10 Critical path analysis

Syllabus topic — N3 Critical path analysis

This topic will develop your awareness of critical path analysis in the optimisation of real-life problems.

Outcomes

- Construct an activity chart to represent a particular project.
- Draw a network diagram to represent the activities to be completed during a project.
- Use critical path analysis to determine the minimum time for a project to be completed.
- Use forward and backward scanning to determine the earliest starting times (EST) and latest starting times (LST) for each activity in a project.
- Calculate float times of non-critical activities.
- Use ESTs and LSTs to locate the critical path for the project.
- Solve small-scale network flow problems.
- Use the 'maximum-flow minimum-cut' theorem to solve a flow problem.
- Convert information presented in a table into a network diagram.
- Determine the flow capacity of a network.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Desmos widgets
- Literacy worksheet
- Spreadsheets
- Quick Quiz
- Study guide
- Solutions (enabled by teacher)

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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10A Introduction to critical path analysis

This chapter will develop your awareness of critical path analysis. Critical path analysis is a technique that identifies the activities required to complete a project or task, including identifying the time necessary to complete each activity and the relationships between the activities. Critical path analysis helps to predict if a project will finish on time.

For example, building a house, manufacturing a product, organising a wedding and other similar projects all require many individual activities to be completed before the project is finished. The individual activities often rely upon each other and some can't be performed until other activities are complete. In the organisation of a wedding, invitations would be sent out to guests, but a plan for seating people at the tables during the reception can't be completed until the invitations are accepted. When building a house, the plastering of the walls can't begin until the house is sealed from the weather.

CRITICAL PATH ANALYSIS

Critical path analysis is a technique that identifies the activities required to complete a project, the time necessary to complete each activity and the relationships between the activities.

Activity chart

An activity chart or precedence table is used to describe the activities in a project. It consists of a table containing the activities of a project, their duration and their immediate predecessors. Let's consider some simple projects and their activity chart.

Project: To make a phone call

Activity	Task	Duration (in min)	Immediate predecessors
A	Look up the number	1	–
B	Dial the number and wait for answer	1	A
C	Deliver the message	10	B
D	Hang up the phone	1	C

Project: To make old-style lemonade

Activity	Task	Duration (in min)	Immediate predecessors
A	Fill a tall glass with cracked ice	3	–
B	Add lemon juice and sugar	2	A
C	Stir well	2	B
D	Top with water	1	C
E	Add a slice of lemon	1	D
F	Serve with drinking straws	2	E

ACTIVITY CHART

A table containing the activities of a project, their duration and their immediate predecessors.

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Immediate predecessors and scheduling

For any project, if activity A must be completed before activity B can begin then activity A is said to be an immediate predecessor of activity B. The activities within a project can have multiple immediate predecessors. A time schedule indicates the length of time or duration of each activity. Allocating time to the completion of activities in a project is called scheduling. It shows the immediate predecessors and duration for each activity. Notice that checking email can only be completed after Sandra gets out of bed, has breakfast, and gets dressed.

Activity	Task	Duration (in min)	Immediate predecessors
A	Get out of bed	1	–
B	Cook breakfast	6	A
C	Eat breakfast	5	B
D	Brush teeth	3	C
E	Have shower	4	A
F	Get dressed	6	E
G	Check email	5	C, F

Example 1: Constructing an activity chart 10A

Project: To catch a train.

The tasks shown below are to catch a train. However, they are in the wrong sequence.

Wait for train (8 minutes)
Go to the correct platform (2 minutes)
Scan your Opal card (1 minute)
Board the train (1 minute)
Travel to the train station (5 minutes)

Correctly sort the above tasks and construct an activity chart for this project with the four headings: Activity, Task, Duration and Immediate predecessors.

SOLUTION:

1. Analyse the tasks and sort into a logical sequence.
2. Draw an activity chart with 4 columns.
3. Label the 5 activities (A to E).
4. Write the sorted tasks in the task column.
5. Record the duration for each activity.
6. Determine the predecessor for each activity.

Correct sequence of tasks: Travel to the train station, scan your Opal card, go to the correct platform, wait for train and board the train.

Activity	Task	Duration (in min)	Immediate predecessors
A	Travel to train station	5	–
B	Scan your opal card	1	A
C	Go to platform	2	B
D	Wait for train	8	C
E	Board the train	1	D

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Example 2: Reading an activity chart 10A

Joshua wants to create a patio in his garden. The activity chart for this project is shown below.

Activity	Task	Duration (in hours)	Immediate predecessors
A	Clean up garden	3	–
B	Measure area of patio	1	–
C	Plan the patio	2	B
D	Buy fencing	1	B
E	Buy plants and pots	3	A, C
F	Insert plants in pots	1	E
G	Buy paving	1	C
H	Make garden	5	D, F, G

Note: Activity A (clean up garden) is an immediate predecessor of activity E (buy plants and pots) as it determines the number plants required to fill the space.

a How many activities in the project?
b What are the first activities?
c Which activity has the longest time interval?
d What is the total time for Joshua to complete this project?
e What is the immediate predecessor for activity C?
f What is the immediate predecessor for activity G?
g Why are there three immediate predecessors for activity H?

SOLUTION:

1 Count the number of activities (A to H).
2 List the activities with no immediate predecessor.
3 Find the largest number in the duration column.
4 Add the time listed in the duration column.
5 List the activity in the predecessor column.
6 List the activity in the predecessor column.
7 Look for the 3 immediate predecessors of activity H.

a There are 8 activities.
b A to B are the first activities.
c Making the garden (H) takes 5 hours.
d Total time $= 3 + 1 + 2 + 1 + \ldots + 5$
$= 17$ h
e Immediate predecessor of C is B.
f Immediate predecessor of G is C.
g Activity H cannot be completed until Joshua buys the materials (fencing, plants, pots and paving).

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Exercise 10A

LEVEL 1

1 Copy and complete the following sentences.

a Critical path analysis identifies the _____ required to complete a project.

b Activity chart contains the activities, _____ and their immediate predecessors.

c If activity A must be completed before activity B can begin then activity A is said to be an _____ of activity B.

d Allocating time to the completion of activities in a project is called _____.

Example 2

2 An incomplete activity chart for washing your hands is shown below.

Activity	Task	Duration (in min)	Immediate predecessors
	Turn on water	1	–
B	Wet hands	1	
	Clean hands with soap	3	B
D	Rinse off soap	2	
	Dry hands	3	D

a Copy and complete the activity chart shown above.

b How many activities in the project?

c Which activity has the longest time interval?

d What is the total time for this project?

e What is the immediate predecessor for activity C?

f What activities precede activity B?

3 An incomplete activity chart for preparing a meal is shown below.

Activity	Task	Duration (in min)	Immediate predecessors
A	Find recipe	2	
	Prepare ingredients	10	A
C	Cook meal	25	
	Serve meal	3	C
	Clean up	10	D

a Copy and complete the activity chart shown above.

b How many activities in the project?

c Which activity has the shortest time interval?

d What is the total time for this project?

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Example 1

4 The tasks for each of the following projects have errors in their sequence. Find these errors and construct an activity chart for each project with the four headings: Activity, Task, Duration and Immediate predecessor.

a Project: To watch your favorite television show
Turn television off (1 minute)
Turn television on (1 minute)
Find remote control (2 minutes)
Watch television show (30 minutes)
Select television show (1 minute)

b Project: To listen to a music playlist
Select play (1 minute)
Turn off phone (1 minute)
Unlock phone (1 minute)
Select the music app (1 minute)
Turn on phone (2 minutes)
Select playlist (3 minutes)
Listen to music (50 minutes)

c Project: To plant a tree in the garden
Water the tree (5 minutes)
Decide on the tree required (10 minutes)
Purchase the tree (8 minutes)
Dig the hole (12 minutes)
Plant the tree (5 minutes)
Mulch the tree (3 minutes)

d Project: To bake a cake
Mix the icing (4 minutes)
Cool the cake (30 minutes)
Bake the cake (35 minutes)
Put cake mixture in cake tin (3 minutes)
Heat oven (10 minutes)
Mix ingredients (6 minutes)
Ice the cake (8 minutes)

e Project: To install a smoke alarm
Select a section of the ceiling (2 minutes)
Screw the mounting plate into the ceiling (5 minutes)
Install the batteries into the alarm (3 minutes)
Remove the mounting plate from the alarm (2 minutes)
Choose a room (2 minutes)
Connect the alarm body to the mounting plate (4 minutes)

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LEVEL 2

5 Jayden is required to organise a birthday party. He has estimated the following time for these tasks: buying the food – 2 hours; preparing the food – 3 hours; serving the meal – 1 hour; cleaning up – 2 hours. Construct an activity chart for this project.

6 Hayley needs her new laptop operating in less than a day. She has estimated the following time for these tasks: buying the laptop – 1 hour; unpacking the laptop – 0.5 hours; installing the software – 4 hours; copying the data – 3 hours; testing the software with the data – 1 hour. Construct an activity chart for this project.

7 Consider each of the following tasks as projects and construct an activity chart. Use the four headings: activity, task, duration and immediate predecessor.

a Make buttered toast
b Pump up a bicycle tyre
c Make a cheese sandwich

LEVEL 3

8 Sophie is renovating her lounge room. She has organised this project into six unordered tasks.

Arrange new furniture – 1 day.
Remove furniture and curtains – 1 day;
Lay new carpet – 1 day;
Hang new curtains – 1 day;
Paint walls – 4 days;
Prepare walls – 2 days;

a What is the first task? Give a reason for your answer.
b What is the immediate predecessor to the following tasks?
  i Prepare walls
  ii Paint walls
  iii Lay new carpet
  iv Hang new curtains
c List the two immediate predecessors for 'arranging new furniture'.
d Construct an activity chart for this project.

9 Samuel is cooking a pizza for dinner. He has divided this project into five tasks: Defrost the pizza base – 5 min; prepare the toppings – 10 min; place the sauce and topping on pizza – 2 min; heat oven – 10 min; cook pizza – 25 min.

a What tasks can be performed at the same time?
b Which tasks are predecessor tasks for cooking the pizza?
c Construct an activity chart for this project.

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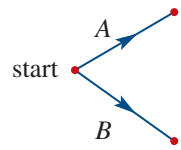
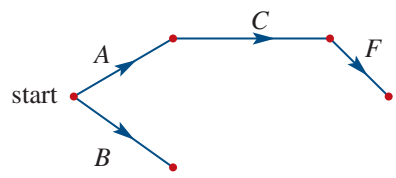
10B Constructing a network diagram given an activity chart

A network diagram is often used for critical path analysis. It can represent the activities required to complete a project, the time necessary to complete each activity and the relationships between the activities. In critical path analysis a network diagram does not have labelled vertices. The activities in the project are represented by the edges of the diagram and so it is the edges that must be labelled, not the vertices. Drawing a network diagram from the activity chart opposite, is shown below.

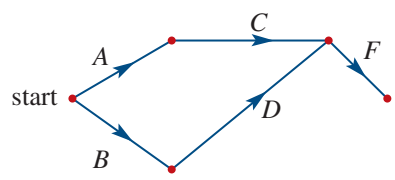
Activity	Immediate predecessors
A	–
B	–
C	A
D	B
E	B
F	C, D
G	E, F



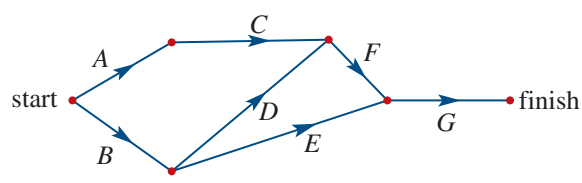
1. Activities A and B have no immediate predecessors. These activities can start immediately and can be completed at the same time.



2. Activity A is an immediate predecessor of activity C, so activity C must follow immediately after activity A. Activity C is an immediate predecessor of activity F, so activity F must follow immediately after activity C.



3. Activity D has immediate predecessor activity B so it follows immediately after activity B. Activity D is also an immediate predecessor of activity F so activity F must follow immediately after activity D.



4. Activity E has immediate predecessor activity B so it will follow immediately after activity B. Activity G has immediate predecessor activity F and activity E and so it must follow immediately after both of these activities.
5. Activity G is not an immediate predecessor for any activity and so the project is finished after this activity is complete.

CONSTRUCTING A NETWORK DIAGRAM GIVEN AN ACTIVITY CHART

Activities that have no immediate predecessors follow from the start vertex.
Activities that are not immediate predecessors for other activities lead to the finish vertex.
For every other activity, look for:

- the activities for which it is an immediate predecessor
- the activities it has as immediate predecessors.

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Example 3: Constructing a network diagram given an activity chart 10B

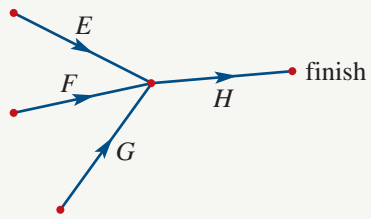
Draw a network diagram from the activity chart shown on the right. In this solution, the network diagram will be drawn from the finish back to the start.

Activity	Immediate predecessors
A	–
B	A
C	A
D	A
E	B
F	C
G	D
H	E, F, G

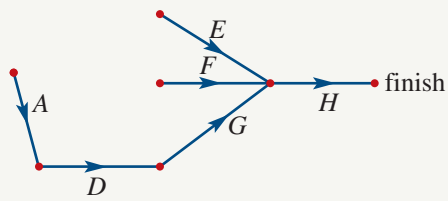
SOLUTION:

1 H is not an immediate predecessor for any other activity so it will lead to the finish of the project.

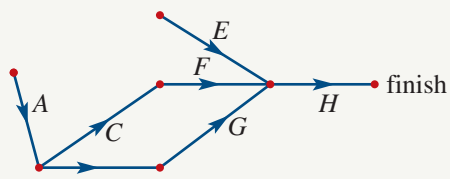
2 H has immediate predecessors E, F and G and so these three activities will lead into activity H.



