Newsvendor Games with Budget Constraint in A Multi-hospital System

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***Abstract*—In this paper, we study a class of cooperative games among multiple hospitals. The optimization problems with the newsvendor games are formulated as deterministic and stochastic programs respectively. We analyze the cooperative allocation decision using the notion of a core. Start with the elegant result of a deterministic linear programming case where we assume the demand is definite larger than the supply, we prove the nonemptiness of the core and use the duality theory to find an imputation in the core. We further investigate it as a stochastic linear programming problem. The proposed method with strong duality theory is applied to show the nonemptiness of the core and suggests a way to find an element in the core.**

***Index Terms*—healthcare; newsvendor games;stochastic; linear programming; duality theory.**

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1. Introduction

Hospitals and care systems face unparalleled pressures in many countries and regions in the current environment. Growing populations, harsh environmental conditions and scarce medical resources are straining the budgets. In a populous country like China, public hospitals are always overcrowded and the time assigned for each patient is far from enough. On the contrary, private care organizations with higher charges need to advertise to improve the utilization of their capacity. Provision of high quality and affordable health care becomes one of the greatest challenges to the society. To solve these problems, health care systems must become more efficient at delivering care and preventing disease. As we know, waiting is the consequence of a mismatch between the needs for service and the availability of resources to provide that service. This mismatch might be the result of having too little of needed resources, or perhaps the needs themselves are excessive. Also it is likely that the resources and needs have not been adequately synchronized with each other.

We believe it is important to explore the resource allocation problem in the long run, especially when information system makes it possible for cooperation between health care sectors. We should strive for a well-coordinated system that delivers care with great efficiency and quality, at reasonable cost, matching the resources for care to where (and when) they are needed most.

The concept of sharing inventory has been studied a lot in centralized news-vendor model. Which can be conjectured that when demands are uncertain, centralization of capacity can help the hospital reduce resource investment and improve patient satisfaction. In our previous work [Luo and Cai(2016b)], we formulate an inter-hospital collaboration problem and derive solutions when capacities are available over multiple hospitals. To operationalize the sharing of capacity in the integrated system, we need to secure the willingness of the various parties to cooperate with each other, which leads to our current research. Specifically, we investigate, based on the concepts and methodologies of cooperative game and an inter-hospital cooperation problem for elective surgery planning. Based on some preliminary exploration [Luo and Cai(2016a)], we will show, this represents a new research effort and many interesting but challenging questions have to be addressed.

In recent years, several collaborative actions have been reported in various industries including pharmaceutical, fashion, automobile, and post-sales support (see [Anupindi and Bassok(1999)] [Chen and Zhang(2009)] for their motivating examples). The main driver of these collaborative activities is the cost benefit due to economies of scale and risk pooling, which can be achieved in several operations including logistics, purchasing, and inventories. Among them, inventories have a special importance because in many supply chains they constitute a big portion of the overall investment and resources [Anupindi et al.(2001)Anupindi, Bassok, and Zemel].

In a two-party situation, the answer would be easy. Any allocation improving the firms' stand-alone profit will be considered as a winwin case. However, if more firms are involved, they want to know that it is not more advantageous

for them to cooperate in a smaller group rather than joining the big group. Existence of such an allocation is crucial for the stability of the cooperation. This problem can be studied using cooperative game theory, which provides, among others, the core as a popular solution concept.

The core is the set of allocations upon which no coalition can improve. Depending on the characteristics of the game, core allocations may exist or may fail to exist, i.e., the core can be empty. Since the lack of related papers in hospital cooperation, we review some of the papers studying inventory cooperation using newsvendor models in combination with the core as solution concept for the allocation problem of anticipated benefits.

The newsvendor model is probably the most celebrated model in inventory literature and it is used extensively to study inventory centralization and cooperation. Newsvendor models are used especially for products with high perishability or short life cycles and they have the ability to capture the basic trade-offs regarding inventory-related decisions.

Inventory cooperation and allocation of benefits has been a subject of inventory literature for a long time. Early papers studied slightly different models and approaches to the problem. [Eppen(1979)] was one of the first to consider the effect of centralization for a multi-location newsboy problem. [Gerchak and Gupta(1991)] proposed different cost allocation rules in centralized inventory systems.

