U-CONTROL CHART SELECTION FLOW CHART

Which control chart?

Is it variables data?

—Yes→

Subgroup size = 1?

—Yes→ Use X-MR chart, page 93.

No↓

Use X̄R-chart, page 73.

No (attributes data)↓

Can the data being collected be larger than the subgroup?

Yes (aspects)↓ — No (items)

Equal sized subgroups?

Yes↓ — No→

Use c-chart, page 157

or u-chart, page 169.

Use u-chart, page 182.

Equal sized subgroups?

←Yes — No→

Use np-chart, page 111

or p-chart, page 125.

Use p-chart, page 138.

WHAT IS IT?

A "u-chart" is a control chart for a particular type of attributes data. The role of any control chart is to monitor how a system changes over time. The attributes data plotted on a u-chart typically monitors the number of nonconformities produced. A nonconformity can be anything present on an item that should not be there, or alternatively, features that should be present, but are not. The number of errors on invoices, the number of scratches or blemishes found on a car window, the number of areas not completed on application forms, could all be plotted on a u-chart. This type of attributes data is defined as having no maximum number to the number of potential nonconformities. If ten invoices are checked for errors, there is no limit to the number of errors that can potentially be found. The data is not constrained by the sample size, unlike other types of attributes data. Unlike the *c-chart*, the u-chart can deal with varying sample sizes. The u-chart with a constant sample size is presented here, and with a varying sample size later in this section.

WHAT DOES IT LOOK LIKE?

A completed example of a u-chart is shown below. In this case, the subgroup size is constant. This chart monitors the quality of radiator grills. Each subgroup or sample represents two radiator grills. This is the same example used in the *c-chart*, converted into a u-chart.

Header information

ATTRIBUTES CONTROL CHART

Product / Service	Chart type		Quality measure	Measurement device
Radiator grill	☐ p ☐ c	☐ np ■ u	Visual concerns	Eyes

User name	Location						Chart no.
Elaine Watson	Injection molding press	AVG. = 0.96	UCL = 3.04	LCL = 0	$\bar{n}$ = 2		

Upper control limit u

Overall average ($\bar{u}$)

Discrepancy tally

Data

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
DATE	Sep 1					Sep 2					Sep 3					Sep 4					Sep 5				
TIME	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00

Type of discrepancy

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1. Black spot		l	l				l		l			l		l		ll			l		l			l	
2. Scratch	l	ll		l		ll			ll	l		ll		ll		l	l	ll		l	l				l
3. Splay								ll					l												
4. Sink											l					l			l						
5. Flow line	l							l						l							l				
6. Other																		l							
7.																									
8.																									
9.																									
10.																									

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Total number (np,c)	2	4	3	1	0	3	1	4	0	2	2	3	0	2	4	0	3	2	4	1	3	0	1	2	1
Proportion (p,u)	1.0	2.0	1.5	0.5	0.0	1.5	0.5	2.0	0.0	1.0	1.0	1.5	0.0	1.0	2.0	0.0	1.5	1.0	2.0	0.5	1.5	0.0	0.5	1.0	0.5
Subgroup size (n)	2 →																								

u-control chart

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◔ When is it used?

Use a u-chart with a constant sample size when you can answer "yes" to all of the following questions:

1. **Do you need to assess the variability in the system?**
   All control charts are used to assess whether the variation in a process is generated by special or common causes. Control charts are maintained on an ongoing basis in order to minimize the chance of reacting inappropriately to the variation by either overcontrolling or undercontrolling the system or process.

2. **Can the data be collected or does a collection of data already exist?**
   The data must be generated on an ongoing basis. Control charts analyze a series of data points, not a single data point.

3. **Is the time order of the data preserved?**
   Since control charts are designed to study the behavior of a system over time, it is critical that the data is recorded in the order it is collected and produced. Mixing the order of the data would be like editing a movie film with no attention given to the sequence of events.

4. **Is the data in attributes format?**
   Usually this data comes from a process whose aim is to judge whether a condition exists. Do parts have defects? Are there any errors in the invoices? Is information missing on the forms? Data that is measured using a measuring instrument, for example, weight or height are "variables," and different types of charts are used.

5. **Can the data value exceed the size of the subgroup?**
   For example, if 50 parts are being checked and they either pass or fail the check, then the maximum number of parts that can pass or fail is 50, the subgroup size. In this case an *np-* or *p-chart* is used, NOT a u-chart. If, however, the check involves counting the number of faults found on the 50 parts, then more than 50 faults could be found, and a *u-chart* is used.

6. **Are all the subgroups the same size?**
   A u-chart can be used with a constant or varying sample size. The u-chart with a constant sample size is presented here, and with a varying sample size later in this section.

7. **Has the characteristic to be charted been operationally defined?**
   Operational definitions are important when collecting attributes data. Each data collector must have the same idea of what makes a nonconformity so the results will be consistent. Without an operational definition data is unreliable and the chart may be meaningless. The operational definition must be set prior to data collection.

⚒ How is it made?

These steps assume that the relevant data has been collected. The u-chart is constructed using the following steps:

1. Complete the header information.

Complete the header information at the top of the chart. Header information includes such items as product/service, user name, chart type (in this case a u-chart), location, quality measure, and measurement device. It is important that this information is completed, so that others can easily read and understand the chart.

2. Record the data.

Record the data on the control chart paper either from the data collection sheet or directly onto the chart paper as the data is collected. Fill in the row marked "Total number" with the number of occurrences of faults or nonconformities. This is known as the c for each subgroup. In the example, the c is the number of faults found on the radiator grills. Fill in the row marked "Subgroup size." Since the subgroup is constant in this chart, it is not necessary to write this number in every box. Write it in the first box on the chart paper and draw an arrow indicating that it is the same for all the subgroups. It is important to maintain the data in the order it was produced. Do not rearrange the order of the subgroups, as this will hide any trends in the data.

Attributes control chart paper has a section labelled "Type of discrepancy." In this section, list the types of faults found in each subgroup during data collection.

During data collection, record any significant changes or observations on the chart paper or on a separate process log (often located on the back of the chart paper) as they occur. These notes help interpret the control chart.

There are two rows in which to record sample identifiers; they are titled "Date" and "Time." Do not be limited by these titles. Use them to record not only the date and time, but also other useful identifiers, such as location or operator.