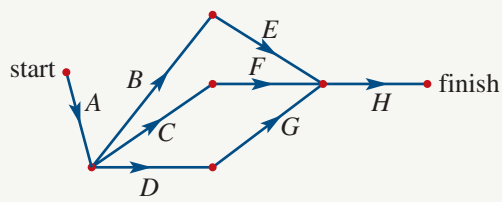
3 Activity D is an immediate predecessor of activity G and has immediate predecessor activity A. There will be a path through activity A, activity D and then activity G.



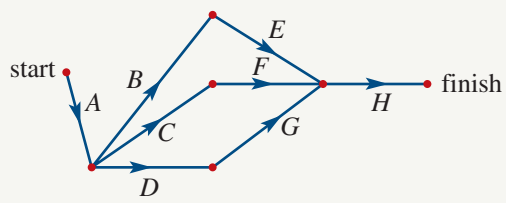
4 Activity C is an immediate predecessor of activity F and has immediate predecessor activity A. There will be a path through activity A, activity C and then activity F.



5 Activity B is an immediate predecessor of activity E and has immediate predecessor activity A. There will be a path through activity A, activity B and then activity E.



6 Activity A has no immediate predecessors, so it is the start of the project.



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Exercise 10B

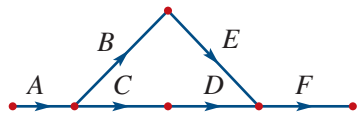
LEVEL 1

1 Answer the following questions for each network diagram in parts **a** to **d**.

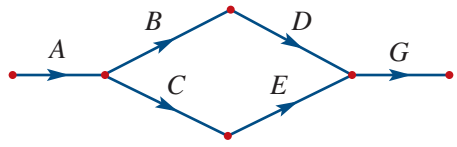
- **i** How many activities in the project?
- **ii** What is the beginning activity?
- **iii** What is the ending activity?
- **iv** Which activity is the immediate predecessor to activity D?
- **v** Which activities have no predecessors?

a

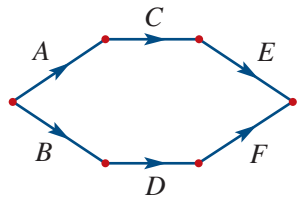
b



c

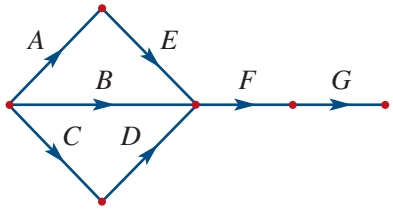


d

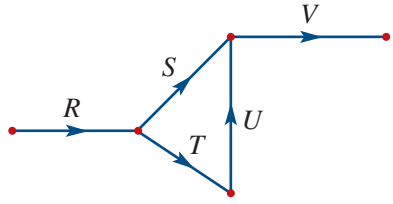


2 Construct an activity chart for each of the following network diagrams.

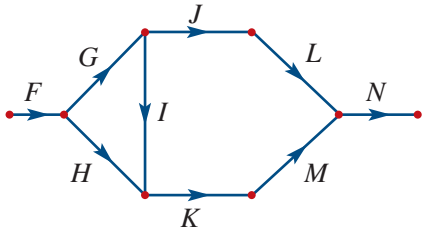
a



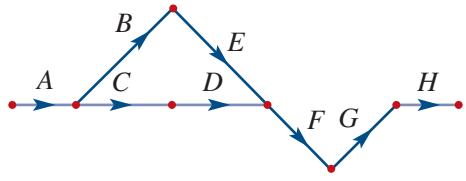
b



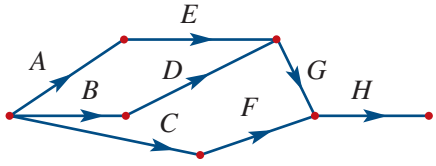
c



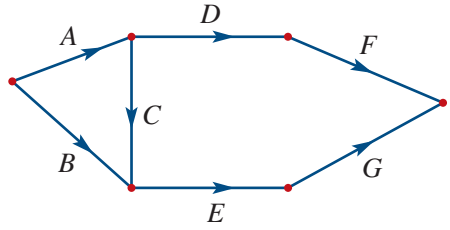
d



e



f



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Example 3 **3** Construct a network diagram for each of the activity charts below.

a

Activity	Immediate predecessors
P	–
Q	–
R	Q
S	R

b

Activity	Immediate predecessors
D	–
R	D
F	R
G	F

c

Activity	Immediate predecessors
D	–
E	–
F	D, E
G	F

d

Activity	Immediate predecessors
W	–
X	W
Y	W
Z	W

e

Activity	Immediate predecessors
A	–
B	A
C	A
D	B

f

Activity	Immediate predecessors
P	–
Q	–
R	P
S	Q

g

Activity	Immediate predecessors
C	–
D	–
E	–
F	C, D, E
G	F

h

Activity	Immediate predecessors
H	–
I	H
J	H
K	I
L	K

i

Activity	Immediate predecessors
A	–
B	A
C	A
D	B
E	C

j

Activity	Immediate predecessors
P	–
Q	–
R	P
S	Q
T	R, S

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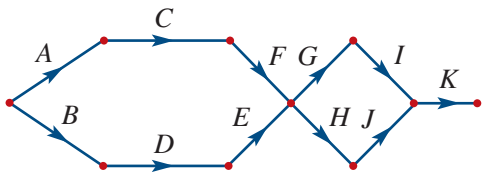
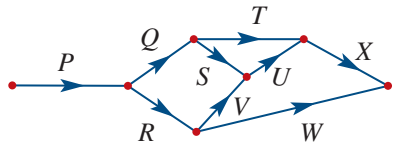
LEVEL 2

4 Construct an activity chart for each of the following network diagrams.

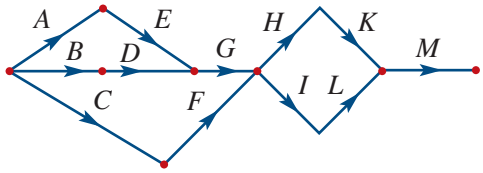
a



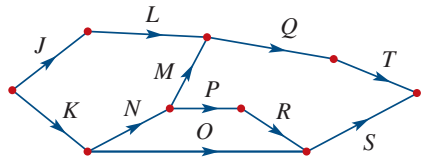
b



c



d



LEVEL 3

5 Construct a network diagram for each of the activity charts below.

a

Activity	Immediate predecessors
T	–
U	–
V	U
W	T
X	V, W
Y	X
Z	Y

b

Activity	Immediate predecessors
F	–
G	–
H	–
I	F
J	G, I
K	H, J
L	K

c

Activity	Immediate predecessors
K	–
L	–
M	K
N	M
O	N, L
P	O
Q	P
R	M
S	R, Q

d

Activity	Immediate predecessors
A	–
B	–
C	–
D	B
E	A, D
F	E, C
G	F
H	G
I	E, C
J	G
K	H, I

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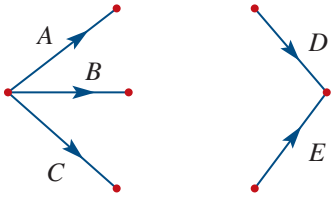
10C Dummy activities

Sometimes two activities will have some of the same immediate predecessors, but not all of them. In this activity chart, activity D and activity E share the immediate predecessor activity B, but they both have an immediate predecessor activity that the other does not. This overlap of predecessors presents some difficulty when constructing the network diagram. That is, you cannot have an edge that represents activity B being drawn to both the edge for activity D and the edge for activity E. This difficulty is easily overcome.

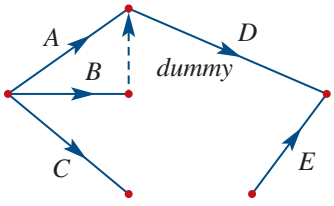
Activity	Immediate predecessors
A	–
B	–
C	–
D	A, B
E	B, C

Activity D and activity E are not immediate predecessors for any other activity, so they will lead directly to the finish vertex of the project. Activities A, B and C have no immediate predecessors, so they will be the starting vertex of the project.

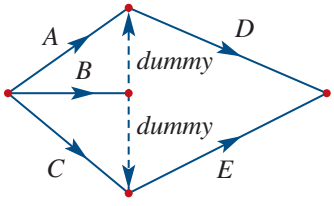


We need to use the precedence information for activity D and activity E to join these two parts together. Activity D needs to follow directly from activity A and activity B, but we can only draw one edge for activity D. Activity E needs to follow directly from both activity B and activity C, but again we only have one edge for activity E, not two.

The solution is to draw the diagram with activity D starting after one of its immediate predecessors, and using a dummy activity for the other. The dummy activities are represented by dotted edges and are, in effect, imaginary. They are not real activities, but they allow all of the predecessors from the table to be correctly represented.



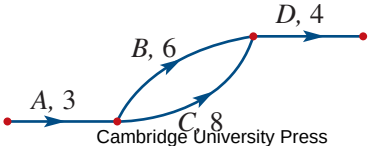
The dummy activity for D allows activity D to directly follow both activity A and B. A dummy activity is also needed for activity E because it, too, has to start after two different activities, activity B and C.



DUMMY ACTIVITIES

- A dummy activity is required if two activities share some, but not all, of their immediate predecessors.
- A dummy activity will be required from the end of each shared immediate predecessor to the start of the activity that has additional immediate predecessors.
- A dummy activity is not real but imaginary, and so it takes no time (its duration is 0).
- Dummy activities are represented in the network diagram using dotted lines so the label 'dummy' on the diagram is not actually needed, and won't be used from now on, but you can include it on your own network diagrams for emphasis.

If two activities share the same immediate predecessor and end at the same node, a dummy activity is not required. Instead use multiple lines between the two nodes.



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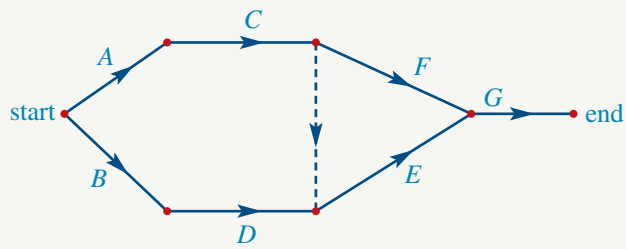
Example 4: Using a dummy activity in a network diagram 10C

Draw a network diagram from the activity chart shown on the right.

Activity	Immediate predecessors
A	–
B	–
C	A
D	B
E	C, D
F	C
G	E, F

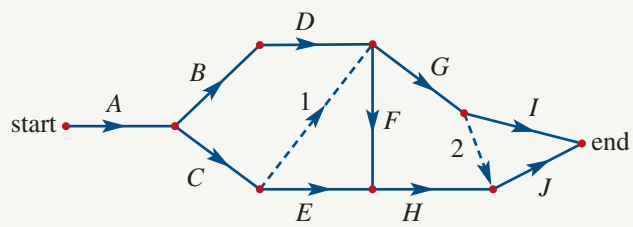
SOLUTION:

1. A and B will lead from the start vertex.
2. G will lead to the end vertex.
3. A dummy will be required the end of activity C (shared immediate predecessor) to the start of activity E (the activity with an additional immediate predecessor).



Example 5: Creating an activity chart from a network diagram 10C

Construct an activity chart for the network diagram shown on the right.



SOLUTION:

1. Create a table with a row for each activity.
2. Look at the start of an activity. Write down all of the activities that lead directly to this activity in the immediate predecessor column.
3. The dummy activity 1 makes activity C a predecessor of activities F and G as well as E.
4. The dummy activity 2 makes activity G a predecessor of activity J as well as I.

Activity	Immediate predecessors
A	–
B	A
C	A
D	B
E	C
F	D, C
G	D, C
H	E, F
I	G
J	G, H

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Scheduling on a network diagram

Activity charts often include a duration to indicate the length of time of each activity. You will recall that allocating time to the completion of activities in a project is called scheduling. Knowing how long individual activities within a project are likely to take allows managers of such projects to hire staff, book equipment and also to estimate overall costs of the project. Scheduling on a network diagram is shown using a weighted edge. The number or weight on the edge indicates the duration of the activity. The weight of dummy activities is always zero.

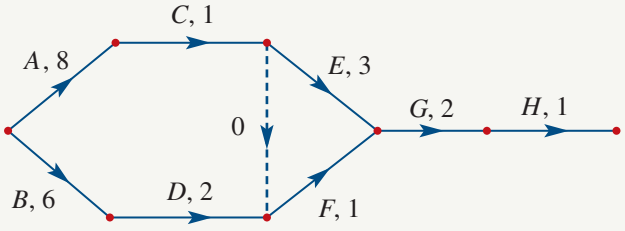
Example 6: Scheduling on a network diagram 10C

Draw a network diagram from the activity chart shown on the right.

Activity	Duration (in days)	Immediate predecessors
A	8	–
B	6	–
C	1	A
D	2	B
E	3	C
F	1	C, D
G	2	E, F
H	1	G

SOLUTION:

1. A and B will lead from the start vertex. Indicate the time as a weight edge.
2. H will lead to the end vertex. Indicate the time as a weighted edge.



3. Both F and G have two predecessors. F shares one of its predecessors (C) with activity E, but F and E do not have exactly the same predecessors. Activity G's immediate predecessors E and F are not shared with any other activity. So a dummy activity is required from the end of activity C to the start of activity F to show that E and F share an immediate predecessor, C.

SCHEDULING ON A NETWORK DIAGRAM

Scheduling on a network diagram is shown using a weighted edge. The number on the edge indicates the duration of the activity. The weight of dummy activities is always zero.

