Statistical Quality Control

What is QUALITY?

Quality is a measure of how closely goods or service conform to specification

Why to inspect?

To assess the actual quality characteristics

Inspection: When and Where?

- Upon receipt of raw material
- Before costly and irreversible operations
- Before a work that could hide defects
- Upon completion of the product
- Before stocking high-value items
- Before shipment to customers

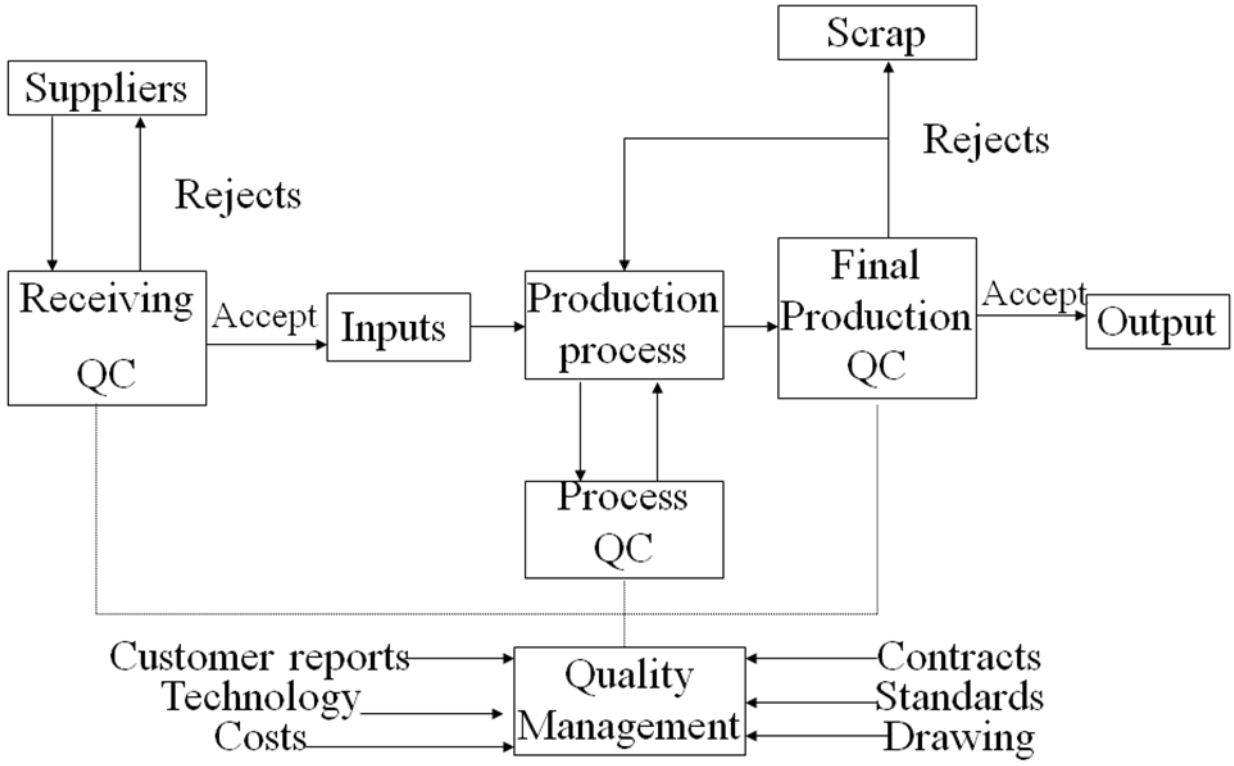
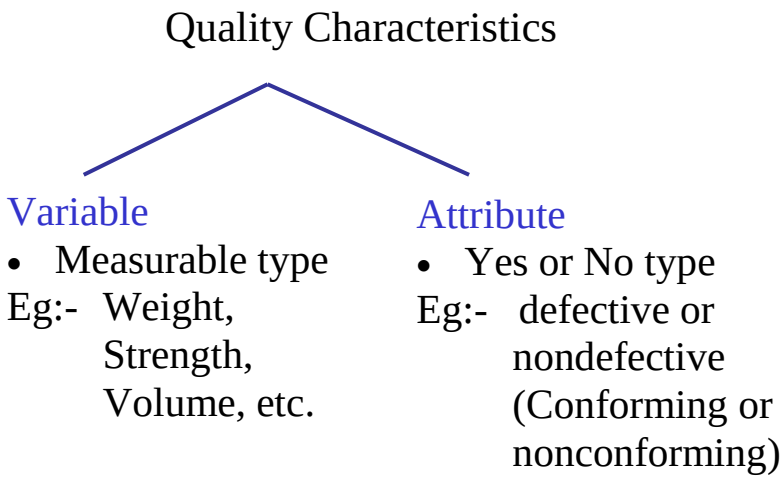