The example u-chart with completed header information and recorded data follows:

ATTRIBUTES CONTROL CHART

Product / Service					Chart type		□p □c	□np ■u		Quality measure					Measurement device			
Radiator grill										*Visual concerns*						*Eyes*		

User name			Location			AVG. =		UCL =		LCL =		$\bar{n}$ =			Chart no.	
Elaine Watson			*Injection molding press*													

DATE	Sep 1					Sep 2					Sep 3					Sep 4					Sep 5				
TIME	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Type of discrepancy

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1. Black spot		I	I					II		I		I		I		I			II		I			I	
2. Scratch	I	II	I		I			II			II	I		II		II			I		II		I		I
3. Splay						II								I				I							
4. Sink		I					I	I					I					II	I						
5. Flow line	I		I								I			I				I			I			I	
6. Other								I						I											
7.																									
8.																									
9.																									
10.																									
Total number (np, c)	2	4	3	1	0	3	1	4	0	2	2	3	0	2	4	0	3	2	4	1	3	0	1	2	1
Proportion (p, u)																									
Subgroup size (n)	2 →																								

3. Calculate the number of faults per unit (u) for each subgroup.

In order to complete the control chart, the number of faults or defects per unit must be calculated. The faults per unit, known as u, is found by dividing the "Total number," c, by the "Subgroup size," n.

$$u = \frac{c}{n}$$

So, for the first subgroup the value of u is:

$$u_1 = \frac{2}{2}$$
$$= 1.0$$

The value of u for the second subgroup is:

$$u_2 = \frac{4}{2}$$
$$= 2.0$$

The value for u should be recorded so there are enough decimal places in the u values to see differences between the values. Often no decimal places are required. However, when recording small u values, two or more decimal places may be required. In the example, one decimal place is sufficient.

Continue calculating the u values for each subgroup. Enter the u values in the row marked "Proportion" on the control chart paper.

4. CALCULATE THE OVERALL AVERAGE.

The average proportion of failures or nonconformities is known as ▯. It is found by adding all the proportions (u's) and dividing by the number of subgroups, known as k.

$$\bar{u} = \frac{\text{total proportions}}{\text{number of subgroups}}$$

$$= \frac{\Sigma u}{k}$$

In the sample $\bar{u}$ is:

$$\bar{u} = \frac{1.0 + 2.0 + 2.5 + \ldots + 0.5}{25}$$

$$= \frac{24}{25}$$

$$= 0.96$$

The value for $\bar{u}$ should be recorded to one more decimal place than the u values. The value for $\bar{u}$ is recorded on the control chart in the header information in the space marked "AVG."

5. CALCULATE THE CONTROL LIMITS.

Control limits are lines drawn on the chart, and are used when interpreting the data. These limits come from variation in the system, not from specifications or requirements. Ideally, at least 25 subgroups should be used to calculate the control limits. However, the more subgroups available, the more accurate the control limits will be. If fewer than 25 subgroups are available, consider the limits as "trial" limits, and recalculate them after enough data has been collected.

The symbol for the upper control limit is UCL_u, and the symbol for the lower control limit is LCL_u. The formula for the upper control limit is:

$$UCL_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

So, UCL$_u$ for the example is:

$$UCL_u = 0.96 + 3 \sqrt{\frac{0.96}{2}}$$
$$= 0.96 + 3 \, (0.69282)$$
$$= 0.96 + 2.078$$
$$= 3.04$$

Control limits should be recorded to one more decimal place than the u values. The formula for the lower control limit is:

$$LCL_u = \bar{\bar{u}} - 3 \sqrt{\frac{\bar{\bar{u}}}{n}}$$

So, LCL$_u$ for the example is:

$$LCL_u = 0.96 - 3 \sqrt{\frac{0.96}{2}}$$
$$= 0.96 - 2.078$$
$$= -1.118 \approx 0$$

Note that the equation for the lower control limit is virtually the same as that for the upper control limit. If the lower control limit turns out to be negative, as in the example, then the limit has no meaning and is not used.

Record the values for the control limits in the spaces marked "UCL" and "LCL" at the top of the chart.

6. SCALE THE CONTROL CHART.

The term "scaling" refers to numbering the lines on the left-hand side of the chart. The chart should be numbered (scaled) evenly so the values can be plotted conveniently, and the resulting chart is uncluttered and easy to read. Scaling can be the most difficult step in constructing a control chart. It is important to understand that the scale for the chart is not determined solely by the calculations performed; it is equally decided by considering ease of plotting and readability of the final chart. The calculations should be seen as a guide in determining the final scale.

Begin by finding the largest u value from the "Proportion" row on the chart. Compare this value to the upper control limit and write down the larger of the two. In the example, the largest u value is 2.0 and the upper control limit is 3.04. So, the larger value is 3.04.

Next, divide the larger value by two-thirds of the total number of lines on the control chart. (Use only two-thirds of the available lines in order to leave space outside the control limits to plot future shifts.) This calculation will provide the increment value for each line on the control chart. In the example, there are 30 lines on the chart paper, so use 20 lines (two-thirds) to find the increment value.

The increment value is: $\dfrac{3.04}{20} = 0.152$

Round the increment value upward as necessary to get an easy-to-work-with number, such as 0.1, 0.2, 0.5, 1, 2, 5, or 10. In the example, 0.152 was round to 0.2. This means that each line represents 0.2.

To scale the chart, label the bottom line as zero and move upward from zero, adding the increment value. Label each dark reference line. In the example, the dark reference lines were labelled 0, 1.0, 2.0, 3.0, 4.0, 5.0, and 6.0. The example completed through this step follows:

ATTRIBUTES CONTROL CHART

Product / Service	Radiator grill		Chart type	☐p ☐c	☐np ■u	Quality measure	Visual concerns		Measurement device	Eyes

User name	Elaine Watson		Location	Injection molding press	AVG. = 0.96	UCL = 3.04	LCL = 0	n̄ = 2	Chart no.

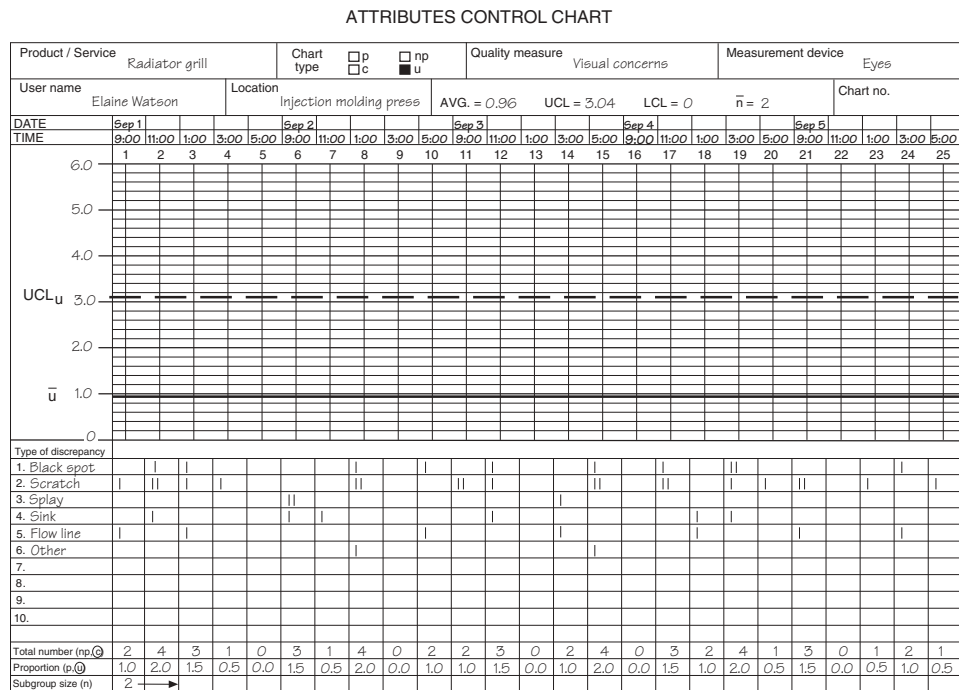| DATE	Sep 1					Sep 2					Sep 3					Sep 4					Sep 5				
TIME	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