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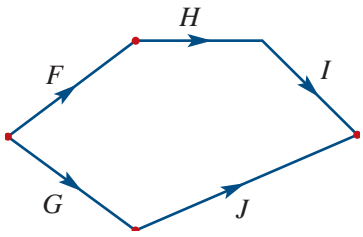
Exercise 10C

LEVEL 1

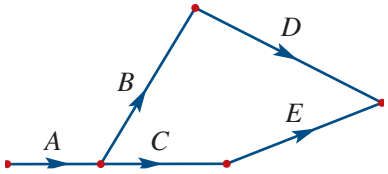
Example 4 **1** Complete each network diagram by inserting a dummy activity for each activity chart.

a



Activity	Immediate predecessors
F	–
G	–
H	F
I	H, G
J	G

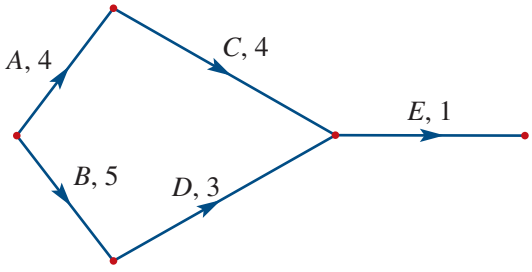
b



Activity	Immediate predecessors
A	–
B	A
C	A
D	B
E	B, C

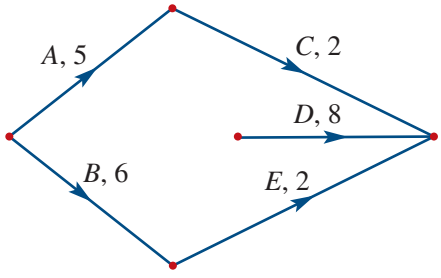
Example 6

c



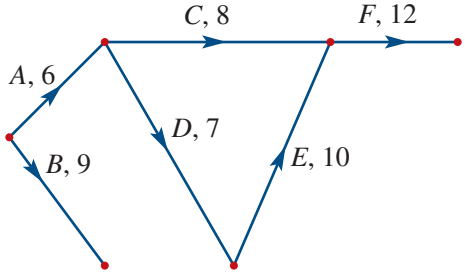
Activity	Duration (in days)	Immediate predecessors
A	4	–
B	5	–
C	4	A, B
D	3	B
E	1	C, D

d



Activity	Duration (in hrs)	Immediate predecessors
A	5	–
B	6	–
C	2	A
D	8	A, B
E	2	B

e



Activity	Duration (in mins)	Immediate predecessors
A	6	–
B	9	–
C	8	A, B
D	7	A, B
E	10	D
F	12	C, E

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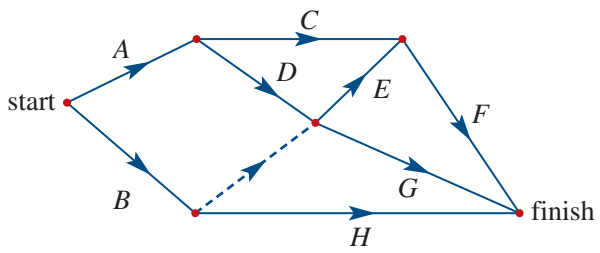
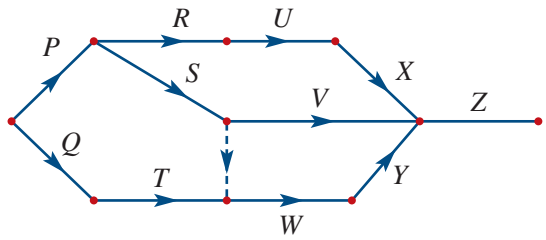
Example 5 **2** Complete each activity chart for the network diagrams shown below.

a



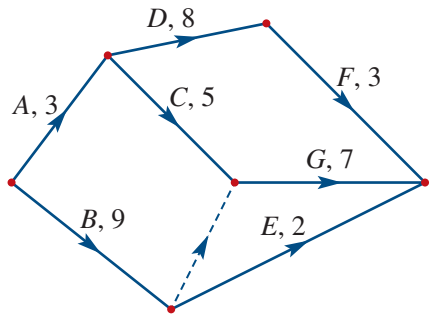
Activity	Immediate predecessors
A	
	–
C	
D	
E	D, B
F	
G	
	B

b



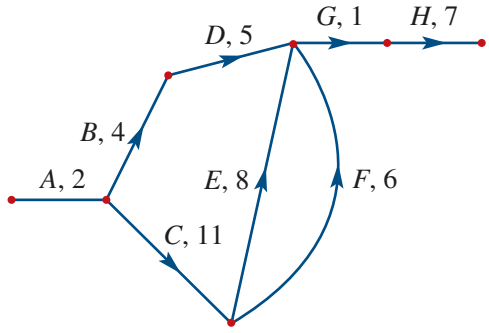
Activity	Immediate predecessors
P	–
R	
S	
	Q
U	
W	
	U
Y	
Z	

c



Activity	Duration (in mins)	Immediate predecessors
A	3	
B		
C		
D		A
E		
F		
G	7	

d



Activity	Duration (in hrs)	Immediate predecessors
A		
B	4	
C		
D		
E		
F		C
G		
H		

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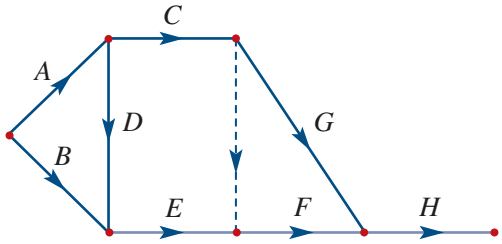
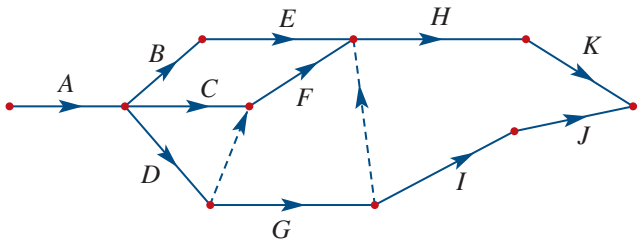
LEVEL 2

3 Draw an activity chart for each network diagram.

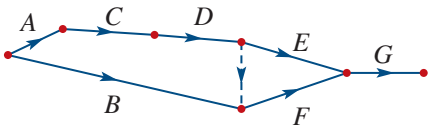
a



b



4 The following network diagram shows the activities in a project to repair a dent in a car panel. The activities are listed in the table on the right.



Activity	Description
A	Remove panel
B	Order component
C	Remove broken component
D	Pound out dent
E	Repaint
F	Install new component
G	Replace panel

a Which activity or activities are the immediate predecessors of the event 'remove broken component'?

b Which activities are the immediate predecessors of the activity 'install new component'?

LEVEL 3

5 Construct a network diagram for each of the activity charts below.

a

Activity	Immediate predecessors
P	–
Q	–
R	P
S	Q
T	Q
U	R, S
V	R, S, T

b

Activity	Immediate predecessors
A	–
B	A
C	A
D	B, C
E	C
F	E
G	D
H	F, G

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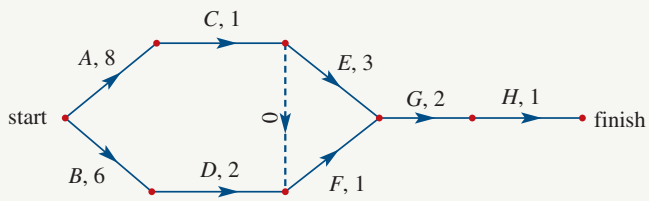
10D Forward and backward scanning

Forward scanning

For a project to be completed in the shortest time possible, it is important to be aware of each activity's earliest starting time (EST) after the start of the entire project. An EST of 8 means an activity cannot start until 8 hours (or whatever time unit is given) after the start of the project. The EST for each activity is found by a process called forward scanning, described below.

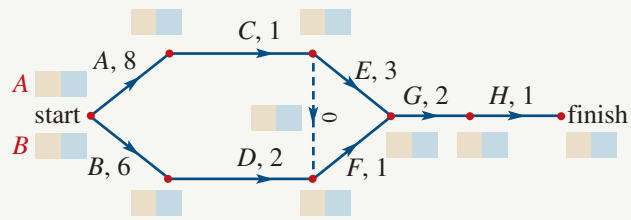
Example 7: Finding the earliest starting time using forward scanning 10D

Find the earliest starting time (EST) for the project represented in the network diagram opposite. The duration of each activity is in days.

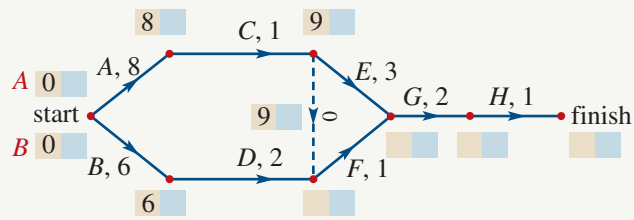


SOLUTION:

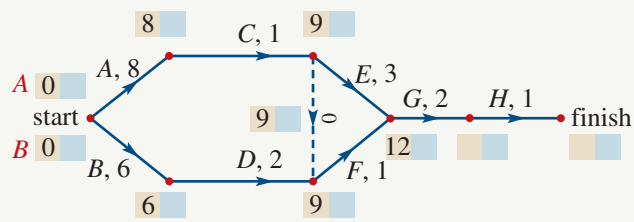
1. Draw a box, split into two cells, next to each vertex of the network diagram, as shown in the diagram opposite. If more than one activity begins at a vertex, draw a box for each of these activities.



2. Activity A and B start the project and have an EST of zero. Write zero in the left box (shaded yellow).
3. Activity C has an EST of 8 (add the EST of the immediate predecessor to the duration of the activity).

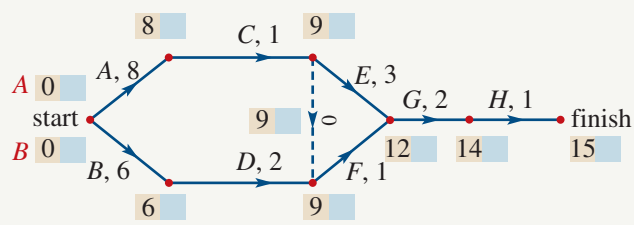


4. Repeat step 3 for each activity. However, if an activity has more than one predecessor, calculate the EST using each of the predecessors and choose the largest value.



Note: EST of F = 6 + 2 = 8 or EST of F = 9 + 0 = 9. Use 9.
EST of G = 9 + 1 = 10 or EST of G = 9 + 3 = 12. Use 12.

5. The EST value at the finish of the project is the minimum time it takes to complete the project.



6. Write the answer in words.

∴ EST of the project is 15 days.

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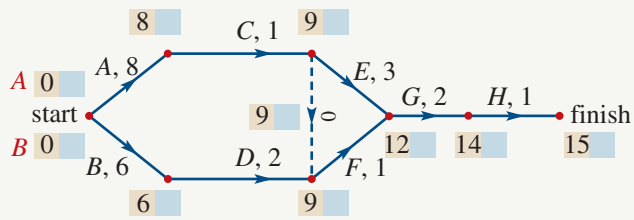
Backward scanning

Some activities have some flexibility around the time they can start. The latest start time, or LST, for each activity is the latest time after the start of the project that the individual activity can start for the project to still be completed in the minimum time. LSTs for each activity are calculated using the reverse of the process used to calculate the ESTs. This process is called backward scanning and is demonstrated below.

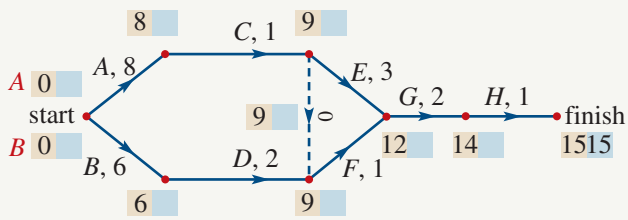
Example 8: Finding the latest starting time using backward scanning 10D

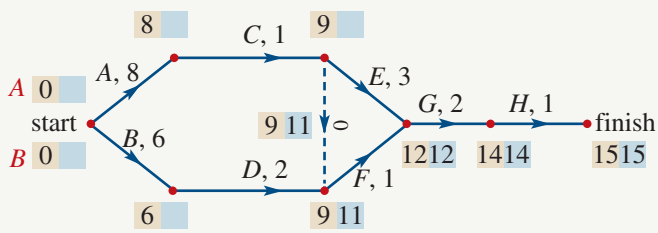
Find the latest starting time (LST) for each activity using the network diagram and completed forward scanning from the previous example. The duration of each activity is in days.



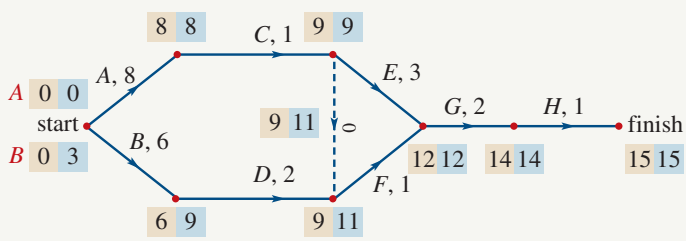
SOLUTION:

1. Copy the minimum time to complete the project into the right cell shown shaded blue in the diagram.
2. Calculate the LST for each activity by subtracting the duration of the activity from the LST of the following activity.
3. If more than one activity has the same predecessor, calculate the LST using each of the activities that follow and choose the smallest value.





4. The LST for each activity is shown in shaded blue cell in the diagram opposite.



EARLIEST STARTING TIME (EST)	LATEST STARTING TIME (LST)
The earliest time any activity can be started after all prior activities have been completed. The EST of an activity is zero if there are no prior activities, otherwise it is greater than zero.	The latest time any activity can be started before delaying the entire project after all prior activities have been completed.

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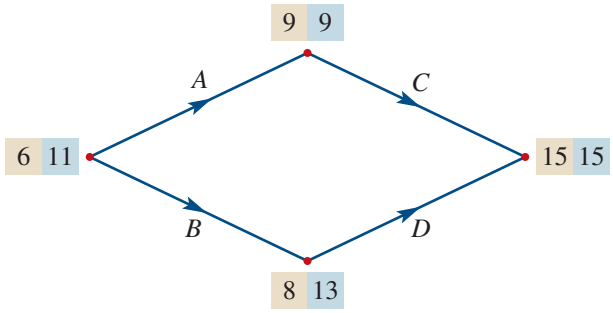
Exercise 10D

LEVEL 1

Example 7, 8

1 A section of a network diagram is shown below.



a What are the earliest start times for each of these activities?