Fig. 1 The quality control system



Statistical Process Control (SPC)

Principle of SPC

- Measured quality of manufactured product is always subject to a certain amount of variation
- Under statistical control the quality characteristic has a particular pattern of variation
- The reason for variation outside this stable pattern may be discovered and corrected

Statistical Control

- Process variations are traceable to two kinds of causes
  - Common (or random or chance) - inherent in the process
  - Special (or assignable) – excessive variation
- Common cause represents a stable and predictable process – minimum variation
- A process that is operating without special cause variation is said to be "in a state of statistical control"
- Control chart distinguish between common and special causes of variation

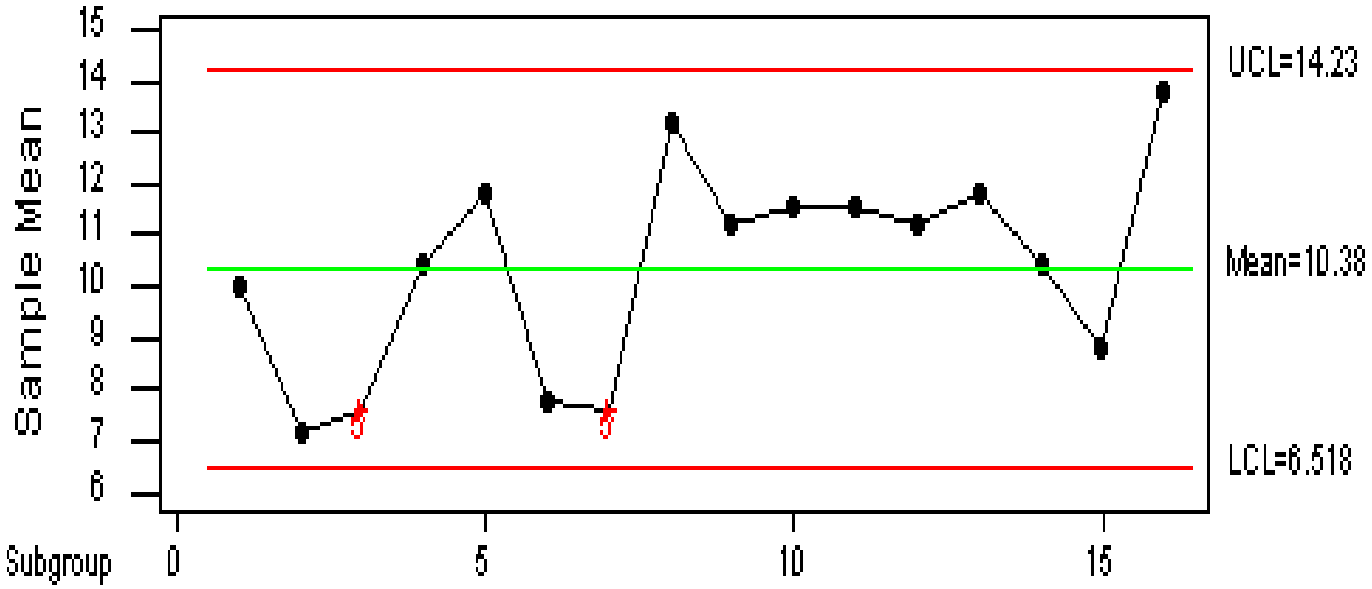


Fig. 2 A control chart for averages

Statistical Process Control

- Uses control charts
- Measure variation of the process during operation
- Reduce process variability and achieve process stability

Constant use of control charts results in

- Substantial improvement in quality
- Reduction in spoilage, rework, error, etc.

And also it tells that

- Quality variations are inevitable (within its pattern of variation)
- When to leave a process alone
- When to search for a causes of variation

Procedure for Establishing and Using Control Limits

- Select the process to be controlled; identify the relevant characteristics and method of measurement
- Take approximately 25 samples (subgroups) of size n, and compute the statistics
- Compute the control limits and make relevant plot on a graph sheet
- Discard any samples which are outside the control limits and recalculate the control limits, and prepare a new plot

- Investigate for assignable causes when the process is out of control, as evidenced by:
  - A point outside the control limits
  - Predominance of points on one side of the centre line
  - Pattern of plotted points

Steps for Average and Range Chart

1. Record N samples of size n measurements

   n around 4 recommended (see fig.2 to get a demonstration of why n = 4 is suitable)

   N >25 recommended

   How to form **RATIONAL SUBGROUPS**?