(Chart y-axis labels: 6.0, 5.0, 4.0, 3.0, 2.0, 1.0, 0)

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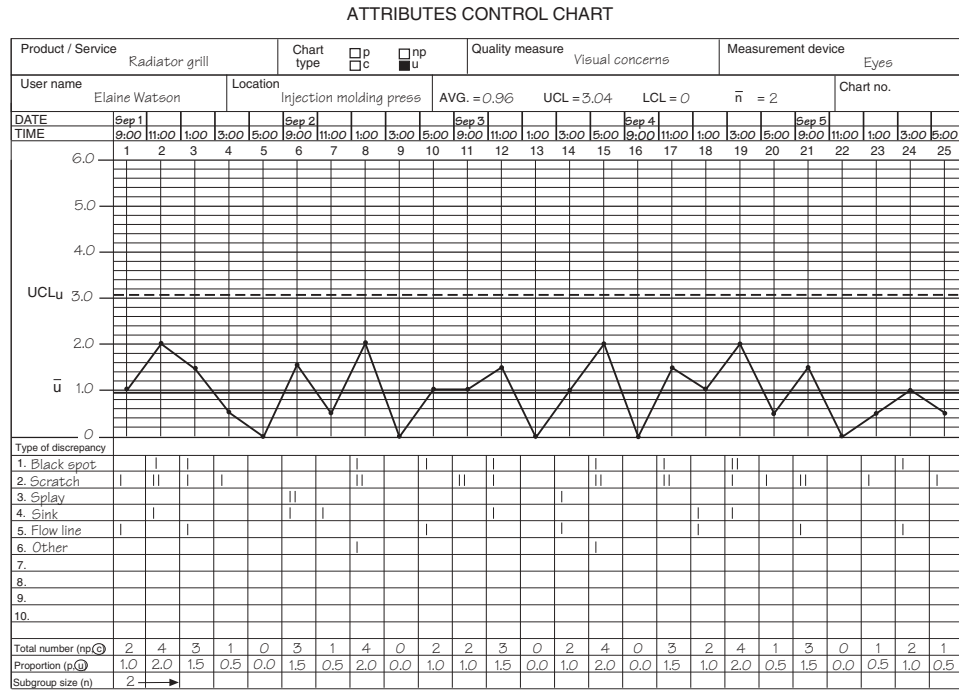u-control chart

7. DRAW THE AVERAGE LINE AND CONTROL LIMITS.

Draw a solid dark horizontal line on the chart to represent $\bar{u}$. Label the line with $\bar{u}$.

Draw dashed horizontal lines to represent the upper and lower control limits. Label each control limit with the correct symbol, UCL_u and LCL_u. The completed example follows:

ATTRIBUTES CONTROL CHART

Product / Service	Radiator grill		Chart type	☐ p ☐ c	☐ np ■ u	Quality measure	Visual concerns			Measurement device	Eyes	
User name	Elaine Watson		Location	Injection molding press		AVG. = 0.96	UCL = 3.04	LCL = 0	$\bar{n}$ = 2		Chart no.	

DATE	Sep 1					Sep 2					Sep 3					Sep 4					Sep 5				
TIME	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

(Chart plotting area with horizontal grid from 0 to 6.0; UCL_u at 3.0 and $\bar{u}$ at 1.0)

Type of discrepancy																									
1. Black spot		I	I					I		I		I			I		I		II		II			I	
2. Scratch	I	II	I		I			II		II	I			II		II			I	I	II		I		I
3. Splay						II						I													
4. Sink		I				I		I					I						I	I					
5. Flow line	I		I						I			I			I			I			I			I	
6. Other								I					I			I									
7.																									
8.																									
9.																									
10.																									

Total number (np,c)	2	4	3	1	0	3	1	4	0	2	2	3	0	2	4	0	3	2	4	1	3	0	1	2	1
Proportion (p,u)	1.0	2.0	1.5	0.5	0.0	1.5	0.5	2.0	0.0	1.0	1.0	1.5	0.0	1.0	2.0	0.0	1.5	1.0	2.0	0.5	1.5	0.0	0.5	1.0	0.5
Subgroup size (n)	2 →																								

8. PLOT THE U VALUES ON THE CHART.

Starting with the first subgroup, plot each u value on the control chart. Connect the dots with straight lines. The completed example follows:

ATTRIBUTES CONTROL CHART

Product / Service	Radiator grill		Chart type	☐p ☐c	☐np ■u	Quality measure	Visual concerns		Measurement device		Eyes

User name: Elaine Watson
Location: Injection molding press
AVG. = 0.96   UCL = 3.04   LCL = 0 $\bar{n}$ = 2
Chart no.

DATE	Sep 1					Sep 2					Sep 3					Sep 4					Sep 5				
TIME	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00	9:00	11:00	1:00	3:00	5:00
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Type of discrepancy

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1. Black spot		I	I					I		I				I		I		II		I		II		I	
2. Scratch	I	II	I	I			II		II	I			II		II		I		I		I		I		I
3. Splay						II								I											
4. Sink		I				I	I									I	I								
5. Flow line	I		I					I				I				I		I			I				
6. Other							I							I											
7.																									
8.																									
9.																									
10.																									

Total number (np/c)	2	4	3	1	0	3	1	4	0	2	2	3	0	2	4	0	3	2	4	1	3	0	1	2	1
Proportion (p/u)	1.0	2.0	1.5	0.5	0.0	1.5	0.5	2.0	0.0	1.0	1.0	1.5	0.0	1.0	2.0	0.0	1.5	1.0	2.0	0.5	1.5	0.0	0.5	1.0	0.5
Subgroup size (n)	2 →																								

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9. Interpret the control chart.

The u-chart is interpreted using the basic rules for control chart interpretation. These are explained fully in "Control chart interpretation" at the end of this section. However, a list of the basic rules for interpreting out-of-control conditions is shown below.

1. Any point lying outside the control limits.

2. Run of seven points:
   a. Seven or more points in a row above or below the center line.
   b. Seven or more points in a row going in one direction, up or down.

3. Any nonrandom pattern, including the following typical cases:
   a. Too close to the average.
   b. Too far from the average.
   c. Cycles.

The example u-chart shows that the faults on the radiator grills are stable, with no special causes of variation. The system can be improved only by reducing the common causes of variation in the radiator grill production process.

When no special causes are evident, as in the example, systems are declared to be stable. The control limits calculated previously are simply drawn onto new chart paper, and as subgroups continue to be collected, they are plotted on the chart. This will allow the user to make decisions about the system's stability as each new subgroup is plotted. New control limits are calculated only when a change is made to the system.

Care must be taken when interpreting a u-chart. Sometimes the data will not behave as expected. In this case, the control limits will look odd in comparison to the data, that is, too wide or too narrow. If this occurs, change the type of control chart to an ***X-MR chart***.

The control limits of attributes charts can be compared directly to specifications or requirements, enabling the user to evaluate the capability of the system to meet the requirements. In the example, the control limits read UCL of 3.04, and $\bar{u}$ of 0.96. This information shows that the radiator grill production process produces many faults. The chart shows that there is an average of one fault per radiator grill, obviously a major problem. The challenge is to reduce that number consistently.