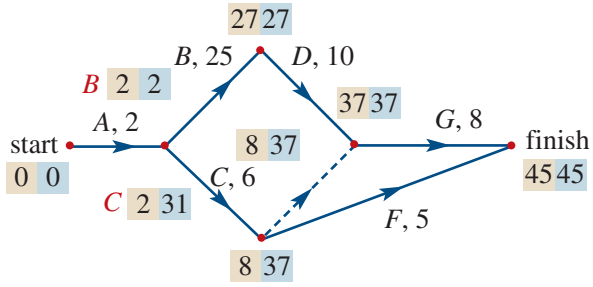
i A **ii** B
iii C **iv** D

b What are the latest start times for each of these activities?

i A **ii** B
iii C **iv** D

c Find the duration of the four activities.

2 A section of a network diagram is shown on the right.



a What are the earliest start times for each of these activities?

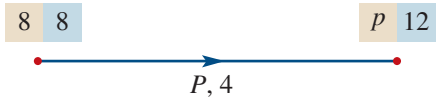
i A **ii** B
iii C **iv** D
v F **vi** G

b What are the latest start times for each of these activities?

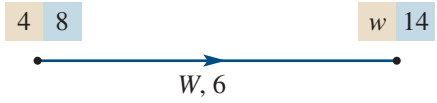
i A **ii** B
iii C **iv** D
v F **vi** G

3 Write down the value of each pronumeral in the sections of network diagrams below.

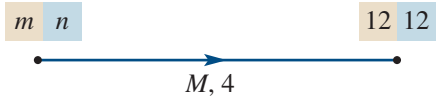
a



b

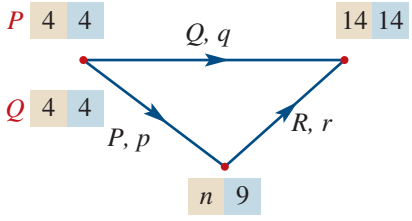


c

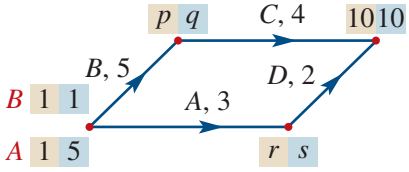


d

e



f



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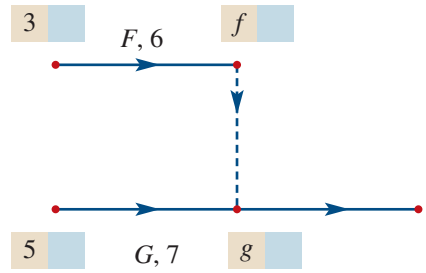
4 What are the values of f and g in the section of a network diagram shown opposite?

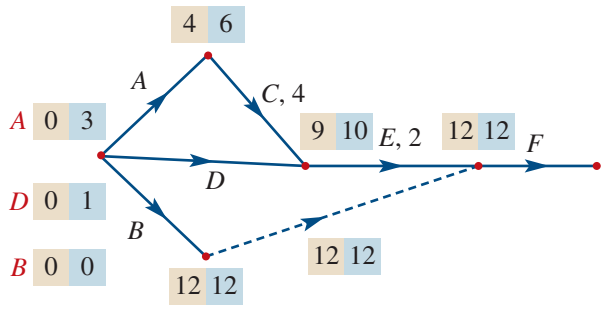


5 Consider the section of a network diagram shown in the diagram on the right.

a What is the duration of activity B?

b What is the latest start time for activity E?

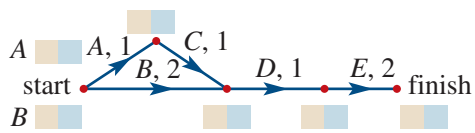
c What is the earliest time that activity E can start?



6 Find the earliest starting times (EST) and latest start times (LST) for all activities given the activity charts and network diagrams below.

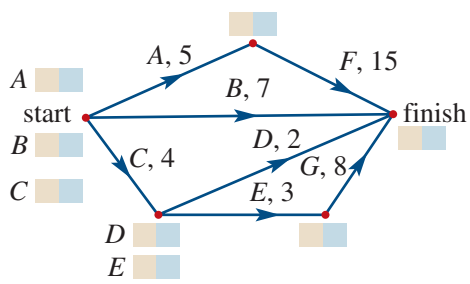
a

Activity	Duration (in hrs)	Immediate predecessors
A	1	–
B	2	–
C	1	A
D	1	B, C
E	2	D



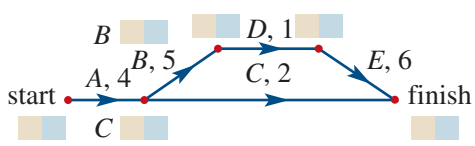
b

Activity	Duration (in hrs)	Immediate predecessors
A	5	–
B	7	–
C	4	–
D	2	C
E	3	C
F	15	A
G	8	E



c

Activity	Duration (in days)	Immediate predecessors
A	4	–
B	5	A
C	2	A
D	1	B
E	6	D



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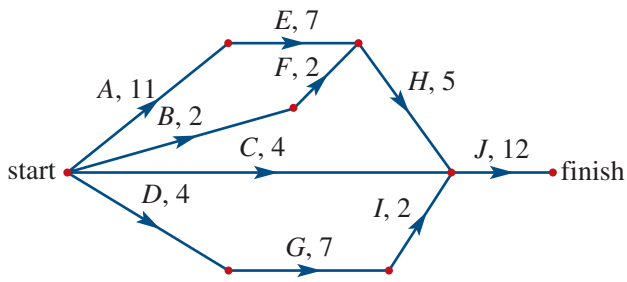
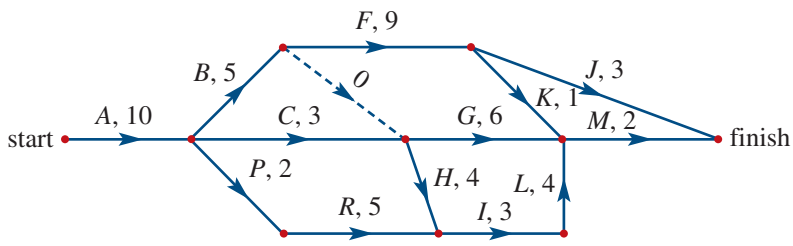
LEVEL 2

7 Determine the earliest starting times (EST) and latest starting time (LST) for each activity in the network diagram.

a



b



LEVEL 3

8 The activity chart for a project is shown opposite.

a Construct a network diagram using the above activity chart.

b Show the earliest starting times (EST) for each activity on the network diagram.

c Show the latest starting times (LST) for each activity on the network diagram.

Activity	Duration (in min)	Immediate predecessors
A	4	–
B	5	–
C	7	A
D	7	B
E	3	B, C
F	6	D, E

9 The activity chart for a project is shown opposite.

a Construct a network diagram using the above activity chart.

b Show the earliest starting times (EST) for each activity on the network diagram.

c Show the latest starting times (LST) for each activity on the network diagram.

Activity	Duration (in min)	Immediate predecessors
A	5	–
B	6	A
C	4	A
D	2	B
E	7	A
F	6	C
G	1	C
H	3	D
I	2	G

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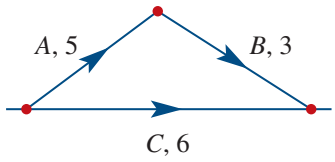
10E Float times and the critical path

The diagram below shows a small section of a network diagram. There are three activities shown, with their individual durations, in hours.



Activity B and activity C are both immediate predecessors to the next activity, so the project cannot continue until both of these tasks are finished. Activity B cannot begin until activity A is finished.

Activity C can be completed at the same time as activity A and activity B. Activity A and B will take a total of $5 + 3 = 8$ hours, while activity C only requires 6 hours. There is therefore some flexibility around when activity C needs to start. There are $8 - 6 = 2$ hours spare for the completion of activity C. This value is called the float time (or slack time) for activity C. The flexibility around the starting time for activity C can be demonstrated with the following diagram.

Hours:	*1st*	*2nd*	*3rd*	*4th*	*5th*	*6th*	*7th*	*8th*
	A	A	A	A	A	B	B	B
Start at same time	C	C	C	C	C	C	Float	Float
Delay C by 1 hour	Float	C	C	C	C	C	C	Float
Delay C by 2 hour	Float	Float	C	C	C	C	C	C

- The five red squares represent the 5 hours it takes to complete activity A.
- The three green squares represent the 3 hours it takes to complete activity B.
- The six yellow squares represent the 6 hours it takes to complete activity C.

Activity C does not have to start at the same time as activity A because it has some float time available (2 hours). Activity C should not be delayed by more than 2 hours because this would cause delays to the project and would affect the EST of the next activity. The next activity requires B and C to be complete before it can begin.

FLOAT TIMES

- Float time is the amount of time that a task in a project network can be delayed without causing a delay to subsequent tasks.
- Float time is calculated by subtracting the earliest start time from the latest start time.
  Float time = LST − EST

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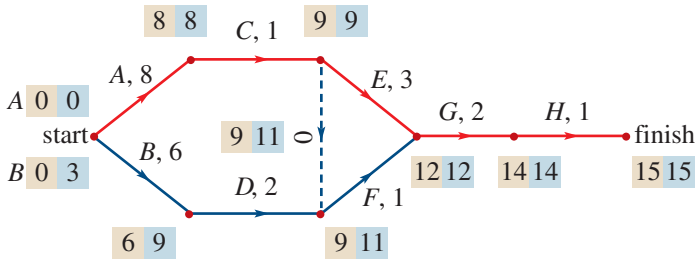
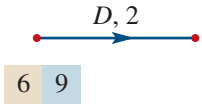
Identifying float times



The boxes at the vertices in the network diagram above give the EST and LST for the activity that begins at that vertex.

The EST for activity D is 6 and the LST for activity D is 9. This means activity D has a float time of $9 - 6 = 3$ hours.



Activity D can be delayed by 3 hours without delaying the rest of the project.

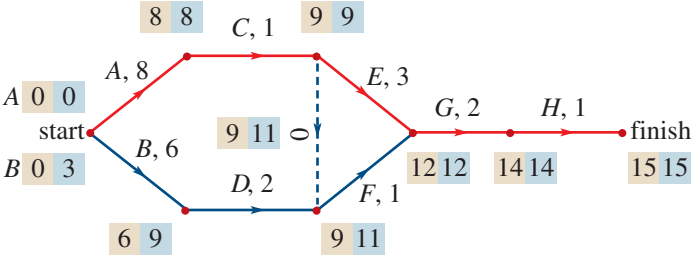
The EST for activity C is 8 and the LST for activity C is 8. This means activity C has a float time of $8 - 8 = 0$ hours.

Activity C has no flexibility around its starting time at all. Any delay to the start of this activity will delay the whole project and extend the minimum time for completion. This means that activity C is a critical activity.

Critical path

All activities that have no float time are critical activities for completion of the project. Tracking through the network diagram along the edges of critical activities gives the critical path for the project. The critical path for the previous project is highlighted in red on the diagram below.

We say the critical path is A → C → E → G → H, and that activities A, C, E, G and H are all critical activities.



The process for determining the critical path is called critical path analysis. The steps involved in critical path analysis are outlined below.

1. Draw a box with two cells next to the start of each edge of the network diagram and at the finish.
2. Calculate the EST for each activity by forward scanning:
   EST = EST of predecessor + duration of predecessor.
3. If an activity has more than one predecessor, the EST is the largest of the alternatives.
4. The minimum overall completion time of the project is the EST value at the finish.
5. The LST of the finish is the same as the EST of the finish. Calculate the LST for each activity by backward scanning:
   LST = LST of following activity − duration of activity.
6. If an activity has more than one following activity, the LST is the smallest of all the alternative following activities.
7. Float time of an activity = LST − EST.
8. If the float time = 0, the activity is on the critical path.

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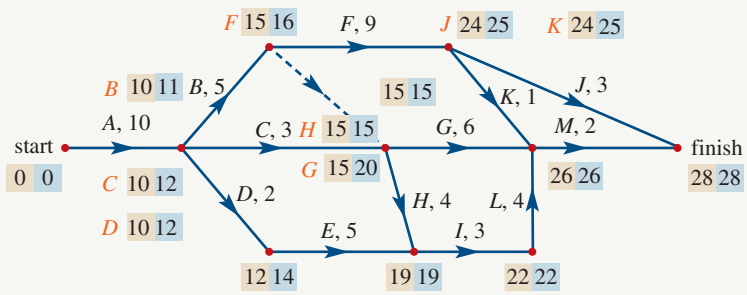
CRITICAL PATH	MINIMUM TIME
The critical path is the sequence of the critical activities. Critical activities are ones that cannot be delayed without affecting the overall completion time of the project.	The minimum overall completion time of the project is the EST value at the finish.

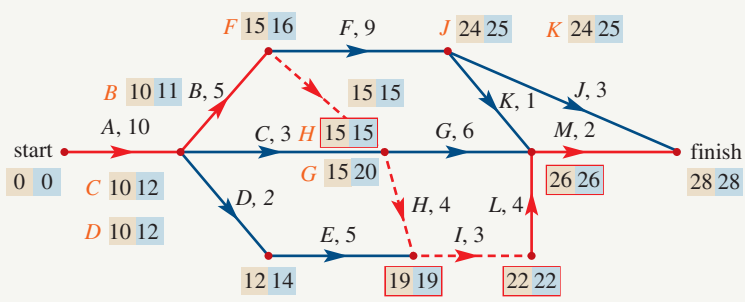
Example 9: Locating the critical path 10E

What is the critical path in the network diagram shown below?



SOLUTION:

1. The critical path has activities with a zero float time.
2. Look for
   Float = LST − EST = 0.
   These instances are highlighted by the red boxes.
3. The critical path is highlighted in red. The dummy activity is not included in the critical path.
4. Write the answer in words.



The critical path of this project is A→B→H→I→L→M.

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Example 10: Finding the float time and the critical path 10E

A project has six activities as shown in the activity chart opposite.

a Draw a network diagram for this project.

b Calculate the EST and LST for each activity.

c What is the float time for each activity?

d Write down the critical path of this project.