In the last decade, cooperative game theory and the core concept became a main method for studying this problem. This part of the literature is the focus of our review paper. [Hartman et al.(2000)Hartman, Dror, and Shaked] showed that the newsvendor game is balanced, and the core of such a game is nonempty in some interesting cases, such as normally distributed demands.

[Müller et al.(2002)Müller, Scarsini, and Shaked] [Slikker et al.(2001)Slikker, Fransoo, and Wouters] showed that any finite newsvendor game is balanced whenever the random demand vectors has finite mean. [Slikker et al.(2005)Slikker, Fransoo, and Wouters] proved balancedness for games with transshipment costs. [Özen et al.(2008)Özen, Fransoo, Norde, and Slikker] dealt with balancedness for games with several warehouses. [Chen and Zhang(2009)] developed a stochastic programming duality approach to find a core element of these games.

For a review of inventory centralization games of deterministic and stochastic models, see [Fiestras-Janeiro et al.(2011)Fiestras-Janeiro, García-Jurado, Meca, and Mosquera]. For a comprehensive review of game theoretical models of supply chain management, see [Nagarajan and Sošić(2008)].

The paper is organized as follows. In Section 2, we introduce some basic concepts in newsvendor games. In Section 3, the basic game are presented. Section 4 deals with the deterministic situation. Section 5 developed a stochastic programming duality approach and find a core element when the hospital faces random demand.

2. Preliminaries

We first give a brief introduction to cooperative game. We refer to [Peleg and Sudhölter(2007)] for an extended treatment of this subject. In the setting of this paper, the hospitals are planning jointly in order to reach a overall service level and achieve the maximum revenue.

These hospitals are referred as *players*, denoted with a finite set $N = \{1, 2, ..., n\}$. A function $v : 2^N \to \mathbb{R}$, is called *characteristic function*. A subset $S \subseteq N$ is called a *coalition* and $v(S)$ is the the maximum revenue the coalition S can achieve by itself. The set N is often called the *grand coalition*. Given a coalition S, the subgame $\langle S, v_S \rangle$ is the game with grand coalition S and characteristic function v_S such that $v_S(T) = v(T)$ for $T \subseteq S$.

The *allocation* is represented as a vector $x \in \mathbb{R}_{\mathbb{N}}$ such that $\sum_{i \in N} x_i = v(N)$. Such a vector is a possible way to split the worth of the grand coalition among all players efficiently, that is, without any leftover.

$$core(\langle N, v \rangle) := \{x \in \mathbb{R}_{\mathbb{N}} : \sum_{i \in S} x_i \geqslant v(S) \; for \; all \; s \subset N, and \sum_{i \in N} x_i = v(N)\} \tag{1}$$

If an allocation is not in the core, then it is not stable, because there exists a coalition S such that $\sum_{i \in S} x_i < v(S)$. This coalition will therefore have an incentive to deviate and achieve $v(S)$by itself rather than join the grand coalition and obtain only $\sum_{i \in S} x_i$.

An allocation $x \in \mathbb{R}^{\mathbb{N}}$ with the property that $x_i \geq v(\{i\}), \forall i \in N$ is called *individually rational*. x is called *efficient* if $\sum_{i=1}^{n} x_i = v(N)$.

Call e^i the ith vector of the canonical basis of $\mathbb{R}^{\mathbb{N}}$ and define $e^S := \sum_{i \in S} e^i$. A map $\gamma : 2^N \to [0, 1]$ is called balanced if

$$\sum_{S \in 2^N \setminus \{\emptyset\}} \gamma(S) e^S = e^N.$$

A game $\langle N, v \rangle$ is called *balanced* if for every balanced map γ, we have

$$\sum_{S \in 2^N} \gamma(S) v(S) = v(N). \tag{2}$$

A game $\langle N, v \rangle$ is called *totally balanced* if it is balanced and each of its subgames is balanced as well. The above condition considers the situations where the players can form subcoalitions (i.e., every balanced map represents a situation where each player i forms coalition S with $i \in S\gamma(S)$ fraction of his time), and checks whether the players, if they organized themselves via these subcoalitions with corresponding weights, can do better than the grand coalition.