See the notes, "Getting the most from control charts," at the beginning of this section for more information on how to use and benefit from this powerful tool.

APPLICATIONS

The applications for the u-chart with a constant sample size are the same as the *c-chart*.

u-Control chart selection flow chart
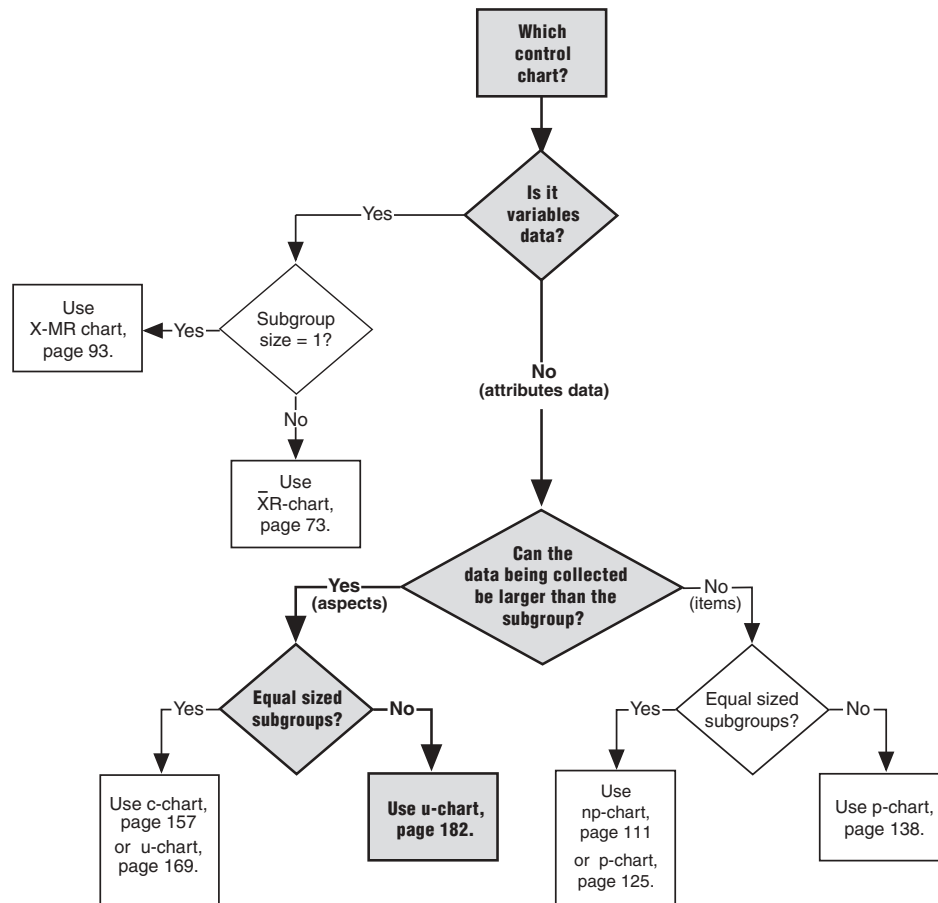
Which control chart?

Is it variables data?
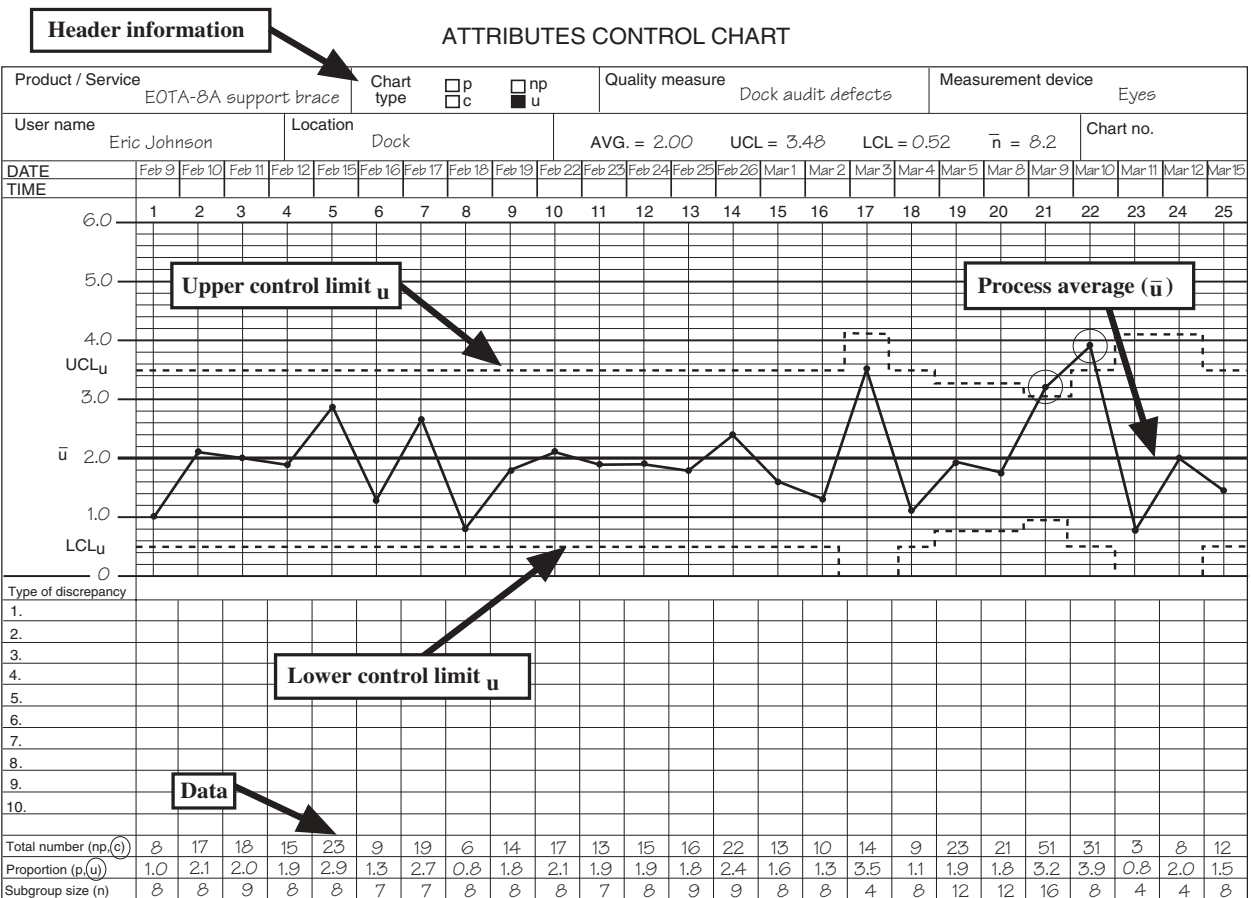
— Yes →

Subgroup size = 1?

— Yes → Use X-MR chart, page 93.

No ↓

Use $\overline{X}$R-chart, page 73.

No (attributes data) ↓

Can the data being collected be larger than the subgroup?