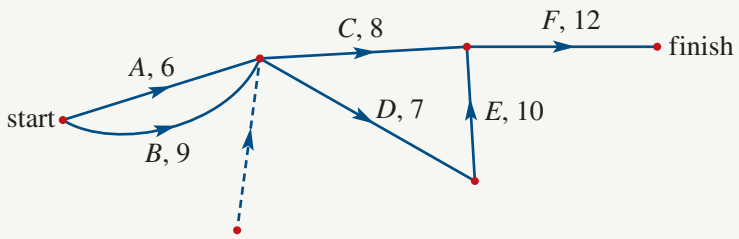
e What is the minimum time required to complete the project?

Activity	Estimated completion time (days)	Immediate predecessors
A	6	–
B	9	–
C	8	A, B
D	7	A, B
E	10	D
F	12	C, E

SOLUTION:

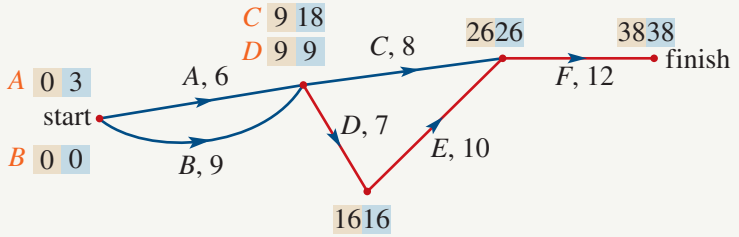
1 A and B have no predecessors and so can begin at the same time. Since A and B are predecessors of C and D, a dummy activity will be required.

a



2 Forward scan to determine the EST (yellow box) and backward scan to determine the LST (blue box).

b



3 Subtract the earliest start time from the latest start time for each activity to calculate their float time.

c A: Float time = 3 − 0 = 3 days
B: Float time = 0 − 0 = 0 days
C: Float time = 18 − 9 = 9 days
D: Float time = 9 − 9 = 0 days
E: Float time = 16 − 16 = 0 days
F: Float time = 26 − 26 = 0 days

4 Look for activities with a zero float time to find the critical path. (Float = LST − EST = 0)

d The dummy is not included in the critical path. Activities with a float time of zero are in the critical path.

5 The critical path is highlighted in red in answer **b**.

The critical path of this project is B→D→E→F.

6 The minimum completion time is in EST of the end box.

e The minimum completion time of this project is 38 days.

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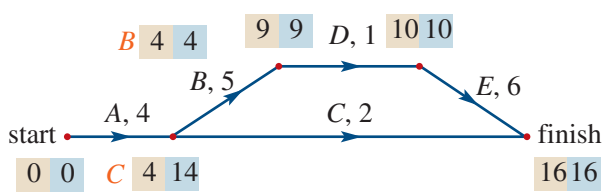
Exercise 10E

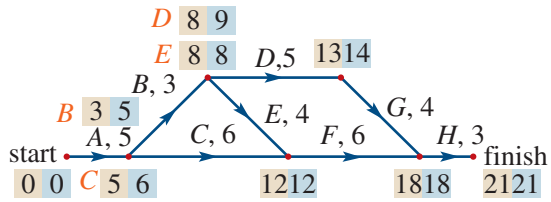
LEVEL 1

1 Find the float time for all activities (measured in minutes) in the network diagrams below.

a



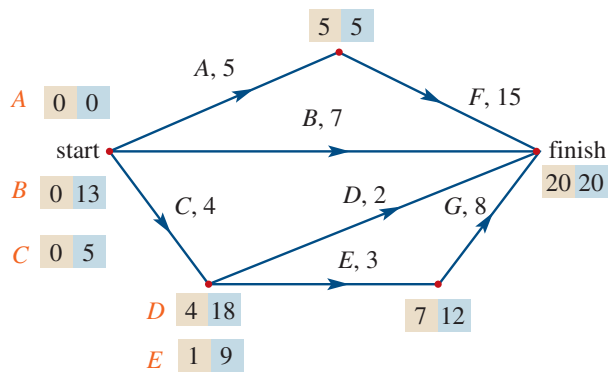
b



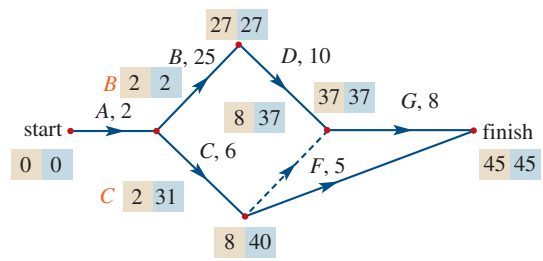
Example 9

2 Identify the critical path in the network diagrams below.

a



b



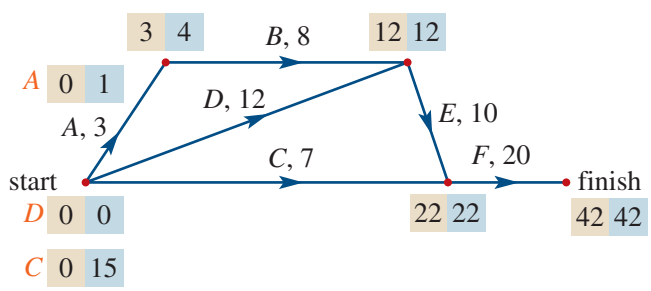
Example 10

3 The network diagram shows the time (in days) for all activities, EST and LST.

a Calculate the float times for all the activities.

b Write down the critical path for this project.

c What is the minimum time for the project to be completed?

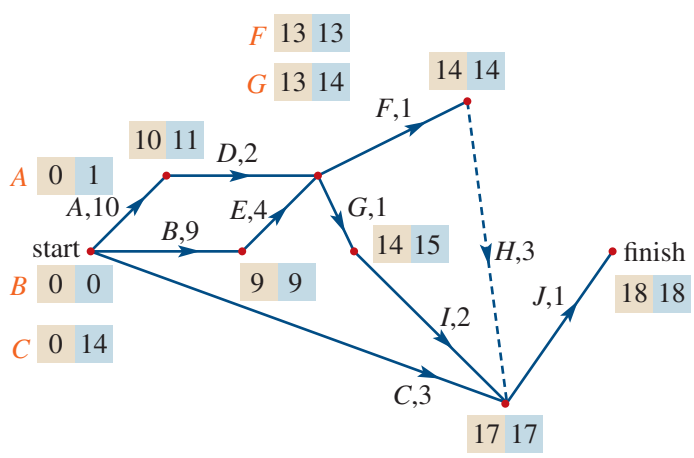


4 The network diagram shows the time (in hours) for all activities, EST and LST.

a Calculate the float times for all the activities.

b Write down the critical path for this project.

c What is the minimum time for the project to be completed?



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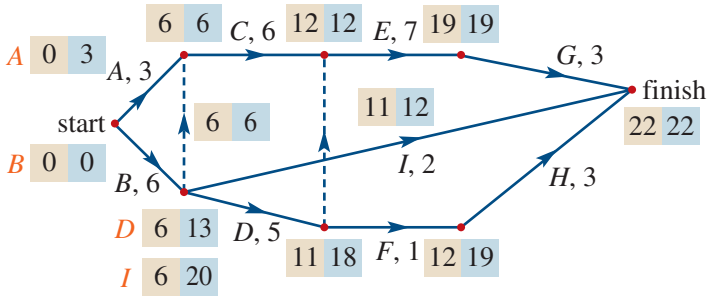
LEVEL 2

5 An incomplete activity chart and network diagram for a project are shown below.



Activity	Duration (in weeks)	Immediate predecessors
A	3	–
B	6	–
C	6	
D		B
E	7	
F	1	D
G		E
H	3	
I	2	B

a Complete the activity chart.
b Find the float times for all activities.
c What is the critical path of this project?
d What is the minimum completion time for the project?

LEVEL 3

6 The activity chart for a project is shown.

Activity	Duration (weeks)	Immediate predecessors
P	4	–
Q	5	–
R	12	–
S	3	P
T	6	Q
U	3	S
V	4	R
W	8	R, T, U
X	13	V
Y	6	W, X

a Draw a network diagram for this project.
b Complete the critical path analysis to calculate the EST and LST for each activity.
c Write down the critical path of this project.
d What is the minimum time required to complete the project?

7 The activity chart for a project is shown.

Activity	Duration (weeks)	Immediate predecessors
I	2	–
J	3	–
K	5	–
L	4	I
M	8	J, N
N	1	K
O	6	L, M
P	6	J, N
Q	7	J, N
R	5	K
S	1	O
T	9	Q, R

a Draw a network diagram for this project.
b Complete the critical path analysis to calculate the EST and LST for each activity.
c Write down the critical path of this project.
d What is the minimum time required to complete the project?

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10F Minimum completion time

The minimum time it takes to complete a project depends upon the time it takes to complete the individual activities of the project, and upon the predecessors for each activity. Critical path analysis can be completed to find the overall minimum completion time.

Completing activities in a shorter time

Sometimes, the managers of a project might arrange for one or more activities within the project to be completed in a shorter time than originally planned. Changing the conditions of an activity within a project, and recalculating the minimum completion time for the project, is sometimes called crashing. An individual activity could be crashed by employing more staff, sourcing alternative materials or simply because weather or other factors allow the activity to be completed in a shorter time than usual.

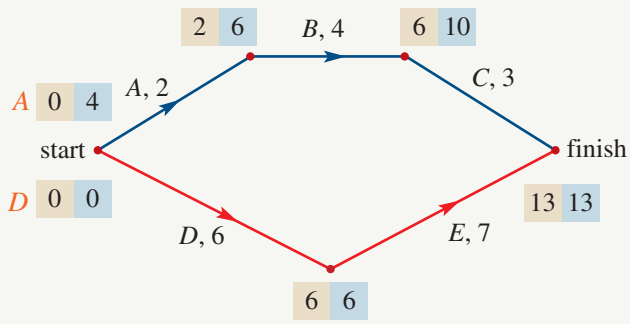
Example 11: Finding the minimum completion time 10F

The network diagram below shows the time (in hours) for all activities, EST and LST. The critical path is shown in red.

a What is the minimum completion time for the project?

b Which of these activities, if reduced in time individually, would not result in an earlier completion of the project?

c Activity D was reduced in time to 4 hours. What is the new minimum completion time?



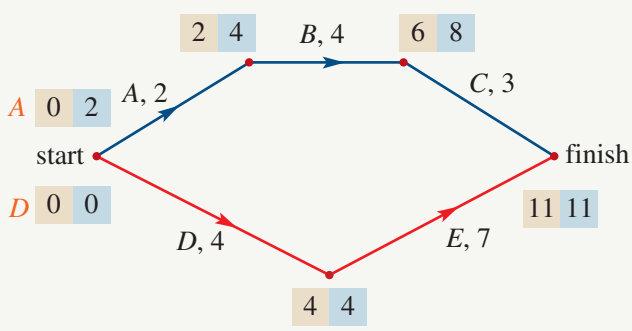
SOLUTION:

1. The minimum completion time is the EST of the end box.
2. The activities not on the critical path will not shorten the time taken to complete the project.
3. Decrease the weighted edge for activity D from 6 to 4 hours.
4. Recalculate the EST and LST for each activity.
5. The minimum completion time is the EST of the end box.

a The minimum completion time of this project is 13 hours.

b Activities A, B or C

c



The minimum completion time is now 11 hours.

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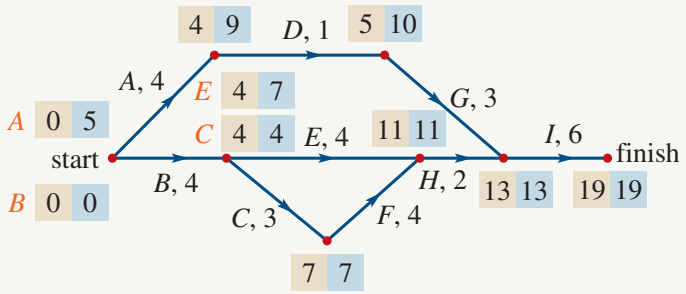
Example 12: Finding the minimum completion time 10F

A community centre is being built. Nine activities have been identified for a building project. The network diagram on the right shows the activities, completion times in weeks, ESTs and LSTs.



a What is the critical path and the minimum time, in weeks, to complete this project?

b The builders of the community centre are able to speed up the project. Activities A, C, F and G can be reduced in time at an additional cost. Which of these activities, if reduced in time individually, would not result in an earlier completion of the project?

c The owner of the estate is prepared to pay the additional cost to achieve early completion. The cost of reducing the time of each activity is \$5000 per week. The maximum reduction in time for each one of the five activities, A, C, F and G is 2 weeks. Find the minimum time, in weeks, for the project to be completed given these activities are reduced in time.

SOLUTION:

1 Look for activities with a zero float time to find the critical path.

2 The minimum completion time is the EST of the end box.

a Critical path is B → C → F → H → I

∴ Minimum completion time is 19 weeks.

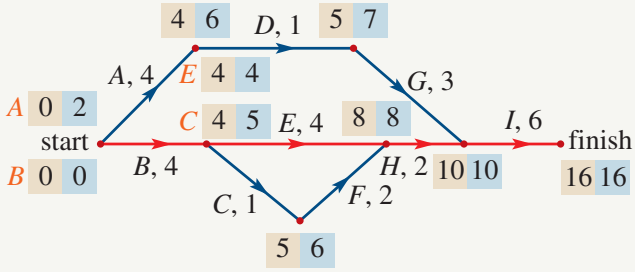
3 Look for activities not on the critical path. Reducing their time will not affect the overall time for the project.

b Activities A, G, D and E are not on the critical path. These activities would not result in an earlier completion of the project.

4 Activities C and F are on the critical path. Reduce the time of each activity by 2 weeks.

5 Recalculate the EST and LST.

c C can now be completed in 1 week.
F can now be completed in 2 weeks.
Update the network diagram (see below).



6 Check for change in the critical path.

7 Look for activities with a zero float time to find the critical path.

New critical path is B→E→ H→I
∴ Minimum completion time is now 16 weeks.