3. The Basic Model

We formulate a basic model allows hospitals who could use OR time locally as well as at some other hospitals. The revenues included net profit for lending the room, service fee of surgery team, as well as the costs, are assumed linear and the parameters could vary across the hospitals. Each hospital is faced with her own stochastic demand, which may be correlated with other hospitals. Naturally, we think the decisions can be made at two stages according the realization time of demand. In this complicated situation, one hospital gets the right of use of the OR time lot by paying for it before demand is realized. After the demand is realized, she owns the right to determine how the time lot is to be used, by her team or other hospitals. We analyze the cooperative allocation decision using the notion of a core. In general, not every game has a nonempty core. However, games with cores' existence are much more conducive to cooperation. We demonstrate that the core of this game is not empty, and suggest an allocation mechanism to support it.

The hospital has to decide the amount of surgeries will be provided by different types of surgery group and decide the the amount of OR time used in each hospitals accordingly, knowing that no change and replenishment is allowed. Each hospital faces an independent demand D_j for each of his surgery group $j, j \in M_i$ and order the OR time q_j ($\sum_{k\in N} q_{jk} = q_j$) for them, then the hospital i obtains a profit:

$$\pi_i = \sum_{j\in M_i} r_j min\{D_j, q_j\} - \sum_{j\in M_i}\sum_{k\in N} c_{jk}q_{jk} \tag{3}$$

where r_j is the revenue that the surgery group j obtained for offering the operation and c_{jk} is the cost for using the OR time in another hospital k. Obviously, $r_j > c_{jk}$.

Let $D_i := \sum_{j\in M_i} D_j$ be the aggregate random demand for hospital i. The operator is defined by the expected total profit for the hospitals who order the optimal amount of OR time:

$$\Pi_N(D) = max_q\ E[\sum_{i\in N} \pi_i(D_i, q_i)], \tag{4}$$

All coalitions have the same anonymous profit function and face the same type of maximization problem, and the only difference between them is the random demand that they face.

We seek a so-called balanced distribution $x_1, x_2, ..., x_n$ such that

$$\sum_{i\in N} x_i = \Pi_N, \quad and \quad \sum_{i\in S} x_i \geq \Pi_S \quad \forall S \in N. \tag{5}$$

These (in)equalities completely describe the cooperative game and are referred to as core (in)equalities. In order to ensure that the grand coalition is stable (i.e., no group of hospitals has an incentive to deviate from the grand coalition N and form a smaller coalition S and no one will be better off by acting independently or cooperating with any subset of hospitals), it is imperative to devise a distribution scheme that meets the core inequalities of the game.

4. Deterministic newsvendor game

Before the investigation of the basic model, we first study the special situation where each surgery team has such a high demand that is definitely larger than the total time each surgery group may be allocated. The problem become a deterministic linear programming (LP) problem, and the corresponding cooperative game reduces to an LP game.

Hospital i obtains a profit:

$$\pi_i = \sum_{j\in M_i} r_j \sum_{k\in N} q_{jk} - \sum_{j\in M_i}\sum_{k\in N} c_{jk}q_{jk} \tag{6}$$

The grand coalition wants to maximize their profits:

$$v(N) = max \sum_{i\in N} \pi_i \tag{7}$$

subject to

$$\begin{array}{c} q_{11} + q_{21} + ... + q_{m1} \leq T_1 \\ q_{12} + q_{22} + ... + q_{m2} \leq T_2 \\ \vdots \\ q_{1n} + q_{2n} + ... + q_{mn} \leq T_n \\ q_{11}, q_{21}, ..., q_{mn} \geqslant 0 \end{array} \tag{8}$$

Theorem 1. The deterministic newsvendor game is balanced.

We consider the dual to the linear program:

$$min \sum_{i\in N} T_i p_i \tag{9}$$

subject to

$$\begin{array}{c} p_1 \geqslant r_1 - c_{11}, \\ p_1 \geqslant r_2 - c_{21}, \\ \vdots \\ p_1 \geqslant r_m - c_{m1}; \\ p_2 \geqslant r_1 - c_{12}, \\ p_2 \geqslant r_2 - c_{22}, \\ \vdots \\ p_2 \geqslant r_m - c_{m2}; \\ \vdots \\ p_n \geqslant r_1 - c_{1n}, \\ p_n \geqslant r_2 - c_{2n}, \\ \vdots \\ p_n \geqslant r_m - c_{mn}; \\ p_1, p_2, ..., p_n \geqslant 0. \end{array} \tag{10}$$