Yes (aspects)

No (items)

Equal sized subgroups?

Yes → Use c-chart, page 157 or u-chart, page 169.

No → **Use u-chart, page 182.**

Equal sized subgroups?

Yes → Use np-chart, page 111 or p-chart, page 125.

No → Use p-chart, page 138.

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What is it?

A "u-chart" is a control chart for a particular type of attributes data. The role of a control chart is to monitor how a system changes over time. The attributes data plotted on a u-chart typically monitors the number of nonconformities produced. A nonconformity can be anything present on an item that should not be there, or alternatively, features that should be present but are not. The number of errors on invoices, the number of scratches or blemishes found on a car window, the number of areas not completed on an application form, can all be plotted on a u-chart. This type of attributes data is defined as having no limit to the number of potential nonconformities. If ten invoices are checked for errors, there is no maximum to the number of potential errors that can be found. The data is not constrained by the sample size, unlike other types of attributes data. Unlike the *c-chart*, the u-chart can deal with a varying sample size.

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WHAT DOES IT LOOK LIKE?

Shown below is an example of a u-chart with a varying subgroup size. Notice how the control limits change for some subgroups. These subgroups vary in size by more than 25 per cent when compared to the other subgroups. This chart shows the number of defects found in dock audits of support braces completed each day.

Header information

ATTRIBUTES CONTROL CHART

Product / Service		Chart type	☐p ☐c	☐np ■u	Quality measure		Measurement device	
EOTA-8A support brace					Dock audit defects		Eyes	

User name		Location							Chart no.
Eric Johnson		Dock		AVG. = 2.00	UCL = 3.48	LCL = 0.52	n̄ = 8.2		

Upper control limit u

Process average (ū)

Lower control limit u

Data

DATE	Feb 9	Feb 10	Feb 11	Feb 12	Feb 15	Feb 16	Feb 17	Feb 18	Feb 19	Feb 22	Feb 23	Feb 24	Feb 25	Feb 26	Mar 1	Mar 2	Mar 3	Mar 4	Mar 5	Mar 8	Mar 9	Mar 10	Mar 11	Mar 12	Mar 15
TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Type of discrepancy
1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

Total number (np, c)	8	17	18	15	23	9	19	6	14	17	13	15	16	22	13	10	14	9	23	21	51	31	3	8	12
Proportion (p, u)	1.0	2.1	2.0	1.9	2.9	1.3	2.7	0.8	1.8	2.1	1.9	1.9	1.8	2.4	1.6	1.3	3.5	1.1	1.9	1.8	3.2	3.9	0.8	2.0	1.5
Subgroup size (n)	8	8	9	8	8	7	7	8	8	8	7	8	9	9	8	8	4	8	12	12	16	8	4	4	8

⊗ When is it used?

Use a u-chart with a varying sample size when you can answer "yes" to all of the following questions:

1. **Do you need to assess the variability in the system?**
    All control charts are used to assess whether the variation in a process is generated by special or common causes. Control charts are maintained on an ongoing basis in order to minimize the chance of reacting inappropriately to the variation by either overcontrolling or undercontrolling the system or process.

2. **Can the data be collected or does a collection of data already exist?**
    The data must be generated on an ongoing basis. Control charts analyze a series of data points, not a single data point.

3. **Is the time order of the data preserved?**
    Since control charts are designed to study the behavior of a system over time, it is critical that the data is recorded in the order it is collected and produced. Mixing the order of the data would be like editing a movie film with no attention given to the sequence of events.

4. **Is the data in attributes format?**
    Usually this data comes from a process whose aim is to judge whether a condition exists. Do parts have defects? Are there any errors in the invoices? Is information missing on the forms? Data that is measured using a measuring instrument, for example weight or height, are "variables," and different types of charts are used.

5. **Can the data value exceed the size of the subgroup?**
    For example, if 50 parts are being checked and they either pass or fail the check, then the maximum number of parts that can pass or fail is 50, the subgroup size. In this case an *np-* or *p-chart* is used, NOT a u-chart. If, however, the check involves counting the number of faults found on the 50 parts, and more than 50 faults could be found, a u-chart is used.

6. **Do the subgroups vary in size?**
    If the sample size is constant, then it is simpler to complete a *c-chart* or a u-chart with constant sample size, both these charts have been presented earlier in this section.

7. **Has the characteristic to be charted been operationally defined?**
    Operational definitions are important when collecting attributes data. Each data collector must have the same idea of what makes a nonconformity so the results will be consistent. Without an operational definition, data is unreliable and the chart may be meaningless. Therefore, the operational definition must be set prior to data collection.

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⟍⟍ How is it made?

These steps assume that the relevant data has been collected. The u-chart is constructed using the following steps:

1. Complete the header information.

Complete the header information at the top of the chart. Header information includes such items as product/service, user name, chart type (in this case a u-chart), location, quality measure, and measurement device. It is important that this information is completed, so that others can easily read and understand the chart.

2. Record the data.

Record the data on the control chart paper either from the data collection sheet or directly onto the chart paper as the data is collected. Fill in the row marked "Total number" with the number of faults or nonconformities. This is known as the c for each subgroup. In the example the c is the number of defects found in the dock audits. Fill in the row marked "Subgroup size" with the number of parts, forms, etc. checked. In the example this is the number of dock audits completed each day. It is important to maintain the data in the order it was produced. Do not rearrange the order of the subgroups, since this will hide any trends in the data.

Attributes control chart paper has a section labelled "Type of discrepancy." In this section list the types of faults found in each subgroup during data collection.

During data collection, record any significant changes or observations on the chart paper or on a separate process log (often located on the back of the chart paper) as they occur. These notes help interpret the control chart.

There are two rows in which to record sample identifiers; they are titled "Date" and "Time." Do not be limited by these titles. Use them to record not only the date and time, but also other useful identifiers, such as location or operator.

The example u-chart with completed header information and recorded data follows:

ATTRIBUTES CONTROL CHART

Product / Service	EOTA-8A support brace	Chart type	☐p ☐c	☐np ■u	Quality measure	Dock audit defects	Measurement device	Eyes

User name	Eric Johnson	Location	Dock		AVG. =	UCL =	LCL =	n̄ =	Chart no.

DATE	Feb 9	Feb 10	Feb 11	Feb 12	Feb 15	Feb 16	Feb 17	Feb 18	Feb 19	Feb 22	Feb 23	Feb 24	Feb 25	Feb 26	Mar 1	Mar 2	Mar 3	Mar 4	Mar 5	Mar 8	Mar 9	Mar 10	Mar 11	Mar 12	Mar 15
TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Type of discrepancy
1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Total number (np, c)	8	17	18	15	23	9	19	6	14	17	13	15	16	22	13	10	14	9	23	21	51	31	3	8	12
Proportion (p, u)																									
Subgroup size (n)	8	8	9	8	8	7	7	8	8	8	7	8	9	9	8	8	4	8	12	12	16	8	4	4	8

3. CALCULATE THE NUMBER OF FAULTS PER UNIT (U) FOR EACH SUBGROUP.

In order to complete the control chart, the number of faults or defects per unit checked must be calculated. The number of faults per unit, known as u, is found by dividing the "Total number," c, by the "Subgroup size," n.

$$u = \frac{c}{n}$$

So, for the first subgroup the value of u is:

$$u_1 = \frac{8}{8}$$
$$= 1.0$$

The value of u for the second subgroup is:

$$u_2 = \frac{17}{8}$$
$$= 2.1$$

The value for u should be recorded so there are enough decimal places in the u values to see differences between the values. Often no decimal places are required. However, when recording small u values, two or more decimal places may be required. In the example, one decimal place was sufficient.