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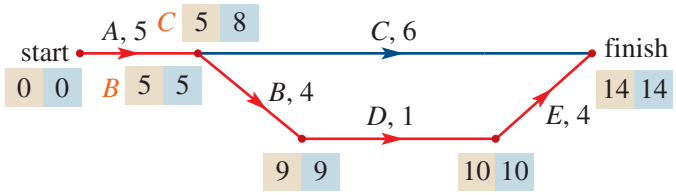
Exercise 10F

LEVEL 1

Example 11

1 The network diagram below shows the time (in days) for all activities, EST and LST. The critical path is shown in red.

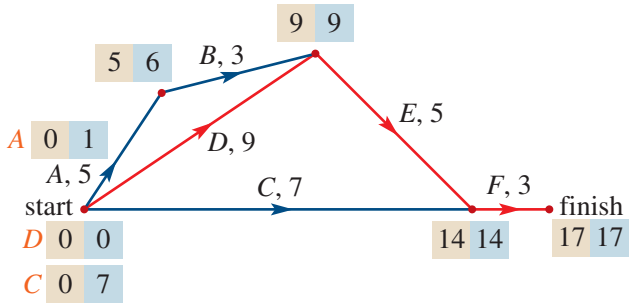


a What is the minimum completion time?

b Reducing the time taken in which activities would not result in a reduction in the minimum completion time?

c Activity A was reduced in time to 3 days. What is the new minimum completion time?

2 The network diagram below shows the time (in weeks) for all activities, EST and LST. The critical path is shown in red.

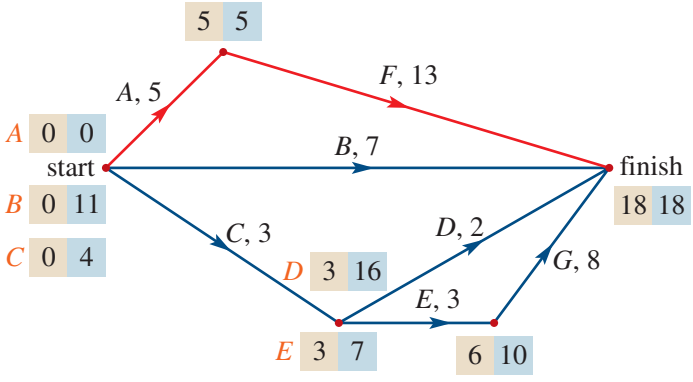


a What is the minimum completion time for the project?

b Reducing the time taken in which activities would not result in a reduction in the minimum completion time?

c Activity E was reduced in time to 1 hour. What is the new minimum completion time?

3 The network diagram below shows the time (in hours) for all activities, EST and LST. The critical path is shown in red.



a What is the minimum completion time for the project?

b Reducing the time taken in which activities would result in a reduction in the minimum completion time?

c Activity B was reduced in time to 5 hours. What is the new minimum completion time?

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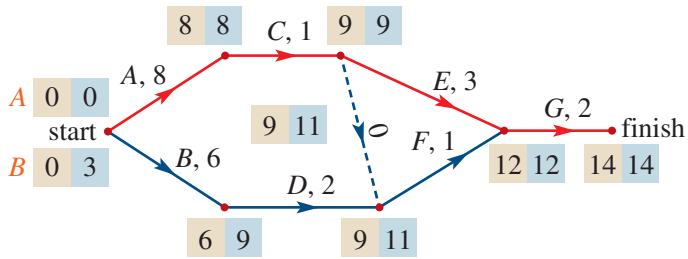
4 The network diagram below shows the time (in minutes) for all activities, EST and LST. The critical path is shown in red.



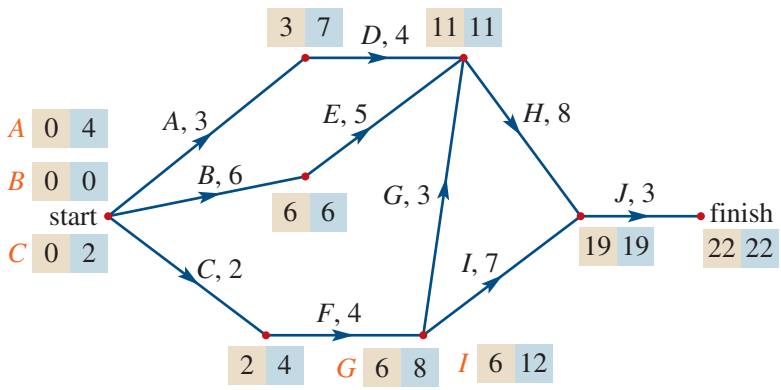
a What is the minimum completion time?

b Reducing the time taken in which activities would result in a reduction in the minimum completion time?

c Both activities A and E are reduced by 2 minutes. What is the new minimum completion time?

LEVEL 2

Example 12 **5** The network diagram for a project is shown in the diagram below. The duration for each activity is in hours.



a Identify the critical path for this project.

b What is the minimum completion time for the project?

c What is the maximum number of hours that the completion time for activity E can be reduced without changing the critical path of the project?

d What is the maximum number of hours that the completion time for activity H can be reduced without affecting the critical path of the project?

e If activity B is reduced from 6 hours to 4 hours, what is the minimum completion time for the project?

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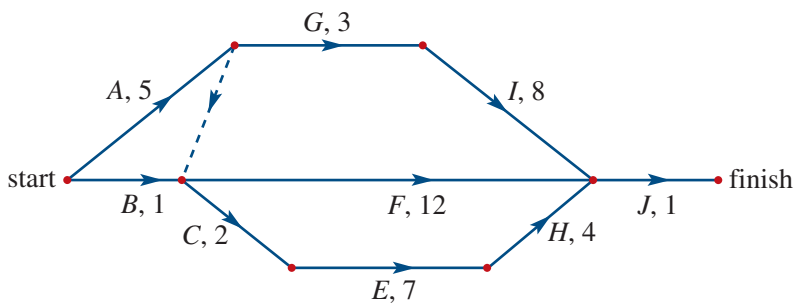
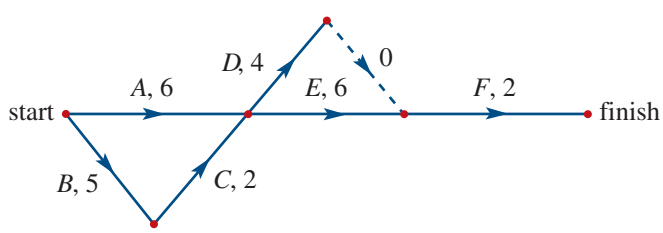
LEVEL 3

6 The network diagram for a project is shown in the diagram below. The duration for each activity is in hours.



a Find the earliest starting times (EST) and latest start times (LST) for all activities.
b Identify the critical path for this project.
c How much time overall is saved if activity F is reduced in duration by 2 hours?
d If only one activity can have its duration reduced, what is the maximum possible reduction in the completion time of the project?

7 The network diagram for a project is shown in the diagram below. The duration for each activity is in hours.



a Find the earliest starting times (EST) and latest start times (LST) for all activities.
b Determine the shortest time in which this project can be completed.
c Write down the critical path for this project.
d The time it takes to complete activity A can be reduced by one hour at a cost of $50. Explain why this will not affect the completion time of this project.
e Activity B can be reduced in time at a cost of $100 per hour. Activity E can be reduced in time at a cost of $50 per hour. What is the cost of reducing the completion time of this project as much as possible?

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10G Network flow problems

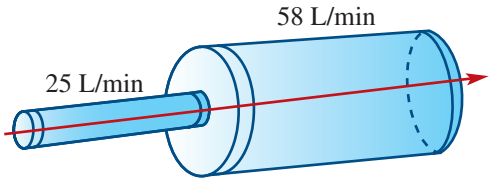
One of the applications of directed graphs to real-life situations is network flow problems which involve the transfer or flow of material from one point, called the source, to another point called the sink. Examples of this include water flowing through pipes, or traffic flowing along one-way roads.

source → flow through network → sink

The source is like the start, and the sink is like the finish, in the critical path analysis section.

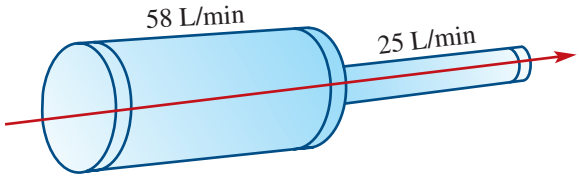
Water flows through pipes in only one direction. In a directed graph representing water flow, the vertices are the origin and destination of the water and the edges represent the pipes connecting them. The weights on the edges would be the amount of water that can flow through the pipe in a given time. The weights of flow problem directed graphs are called capacities.

The diagram on the right shows two pipes that are joined together, connecting the source of water to the sink. There is a small pipe with capacity 25 litres per minute joined to a large pipe with capacity 58 litres per minute. When an edge is at its maximum flow we say it is saturated.



Even though the large pipe has a capacity greater than 25 litres per minute, the small pipe will only allow 25 litres of water through each minute. The flow through the large pipe will never be more than 25 litres per minute. The large pipe will experience flow below its capacity. We say that it has an excess flow capacity, which is like the the float time from the critical path analysis section.

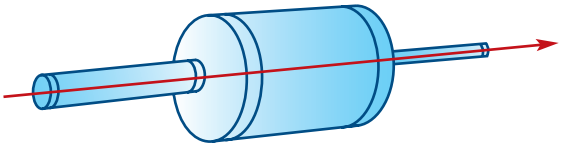
If we reverse the connection and direct water through the large capacity pipe into the smaller capacity pipe, there will be a 'bottleneck' of flow at the junction.



The large capacity pipe is delivering 58 litres of water every minute to the small pipe, but the small pipe will only allow 25 litres per minute to pass.

In both of these situations, the flow through the entire pipe system (both pipes from source to sink) is restricted to a maximum of 25 litres per minute. This is the capacity of the smallest pipe in the connection.

If we connect more pipes together, one after the other, we can calculate the overall flow capacity or maximum flow of the pipe system by looking for the smallest capacity pipe in that system.



MAXIMUM FLOW

If pipes of different capacities are connected one after the other, the maximum flow (or flow capacity) through the pipes is equal to the maximum capacity of the individual smallest pipe.

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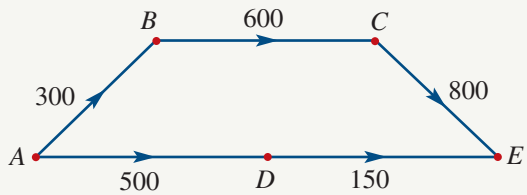
Example 13: Calculating the maximum flow 10G

In the directed graph shown on the right, the vertices A, B, C, D and E represent towns. The edges of the graph represent roads and the weights of these edges are the maximum number of cars that can travel on the road each hour. The roads allow only one-way travel.



a Find the maximum traffic flow from A to E through town C.

b Find the maximum traffic flow from A to E overall.

c A new road is being built to allow traffic from town D to town C. This road can carry 500 cars per hour.

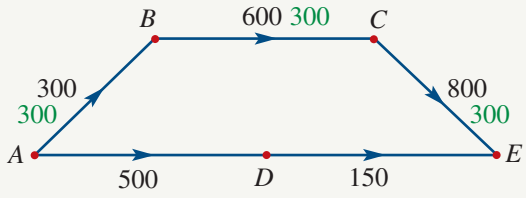
i Add this road to the directed graph.

ii Find the maximum traffic flow from A to E overall after this road is built.

SOLUTION:

1 Look at the path A→B→C→E. The smallest capacity of the individual roads is 300 cars per hour. This will be the maximum flow through this path. Note that at maximum flow, the path A→B is saturated, and path B→C and C→E have excess flow capacity.

a



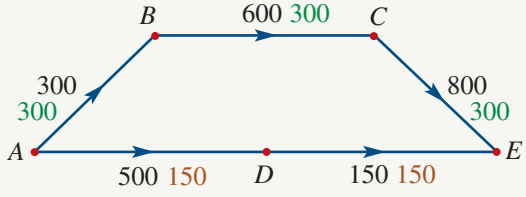
2 Write the maximum flow (300) for this path near the edge's weight.

3 Write answer in words.

The maximum flow is 300 cars per hour.

4 Look at the path A→D→E. The smallest capacity of the individual roads is 150 cars per hour. This will be the maximum flow through this path.

b



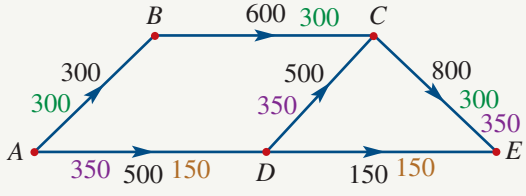
5 Write the maximum flow (150) for this path near the weight edge.

6 Add the maximum flow for path A→B→C→E and path A→D→E.

The maximum flow from A to E overall is: 300 + 150 = 450 cars per hour.

7 Add the edge to the network diagram.

ci



8 Look at the path A→D→C→E. The smallest capacity of the individual roads is 350 cars per hour. This will be the maximum flow through this path.

9 Write the maximum flow (350) for this path near the weight edge.

10 Add the maximum flow for path A→B→C→E, path A→D→E and path A→D→C→E.

ii The new maximum flow is:
300 + 150 + 350 = 800 cars per hour.

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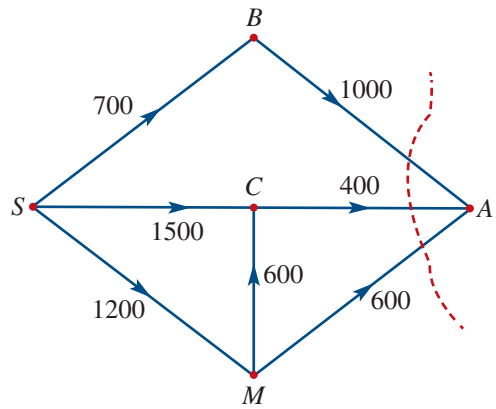
Cuts

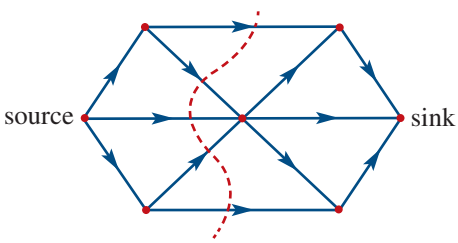
It is difficult to determine the maximum flow by inspection for directed networks that involve many vertices and edges. We can simplify the search for maximum flow by searching for cuts within the directed graph.

