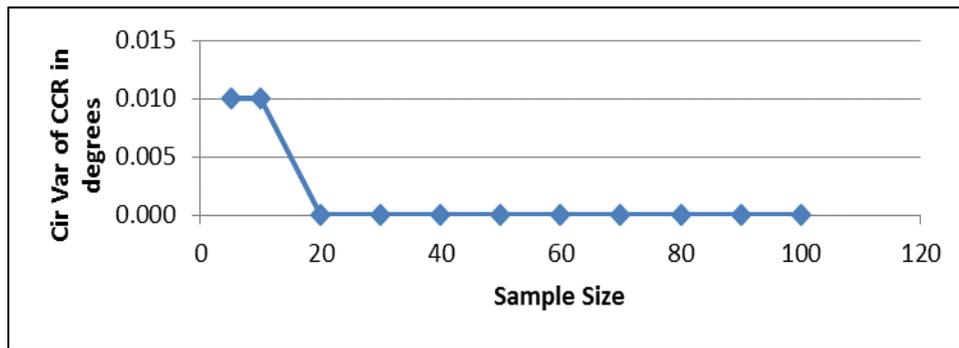


Fig. 14: Circular Variance of CCR Angles for Case 1

From the figure 12, it is clear that with increasing sample size the ACR and CCR angles are coming close to CR angle. Also, from figures 13 and 14 the circular variances of ACR and CCR are more when the sample size is around 5 and 10 but with the increasing sample size, especially after 10, it can be noticed that after a sudden fall the circular variances reaching zero for both ACR and CCR angles. The results obtained for Cases 2, 3, 4 and 5 also confirms the tendencies of CR, ACR and CCR angles, observed in the Case 1.

VI. CONCLUSIONS

A. Effects of Parameters on Acceptance Region:

For both the circular distribution, the acceptance region in the control charts is shrinking with the increasing value of the shape parameter c . On the other hand with the increasing value of λ the acceptance region is expanding. In the Wrapped New Weibull Pareto distribution, when δ is increasing, the acceptance region is decreasing.

B. Effects of Simulation Size on CR, ACR and CCR angles:

In both the circular distributions, it can be noticed that with the increasing simulation size, the CR, ACR and CCR angles are close to their respective theoretical values and their respective circular variances are approaching to zero. Also, it is noticed that the simulation size 1000 would be a reasonable simulation size for both the circular distributions.

C. Effects of Sample size on CR, ACR and CCR angles:

In both the circular distributions, it can be noticed that with the increasing sample size, the ACR and CCR angles are coming close to their respective CR angles and also their respective circular variances are approaching zero. Also, it is noticed that the sample size 20 would be a reasonable sample size for both the circular distributions.

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