Continue calculating the u values for each subgroup. Enter the u values in the row marked "Proportion" on the control chart paper.

4. CALCULATE THE OVERALL AVERAGES.

a. Calculate the average number.

The average proportion of failures or nonconformities is known as $\bar{u}$. It is found by totalling the "Total number" row (c's) and dividing it by the total of the "Subgroup size" row (n's).

$$\bar{u} = \frac{\sum c}{\sum n}$$

In the example $\bar{u}$ is:

$$\bar{u} = \frac{8 + 17 + 18 + \ldots + 8 + 12}{8 + 8 + 9 + \ldots + 4 + 8}$$

$$= \frac{407}{204}$$

$$= 2.00$$

The value for $\bar{u}$ should be recorded to one more decimal place than the u values. The value for $\bar{u}$ is recorded on the control chart in the header information in the space marked "AVG."

Caution: Calculating $\bar{u}$ by totalling the subgroup proportion (u's) and dividing by the number of subgroups is not correct. This will give a different value based on the subgroups, not on the data.

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b. Calculate the average subgroup size.

The average subgroup size is known as $\bar{n}$. It is found by totalling the "Subgroup size" row (n's) and dividing it by the number of subgroups (k).

$$\bar{n} = \frac{\sum n}{k}$$

In the example $\bar{n}$ is:

$$\bar{n} = \frac{8 + 8 + 9 + \ldots + 4 + 8}{25}$$

$$= \frac{204}{25}$$

$$= 8.2$$

The value for $\bar{n}$ should be recorded to one decimal place. The value for $\bar{n}$ is recorded on the control chart in the headers in the space marked " $\bar{n}$."

5. CALCULATE THE CONTROL LIMITS.

Control limits are lines drawn on the chart, and are used when interpreting the data. These limits come from variation in the system, not from specifications or requirements. Ideally, at least 25 subgroups should be used to calculate the control limits. However, the more subgroups available, the more accurate the control limits will be. If fewer than 25 subgroups are available, consider the limits as "trial" limits, and recalculate them after enough data has been collected.

Calculating control limits for a u-chart with varying sample sizes involves a number of steps. First, calculate general control limits for all the data. Second, identify subgroups where the size of the sample taken is significantly different from the rest of the subgroups. "Significant" means that the general control limits will be unsuitable for the data, and revised control limits are required. The revision of the control limits needs to occur if the subgroup size varies by more than 25 percent from the average subgroup size. Third and finally, perform specific control limit calculations for the subgroups that have been identified as varying significantly.

a. Calculate the control limits for all the data.

The symbol for the upper control limit is UCL$_u$, and the symbol for the lower control limit is LCL$_u$. The formula for the upper control limit is:

$$UCL_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

So, UCL$_u$ for the example is:

$$UCL_u = 2.00 + 3\sqrt{\frac{2.00}{8.2}}$$
$$= 2.00 + 3\,(0.49386)$$
$$= 2.00 + 1.48159$$
$$= 3.48$$

Control limits should be recorded to one more decimal place than the u values. The formula for the lower control limit is:

$$LCL_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

So, LCL$_u$ for the example is:

$$LCL_u = 2.00 - 3\sqrt{\frac{2.00}{8.2}}$$
$$= 2.00 - 1.48159$$
$$= 0.52$$

Note that the equation for the lower control limit is virtually the same as that for the upper control limit. If the lower control limit turns out to be negative, then the limit has no meaning and is not used.

Record the values for the control limits in the spaces marked "UCL" and "LCL" at the top of the chart.

b. Identify the subgroups requiring re-calculated control limits.

To calculate the subgroups requiring separate control limits, it is necessary to calculate the maximum and minimum subgroup sizes that are acceptable. These are the sizes that vary by 25 per cent either side of the average subgroup size, $\bar{n}$. The calculations are as follows:

Maximum acceptable subgroup size $= \bar{n} \times 1.25$

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For the example:

Maximum acceptable subgroup size = 8.2 x 1.25
                                  = 10.25

This means that any subgroup size that is larger than 10.25 will require re-calculated control limits.

Minimum acceptable subgroup size = $\bar{n}$ x 0.75

For the example:

Minimum acceptable subgroup size = 8.2 x 0.75
                                 = 6.15

This means that any subgroup size that is smaller than 6.15 will require re-calculated control limits.

Next, go through the row marked "Subgroup size" and look for subgroups outside the acceptable sizes. In the example the subgroup sizes being identified as requiring recalculated limits, are when the subgroup size is 11 or larger and 6 or smaller. The subgroups identified in the example are 17 (4), 19 (12), 20 (12), 21 (16), 23 (4), and 24 (4). Identify them with a cross or mark for reference.

c. Calculate the control limits for the subgroups with too large or too small a subgroup size.

When calculating these control limits the subgroups whose subgroup size is larger will have control limits that are narrower than the control limits for the other subgroups. Conversely, subgroup sizes that are smaller will have control limits that are wider than the control limits for the other subgroups.

The control limits formula is similar to the general one stated earlier, except that instead of dividing by $\bar{n}\bar{p}$, divide by the actual subgroup size n. The formulas are as follows:

$$UCL_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$LCL_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

In the example, the recalculations for subgroup 17 are:

$$UCL_{u(k=17)} = 2.00 + 3\sqrt{\frac{2.00}{4}}$$

$$= 2.00 + 3\sqrt{0.50}$$

$$= 2.00 + 2.121$$

$$= 4.12$$

$$LCL_{u(k=17)} = 2.00 - 3\sqrt{\frac{2.00}{4}}$$

$$= 2.00 - 2.121$$

$$= -0.121 \approx 0$$

Remember, the control limits should be recorded to one more decimal place than the u values, and if the lower control limit turns out to be negative, as in the example, then the limit has no meaning and is not used.

Continue to calculate the control limits for the other subgroups identified. The results can be placed in a table. This makes for easy reference when constructing the control chart. The example completed through this step follows:

Subgroup	n	UCL$_u$	LCL$_u$
17	4	4.12	–
19	12	3.22	0.78
20	12	3.22	0.78
21	16	3.06	0.94
23	4	4.12	–
24	4	4.12	–

The dash indicates a negative control limit, which is not used.

6. Scale the control chart.

The term "scaling" refers to numbering the lines on the left-hand side of the chart. The chart should be numbered (scaled) evenly so the values can be plotted conveniently, and the resulting chart is uncluttered and easy to read. Scaling can be the most difficult step in constructing a control chart. It is important to understand that the scale for the chart is not determined solely by the calculations performed; it is equally decided by considering ease of plotting and readability of the final chart. The calculations should be seen as a guide in determining the final scale.