A cut divides the network into two parts, completely separating the source from the sink. It is helpful to think of cuts as imaginary breaks within the network that completely block the flow through that network. For the network or water pipes shown in this diagram, the dotted line is a cut. This cut completely blocks the flow of water from the source (S) to the sink (A).



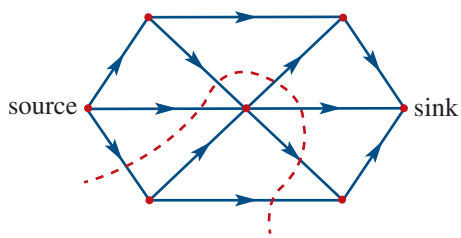
Cut

The dotted line on the graph below is a cut because it separates the source and the sink completely. No material can flow from the source to the sink.



Not a cut

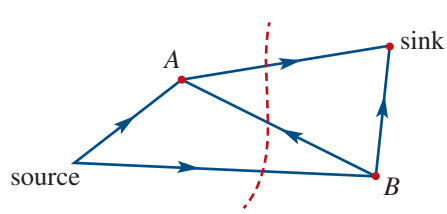
The dotted line on the graph below is not a cut because material can still flow from the source to the sink. Not all of the pathways from source to sink have been blocked by the cut.



CUTS

- A cut is an imaginary line across a directed graph that completely separates the source (start of the flow) from the sink (destination of the flow).
- Cut capacity is the sum of all the capacities of the edges that the cut passes through, taking into account the direction of flow. The capacity of an edge is only counted if it flows from the source side to the sink side of the cut.
- A cut is like a snapshot across the system of the flow at that time.

In the simple network shown, the cut passes through three edges. The edge B to A is not counted in the capacity of the cut because the flow for that edge is from the sink side to the source side of the cut.



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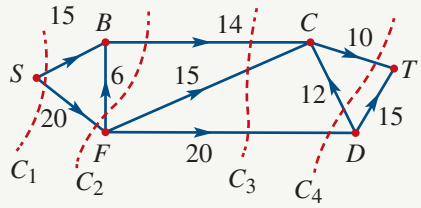
Example 14: Calculating the cut capacity 10G

Calculate the capacity of the four cuts shown in the network on the right. The source is vertex S and the sink is vertex T.



SOLUTION

1. All edges in C_1 are counted. The capacity of $C_1 = 15 + 20 = 35$.
2. In C_2 the edge from F to B is not counted. The capacity of $C_2 = 14 + 20 = 34$.
3. All edges in C_3 are counted. The capacity of $C_3 = 14 + 15 + 20 = 49$.
4. In C_4 the edge from D to C is not counted. The capacity of $C_4 = 20 + 10 = 30$.

Maximum-flow minimum-cut theorem

The capacity of a cut is important to help determine the maximum flow through any directed graph. Look for the smallest, or minimum, cut capacity that exists in the graph. This will be the same as the maximum flow that is possible through that graph. This is known as the maximum-flow minimum-cut theorem.

MAXIMUM-FLOW MINIMUM-CUT THEOREM

The flow through a network cannot exceed the value of any cut in the network and the maximum flow equals the value of the minimum cut.

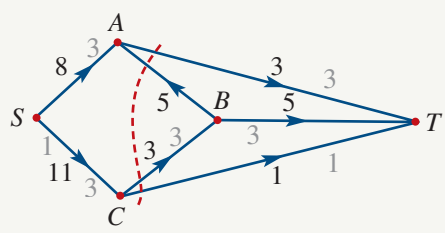
Example 15: Calculating the maximum flow 10G

The table shows the flow capacities (FC) of the edges of a network. Determine the maximum flow from S to T.

From	To	FC
S	A	8
S	C	11
A	T	3
B	A	5
B	T	5
C	B	3
C	T	1

SOLUTION:

1. Construct a network graph using the values in the cells as the flow capacity of the edges.
2. Mark the possible paths and their capacity (trial and error process). These are: Green — 3; brown — 3; purple — 1. Purple path has a capacity of 1.
3. Identify the minimum cut (red dotted line).
4. Find the minimum cut capacity. Minimum cut capacity $= 3 + 3 + 1 = 7$.
5. Find the maximum flow capacity. Maximum flow $= 3 + 3 + 1 = 7$.
6. Write the answer in words. Therefore the maximum flow is 7.



Note: The maximum flow must equal the minimum cut.

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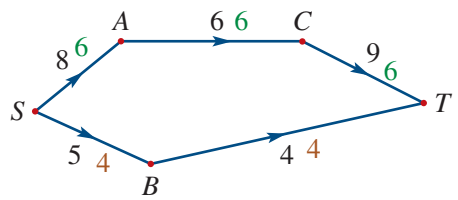
Exercise 10G

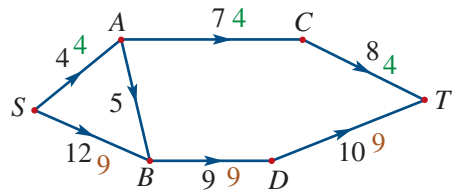
LEVEL 1

Example 13 **1** Find the maximum flow for each of the following network diagrams. Each colour on the weighted edge represents the flow capacity of a path from the source (S) to the sink (T).

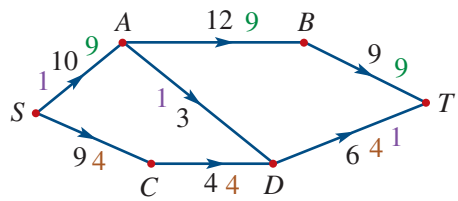
a



b



c



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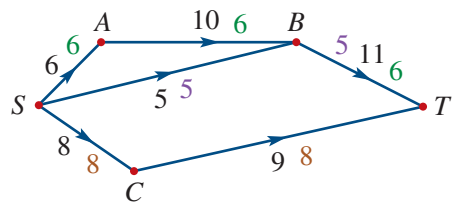

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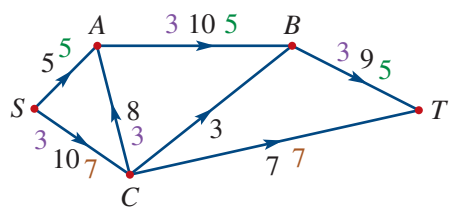

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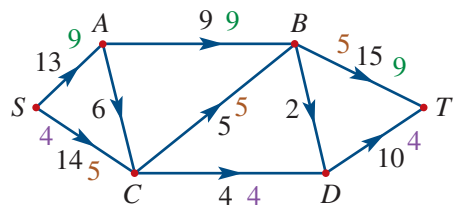
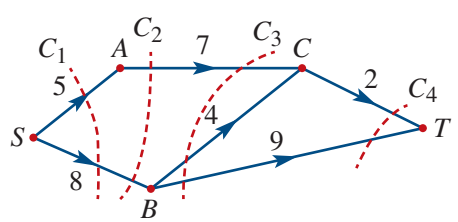


Example 14 **2** Find the capacity of each of the cuts in the following network diagrams. The source is vertex S and the sink is vertex T.

a



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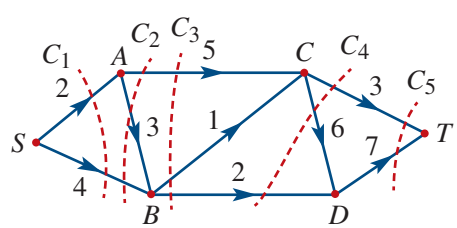

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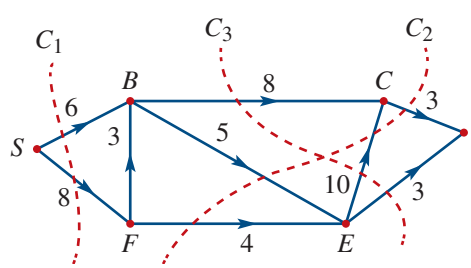

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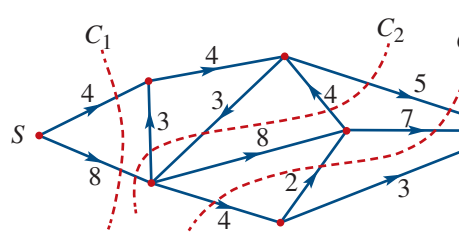

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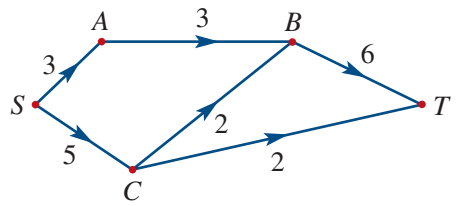
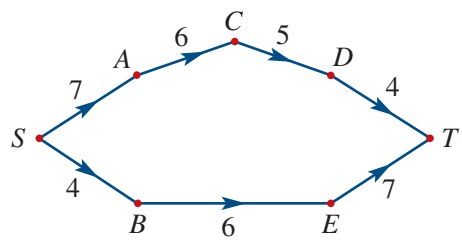
Example 15 **3** Find the maximum flow for each of the following network diagrams. The source is vertex S and the sink is vertex T. Check your answer by finding the minimum cut.

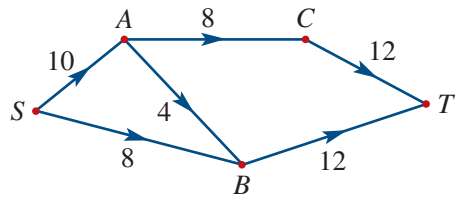
a



b



c



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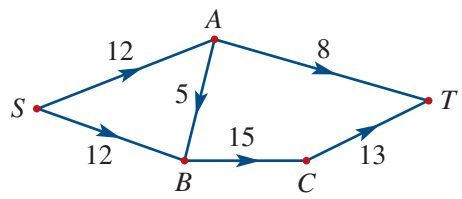

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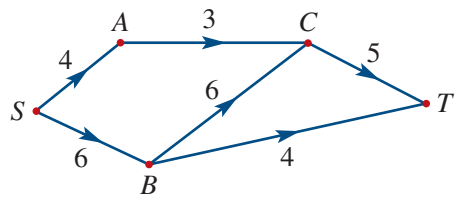

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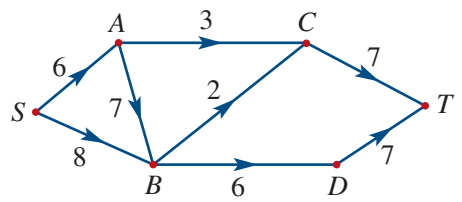

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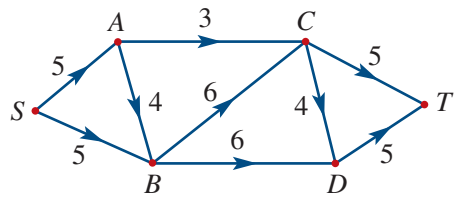

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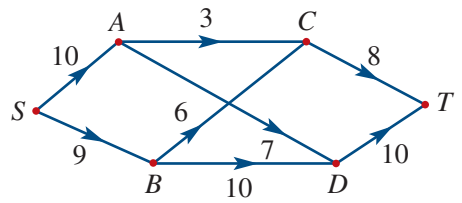

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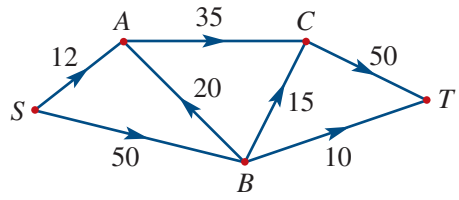

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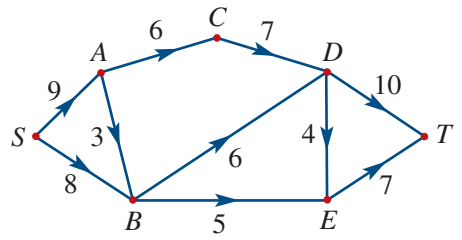

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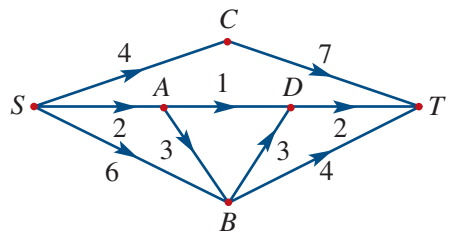

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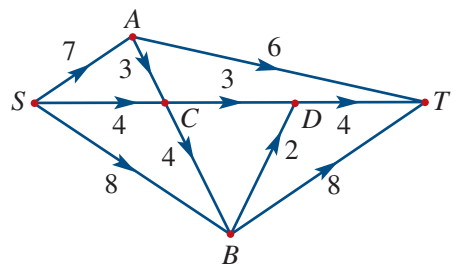

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LEVEL 2

4 The table opposite shows the flow capacities of the edges of a network.

a Construct a network diagram showing the direction and quantity of the flow. The source is vertex A and the sink is vertex E.

b Find the maximum flow through the network. Check your answer by finding the minimum cut.

c Find one example of the maximum flow through the network.

From	To	Capacity
A	B	5
B	C	4
C	D	2
C	E	2
D	E	1

5 The table opposite shows the flow capacities of the edges of a network.

a Construct a network diagram showing the direction and quantity of the flow. The source is vertex C and the sink is vertex H.

b Find the maximum flow through the network. Check your answer by finding the minimum cut.

c Find one example of the maximum flow through the network.

From	To	Capacity
C	D	9
C	E	9
D	G	7
D	F	3
E	F	6
E	G	4
F	H	9
G	H	9

6 The table opposite shows the flow capacities of the edges of a network.

a Construct a network diagram showing the direction and quantity of the flow. The source is vertex A and the sink is vertex E.

b Find the maximum flow through the network. Check your answer by finding the minimum cut.

c Find one example of the maximum flow through the network.