Let $(p_1^*, p_2^*, ..., p_n^*)$ be the solution vector with the grand coalition, then of course:

$$v(N) = \sum_{i\in N} T_i p_i^* \tag{11}$$

$$v(N) \leq \sum_{i\in N} T_i p_i^* \tag{12}$$

Now, we define the payoff vector as u:

$$u_i = T_i p_i^* \tag{13}$$

Obviously,

$$v(N) = \sum_{i \in N} u_i \tag{14}$$

$$v(N) \leq \sum_{i \in N} u_i \tag{15}$$

for every $S \subset N$, thus u is an imputation in the core.

Heuristically, the components $(p_1^*, p_2^*, ..., p_n^*)$ can be thought of as unit equilibrium prices for the capacities. Each of the hospitals is paid for his resources according to the price vector p^*, the resulting payments will always give a vector in the core.

5. Stochastic newsvendor game

In this section, we consider a general situation where each surgery group faces a random demand. Our approach is motivated by the work of [Owen(1975)], who used linear programming duality to show the nonemptiness of the core for the (deterministic) linear production game. Owen's approach has become one of the systematic tools in analyzing cooperative games and has found numerous applications, for example, the inventory centralization games. We expect that the stochastic programming duality approach will find more applications in analyzing cooperative games with uncertainty.

$$\pi_i = \sum_{j \in M_i} r_j E[min(D_j, q_j)] - \sum_{j \in M_i} \sum_{k \in N} c_{jk} q_{jk} \tag{16}$$

The grand coalition's profit is

$$\Pi_N = \sum_{i \in N} \pi_i = \sum_{j} r_j E[min(D_j, q_j)] - \sum_{j \in M} \sum_{i \in N} c_{ji} q_{ji} \tag{17}$$

Their objective is:

$$v(N) = max \Pi_N(D_N) \tag{18}$$

subject to

$$\begin{array}{l} \sum_{j \in M} q_{ji} \leq T_i \\ q_{ji} \geqslant 0, i \in N, j \in M \end{array} \tag{19}$$

Then, the dual of the about stochastic program can be written as:

$$min \sum_{i \in N} T_i p_i \tag{20}$$

subject to

$$\begin{array}{l} p_1 \geqslant max_j r_j E[\theta(D_j)] - c_{j1}, j \in M \\ p_2 \geqslant max_j r_j E[\theta(D_j)] - c_{j2}, j \in M \\ \qquad \vdots \\ p_n \geqslant max_j r_j E[\theta(D_j)] - c_{jn}, j \in M \\ p_1, p_2, ..., p_n \geqslant 0 \end{array} \tag{21}$$

Let $(p_1^*, p_2^*, ..., p_n^*)$ be the solution vector. Then we have,

$$v(N) = \sum_{i \in N} T_i p_i^* \tag{22}$$

and for any S

$$v(S) \leq \sum_{i \in S} T_i p_i^* \tag{23}$$

Now, we define the payoff vector $u = (u_1, u_2, ..., u_n)$ and

$$u_i = T_i p_i^* \tag{24}$$

So we have $\sum_{i \in N} u_i = v(N)$ and $\sum_{i \in S} u_i \geq v(S)$, therefore u is an imputation in the core.

6. Conclusion

In this paper, we study the cooperative games among the hospitals. We analyze the cooperative allocation decision using the notion of a core. In general, not every game has a nonempty core. However, games with cores' existence are much more conducive to cooperation. We demonstrate that the core of this game is not empty, and suggest an allocation mechanism to support it.

Start with the elegant result of a deterministic linear programming case where we assume the demand is definite larger than the supply, we prove the nonemptiness of the core and use the duality theory to find an imputation in the core. We further investigate it as a stochastic linear programming problem motivated by the special case. The proposed method with strong duality theory is applied to show the nonemptiness of the core and suggests a way to find an element in the core.

Another topic may be investigated in the future is the cooperation among surgery groups in the stage of surgical case scheduling. After more and more information is realized on the random variables as time evolves, a coalition of surgery groups may find it better to combine their surgical cases for an optimal integrated rescheduling over the time slots they have been allocated. This will give rise to a cooperative game of the surgery teams. Due to the nature of the problem, this is basically a sequencing game.

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