Begin by finding the largest u value from the "Proportion" row on the chart. Compare this value to the largest upper control limit and write down the larger of the two. In the example, the largest u value is 3.9 and the largest upper control limit is 4.12. So, the larger value is 4.12.

Next, divide the larger value by two-thirds of the total number of lines on the control chart. (Use only two-thirds of the available lines in order to leave space outside the control limits to plot future shifts). This calculation will provide the increment value for each line on the control chart. In the example, there are 30 lines on the chart paper, so use 20 lines (two-thirds) to find the increment value.
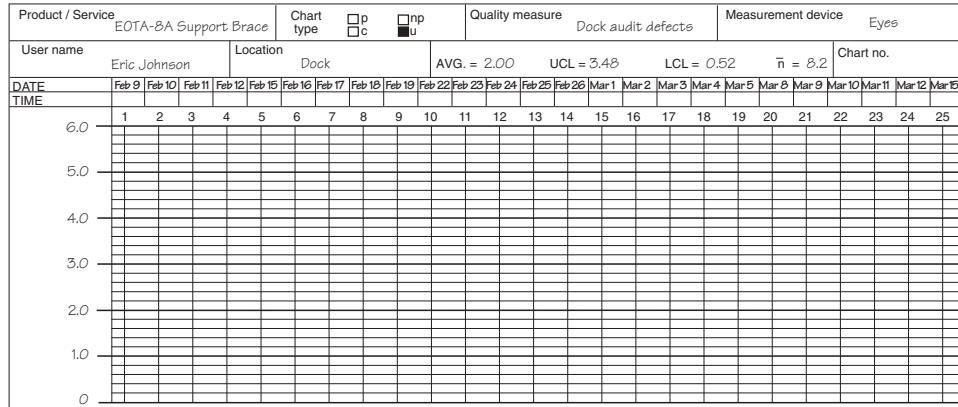
The increment value is: $\dfrac{4.12}{20} = 0.2$

Round the increment value upward as necessary to get an easy-to-work-with number, such as 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, or 10. In the example, there was no need to round up the increment value as 0.2 is easy to use. This means that each line represents 0.2.

To scale the chart, label the bottom line as zero and move upward from zero, adding the increment value. Label each dark reference line. In the example, the dark reference lines were labelled 0, 1, 2, 3, 4, 5, and 6. The example completed through this step follows on the next page:

ATTRIBUTES CONTROL CHART

Product / Service EOTA-8A Support Brace	Chart type	☐p ☐c	☐np ■u	Quality measure Dock audit defects		Measurement device Eyes	
User name Eric Johnson	Location Dock		AVG. = 2.00	UCL = 3.48	LCL = 0.52	$\bar{n}$ = 8.2	Chart no.

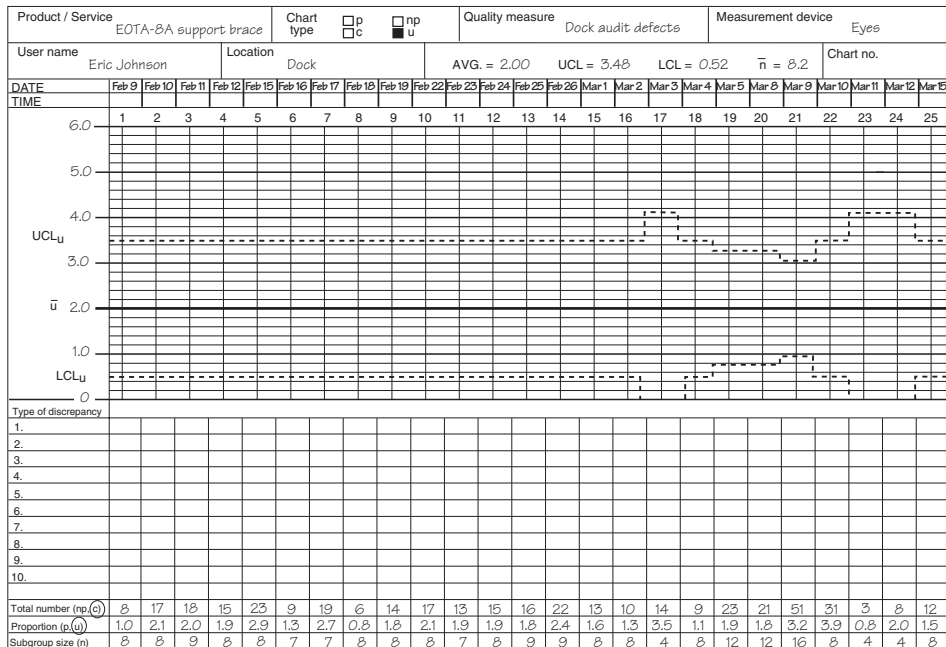| DATE	Feb 9	Feb 10	Feb 11	Feb 12	Feb 15	Feb 16	Feb 17	Feb 18	Feb 19	Feb 22	Feb 23	Feb 24	Feb 25	Feb 26	Mar 1	Mar 2	Mar 3	Mar 4	Mar 5	Mar 8	Mar 9	Mar 10	Mar 11	Mar 12	Mar 15
TIME																									
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

7. DRAW THE AVERAGE LINE AND CONTROL LIMITS.

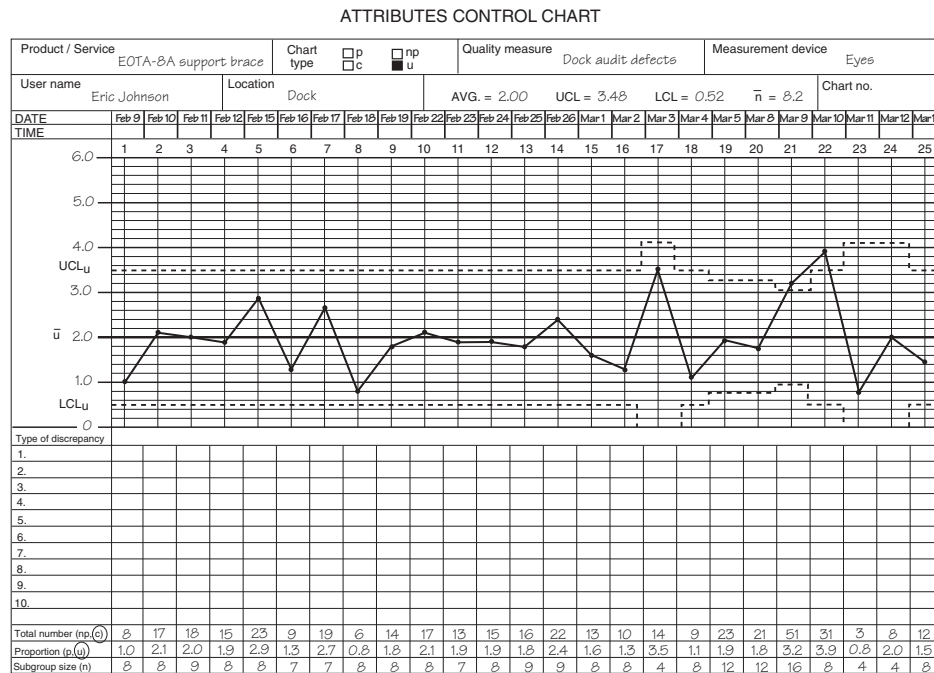Draw a solid dark horizontal line on the chart to represent $\bar{u}$. Label the line with $\bar{u}$.