From	To	Capacity
A	B	14
B	C	8
B	D	6
C	E	6
C	F	4
D	F	5
E	G	7
F	G	8
G	H	14

7 The table opposite shows the flow capacities of the edges of a network.

a Construct a network diagram showing the direction and quantity of the flow. The source is vertex P and the sink is vertex V.

b Find the maximum flow through the network. Check your answer by finding the minimum cut.

c Find one example of the maximum flow through the network.

From	To	Capacity
P	Q	5
P	R	8
P	S	9
Q	R	5
Q	U	4
R	U	7
R	T	4
S	R	5
S	T	3
U	V	10
T	V	12

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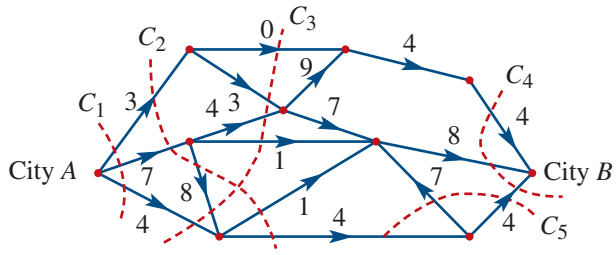
LEVEL 3

8 A plane journey consists of a connected sequence of stages formed by edges on the following directed network from city A to city B. The number of available seats for each stage is indicated beside the corresponding edge, as shown on the diagram on the right.



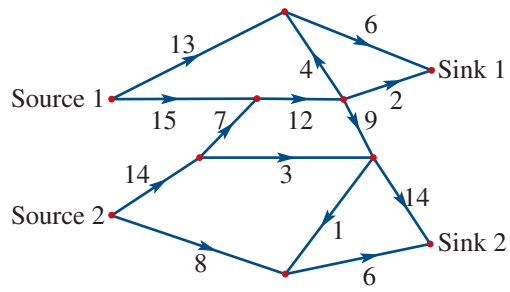
a What is the capacity of each of the five cuts shown?

b Explain why cut 5 is not a valid cut when trying to find the minimum cut between city A and city B.

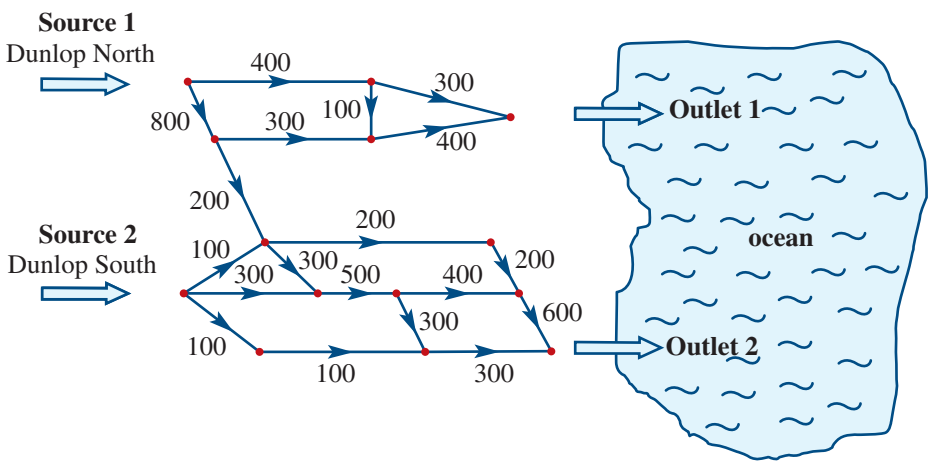
c Find the maximum number of available seats for a plane journey from city A to city B.

d Find one example of the maximum number of available seats for a plane journey from city A to city B.

9 Water pipes of different capacities are connected to two water sources and two sinks. Network of water pipes is shown in the diagram opposite. The numbers on the edges represent the capacities, in kilolitres per minute, of the pipes. Find the maximum flow, in kilolitres per minute, to each of the sinks in this network.



10 Storm water enters a network of pipes at either Dunlop North (Source 1) or Dunlop South (Source 2) and flows into the ocean at either Outlet 1 or Outlet 2. On the network diagram below, the pipes are represented by straight lines with arrows that indicate the direction of the flow of water. Water cannot flow through a pipe in the opposite direction. The numbers next to the arrows represent the maximum rate, in kilolitres per minute, at which storm water can flow through each pipe.



Determine the maximum rate, in kilolitres per minute, at which water can flow from these pipes into the ocean at Outlet 1 and Outlet 2.

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PPT STUDY GUIDE

Key ideas and chapter summary

Critical path analysis

Critical path analysis is a technique that identifies the activities required to complete a project, the time necessary to complete each activity and the relationships between the activities.

Activity chart

A table containing the activities of a project, duration and their immediate predecessors.

Constructing a network diagram given an activity chart

Activities that have no immediate predecessors follow from the start vertex. Activities that are not immediate predecessors for other activities lead to the finish vertex. For every other activity, look for:

- the activities for which it is an immediate predecessor
- the activities it has as immediate predecessors.

Dummy activities

A dummy activity is required if two activities share some, but not all, of their immediate predecessors.

Scheduling on a network diagram

Scheduling on a network diagram is shown using a weighted edge. The number on the edge indicates the duration of the activity. The weight of dummy activities is always zero.

Earliest starting time (EST)

The earliest time any activity can be started after all prior activities have been completed.

Latest starting time (LST)

The latest time any activity can be started after all prior activities have been completed.

Float times

Float time is the amount of time that a task in a project network can be delayed without causing a delay to subsequent tasks.

$$\text{Float time} = \text{LST} - \text{EST}$$

Critical path

The critical path is the sequence of activities that cannot be delayed without affecting the overall completion time of the project.

Minimum time

The minimum overall completion time of the project is the EST value at the end vertex.

Crashing

Crashing involves recalculating the minimum completion time for the project after the conditions of an activity within a project have changed.

Maximum flow

If pipes of different capacities are connected one after the other, the maximum flow (or flow capacity) through the pipes is equal to the minimum capacity of the individual pipes.

Cuts

A cut is an imaginary line across a directed graph that completely separates the source (start of the flow) from the sink (destination of the flow).

Maximum-flow minimum-cut theorem

The flow through a network cannot exceed the value of any cut in the network and that the maximum flow equals the value of the minimum cut.

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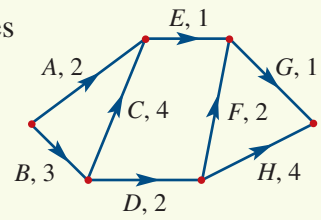
Review

Multiple-choice

1 The network diagram represents a manufacturing process with activities and their duration (in hours) listed on the edges of the graph. What is the earliest time (in hours) after the start that activity G can begin?



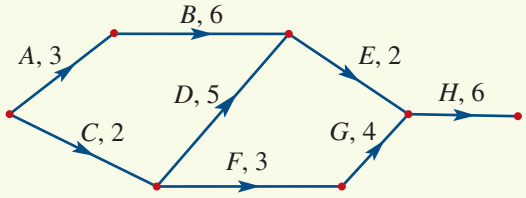
A 3 **B** 6 **C** 7 **D** 8

2 The table opposite lists the six activities in a project and the earliest start time, in hours, and the predecessor(s) of each task. The time taken for activity E is two hours. Without affecting the time taken for the entire project, how much could the time taken for activity C be increased by?

Activity	Immediate predecessors	EST
A	–	0
B	–	0
C	A	8
D	B	15
E	C	22
F	D, E	35

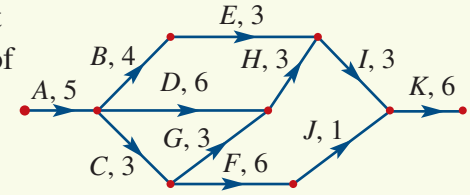
A 0 hours **B** 8 hours
C 11 hours **D** 22 hours

3 The edges in this network diagram correspond to the tasks involved in the preparation of an examination. The numbers indicate the time, in weeks, needed for each task. What is the minimum completion time to prepare this examination?



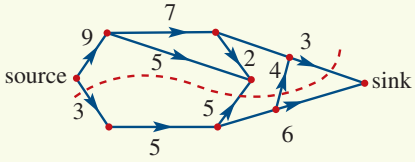
A 14 **B** 15 **C** 16 **D** 17

4 This network diagram is for a project where the component times are shown in days. The critical path for the network of this project is given by:



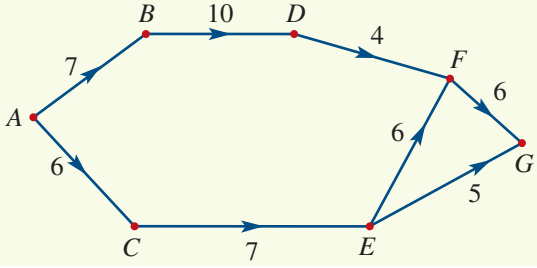
A A–B–E–I–K **B** A–D–H–I–K
C A–C–G–H–I–K **D** A–C–F–J–K

5 For the network shown on the right, the capacity of the cut is:



A 3
B 7
C 10
D 15

6 The maximum flow in the network diagram opposite is:



A 10
B 11
C 12
D 13

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Short-answer

1 Jake is renovating his family room. He has organised this project into six tasks.

Buy hardware –1 day
Prepare walls – 1 day
Paint walls – 4 days
Sand timber floor – 3 days
Seal floor – 2 days
Put furniture in the room – 1 day.

a Which task has no immediate predecessor? Give a reason for your answer.

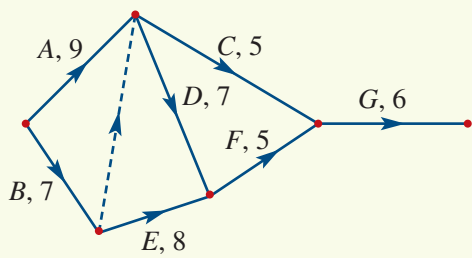
b What is the immediate predecessor to the following tasks?

i Buy hardware
ii Prepare walls
iii Paint walls
iv Sand timber floor
v Seal floor
vi Put furniture in the room

c Construct an activity chart for this project. Use the headings; Activity, Task, Duration (in days) and Immediate predecessors. Label the above tasks as activities A to F respectively.

d Draw a network diagram from this activity chart.

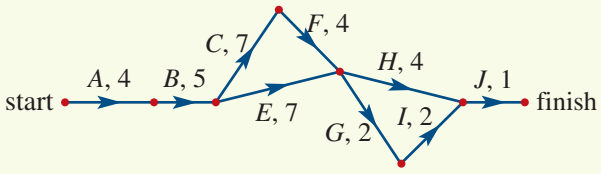
2 Draw an activity chart for the network diagram shown opposite.



3 Jet 12 makes light aircraft for the civil aviation industry. They have identified 10 activities required for production of their new aircraft. These activities and their durations are given in the table opposite.

Activity	Duration (weeks)	Immediate predecessors
A	4	–
B	5	A
C	7	B
D	6	B
E	7	B
F	4	C
G	2	E, F
H	4	E, F
I	2	D, G
J	1	H, I

a Copy and complete the network diagram shown below for this project by inserting and labelling activity D.



b Find the earliest starting times (EST) for all activities.

c Find the latest starting times (LST) for all activities.

d What is the critical path for this network?

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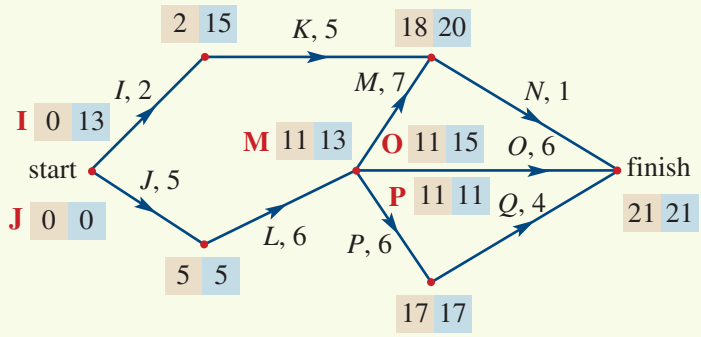
Review

4 A network diagram for project is shown. The duration for each activity is in hours.

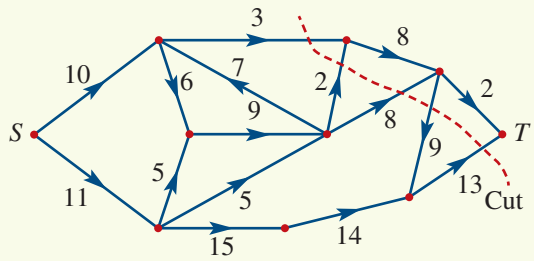


a Calculate the float times for all the activities.

b Identify the critical path for this project.

c What is the minimum completion time for the project?

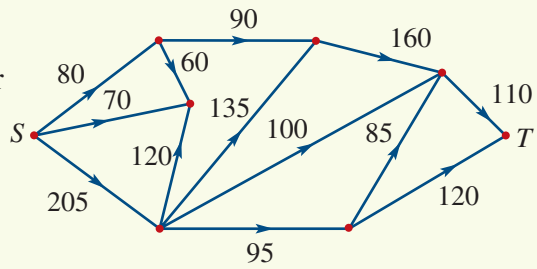
5 In the network opposite, the values on the edges give the maximum flow possible between each pair of vertices. The arrows show the direction of flow in the network. Also shown is a cut that separates the source (S) from the sink (T).



a Find the capacity of the cut shown.

b Determine the maximum flow through this network.

6 Determine the maximum flow from S to T for the network graph shown on the right. Check your answer by finding the minimum cut.



Extended-response

7 A development project involves completing a number of activities as shown in the table. Each activity is shown with its duration. Time is measured in days.

Activity	Duration (days)	Immediate predecessors
A	4	–
B	2	–
C	1	A
D	6	B
E	5	B
F	7	B
G	5	C, D
H	1	E
I	2	F
J	10	G, H
K	6	I

a Construct a network diagram for this project, labelling the activities on the edges with their associated shortest durations.

b Determine the earliest start time for each activity from your graph.

c Determine the latest start time for each activity from your graph.

d State the critical path.

e How long is the estimated project time under this set of activity durations?

f If the final activity, K, had to be delayed, how many days could this delay take before the project schedule was disrupted?

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