Draw dashed horizontal lines to represent the upper and lower control limits. Label each control limit with the correct symbol, UCL_u and LCL_u. Remember to draw the control limits so they vary according to the correct subgroup. The example completed through this step follows:

ATTRIBUTES CONTROL CHART

Product / Service EOTA-8A support brace	Chart type	☐p ☐c	☐np ■u	Quality measure Dock audit defects		Measurement device Eyes	
User name Eric Johnson	Location Dock		AVG. = 2.00	UCL = 3.48	LCL = 0.52	$\bar{n}$ = 8.2	Chart no.

DATE	Feb 9	Feb 10	Feb 11	Feb 12	Feb 15	Feb 16	Feb 17	Feb 18	Feb 19	Feb 22	Feb 23	Feb 24	Feb 25	Feb 26	Mar 1	Mar 2	Mar 3	Mar 4	Mar 5	Mar 8	Mar 9	Mar 10	Mar 11	Mar 12	Mar 15
TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Total number (np, c)	8	17	18	15	23	9	19	6	14	17	13	15	16	22	13	10	14	9	23	21	51	31	3	8	12
Proportion (p, u)	1.0	2.1	2.0	1.9	2.9	1.3	2.7	0.8	1.8	2.1	1.9	1.9	1.8	2.4	1.6	1.3	3.5	1.1	1.9	1.8	3.2	3.9	0.8	2.0	1.5
Subgroup size (n)	8	8	9	8	8	7	7	8	8	8	7	8	9	9	8	8	4	8	12	12	16	8	4	4	8

8. PLOT THE U VALUES ON THE CHART.

Starting with the first subgroup, plot each u value on the control chart. Connect the dots with straight lines. The completed example follows:

ATTRIBUTES CONTROL CHART

Product / Service: EOTA-8A support brace	Chart type: □p □c □np ■u	Quality measure: Dock audit defects	Measurement device: Eyes

User name: Eric Johnson | Location: Dock | AVG. = 2.00 UCL = 3.48 LCL = 0.52 $\bar{n}$ = 8.2 | Chart no.

DATE	Feb 9	Feb 10	Feb 11	Feb 12	Feb 15	Feb 16	Feb 17	Feb 18	Feb 19	Feb 22	Feb 23	Feb 24	Feb 25	Feb 26	Mar 1	Mar 2	Mar 3	Mar 4	Mar 5	Mar 8	Mar 9	Mar 10	Mar 11	Mar 12	Mar 15
Subgroup	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Total number (np,c)	8	17	18	15	23	9	19	6	14	17	13	15	16	22	13	10	14	9	23	21	51	31	3	8	12
Proportion (p,u)	1.0	2.1	2.0	1.9	2.9	1.3	2.7	0.8	1.8	2.1	1.9	1.9	1.8	2.4	1.6	1.3	3.5	1.1	1.9	1.8	3.2	3.9	0.8	2.0	1.5
Subgroup size (n)	8	8	9	8	8	7	7	8	8	8	7	8	9	9	8	8	4	8	12	12	16	8	4	4	8

9. INTERPRET THE CONTROL CHART.

The u-chart is interpreted using the basic rules for control chart interpretation. These are explained fully in "Control chart interpretation" at the end of this tool. However, a list of the basic rules for interpreting out-of-control conditions is shown below.

1. Any point lying outside the control limits.

2. Run of seven points:
   a. Seven or more points in a row above or below the center line.
   b. Seven or more points in a row going in one direction, up or down.

3. Any nonrandom pattern, including the following typical cases:
   a. Too close to the average.
   b. Too far from the average.
   c. Cycles.

The example u-chart shows two points outside the control limits (subgroups 21 and 22). These indicate that special causes of system variation are present and

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should be investigated. In this case, the team discovered both shipments had been damaged in transit. The way the goods had been handled was different than normal. The team put a system in place to prevent this from reoccurring.

When no special causes are evident, the control limits calculated previously are simply drawn onto new chart paper. As subgroups continue to be collected they are plotted on the chart. This will allow the user to make decisions about the system's stability as each new subgroup is plotted. New control limits are calculated only when a change is made to the system.

Care must be taken when interpreting a u-chart. Sometimes the data will not behave as expected. In this case, the control limits look odd in comparison to the data, that is, too wide or too narrow. If this occurs, change the type of control chart to an *X-MR chart*.

The control limits of attributes charts can be compared directly to specifications, or requirements, enabling the user to evaluate the capability of the system to meet the requirements. In the example, the control limits read: UCL of 3.48, $\bar{u}$ of 2.00 and LCL of 0.52. This information shows that each audit on average finds 2 defects. The challenge is to reduce the number of defects consistently, in this case by working with the supplier.

See the notes, "Getting the most from control charts," at the beginning of this section for more information on how to use and benefit from this powerful tool.

APPLICATIONS

Remember to use a u-chart if your sample size varies; if the sample size is constant either a *c-* or a u- chart can be used.

ALL INDUSTRIES.

- Safety incidents per hours worked in a week or month
- Orders received per operator or sales representative per day, week, or month
- Number of on-site sales calls made per representative per day or week
- Phone calls taken per operator per hour or day
- Audits completed per auditor over a week or month
- Nonconformities found on ISO 9000, ISO 14000, or safety audits completed in a week
- Sales calls per sales person per day or week
- Inquiries per advertisement placed per month
- Procedures viewed per assessor per day, week, or month

Administration

- Documents found misfiled per day, week, or month
- Errors in documents completed in a day or week
  For example: letters, invoices, purchase requisitions, purchase orders
- Calls to a help desk in comparison to the number of employees, or number of computer operators
- Number of responses to job advertisements placed in a month
- Number of adjustments made to invoices per week or month
- Number of supplier deliveries per supplier per week or month
- Number of checks raized per supplier per month

Manufacturing

- Errors on drawings produced per week
- Number of times a drawing is reworked per week or month
- Printing errors on all jobs produced per day or week
- Number of deviations from the production schedule in comparison to the number of jobs scheduled per day or week
- Number of times equipment failed in comparison to the number of pieces of equipment, per day, week, or month
- Parts produced per hour worked over a day or week
- Number of times batches are reworked in comparison to the total produced per day, shift, week, or month (where they can be reworked more than once)
- Defects on a product sample
  For example: scratch, blemish, printing error, dent, incorrect color, marks
- Number of picking errors when putting together a multiple product order in comparison to the number of orders per shift, day, or week

Service

- Registrations for training courses per week or month
- Errors per documents produced per day or week
  For example: legal documents, application forms, bank transactions, a book, newspaper, or magazine
- Technical support calls per product to a software house per day
- Errors per line of code in software programs
- Nonconformities found in a clean audit sample
  For example: dishes, plane, train, bus, restaurant, hotel room, clothes in a dry cleaner
- Customers served per employee per hour, day or week
  For example: in a bank, in a restaurant, at a store checkout, in a hotel, in an airport

u-control chart