   - Samples (subgroups) should be chosen so that the units
     - Within a subgroup have the greatest chance of being alike
     - Between subgroups have the greatest chance of being different
2. Calculate sample averages
3. Calculate sample Range *R*

   NOTE: R can never be negative!
4. Calculate average Range

$$\overline{R} = \frac{\sum_{i}^{N} R_i}{N}$$

5. Calculate control limits for R

$$UCL_R = D_4\overline{R}$$
$$CL_R = \overline{R}$$
$$LCL_R = D_3\overline{R}$$

   D_3 and D_4 are constants obtained from tables
6. Check: all R within control limits?

   If NOT, delete such samples and repeat from Step 4
7. Calculate grand average $\overline{\overline{x}}$

$$\overline{\overline{x}} = \frac{\sum_{i}^{N} \overline{x}_i}{N}$$

8. Calculate control limits for $\overline{\overline{x}}$

$$UCL_{\overline{x}} = \overline{\overline{x}} + A_2\overline{R}$$
$$CL_{\overline{\overline{x}}} = \overline{\overline{x}}$$
$$LCL_{\overline{x}} = \overline{\overline{x}} - A_2\overline{R}$$

10. Choose scales and plot $\overline{x}$ & R

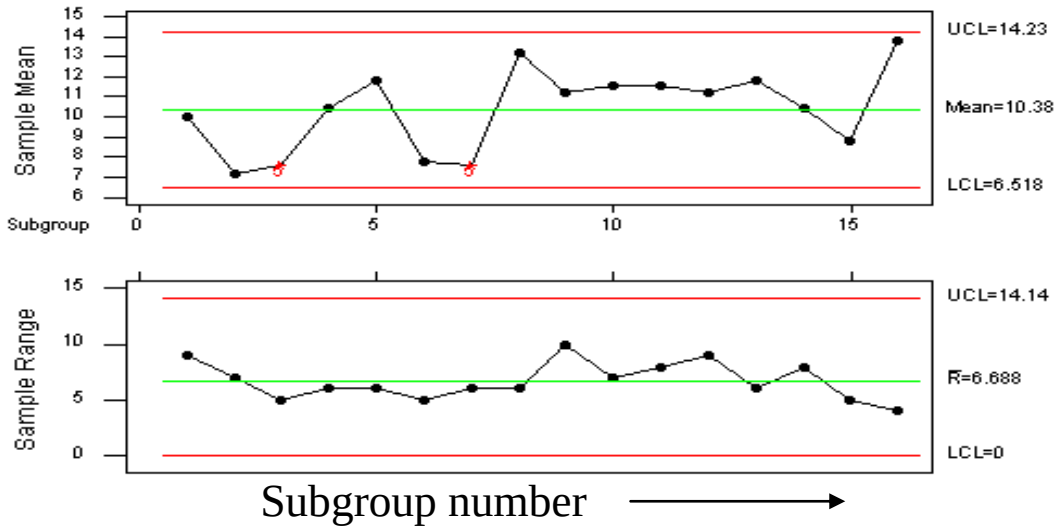


Fig. 3 A Typical X-bar and R chart

11. Check: all points **RANDOMLY** within control limits?

- If **YES,** the process was in **STATISTICAL CONTROL**
- Calculated control limits are suitable for future use

Interpretation of Process in Control

- The control-chart data provide estimates for
  - The centering of the process
    - μ may be estimated as X-double bar
  - The dispersion of the process

- σ may be estimated as

$$\sigma = \frac{\overline{R}}{d_2}$$

Indication of Control or Lack of Control

- When to look for Lack of control
  - Point falling outside the limits on either X-bar or R charts
  - Runs of points above or below the central line
  - Particular patterns of variation of X-bar and R charts

Process Control and Occasional Error

- In a best manufacturing process occasional errors occurs that constitute assignable causes of variation as per control charts
- This may not constitute a basis of action
- This fact may lead to various practical working rules
  - 1 out of 35 or 2 out 100 points outside the control limits as evidence of control

Rules to detect the out of control based on extreme runs

- 7 Successive points on the control chart, all are on the same side of the central line
- In 11 successive points on the control chart at least 10 are on the same side of the central line
- In 14 successive points on the control chart, at least 12 are on the same side of the central line
- In 17 successive points on the control chart, at least 14 are on the same side of the central line
- In 20 successive points on the control chart, at least 16 are on the same side of the central line

Caution

- The sequences as per theory runs will occur as a matter of chance, with no change in the universe, more frequently than a point outside of 3-sigma limits
- For this reason the rules provide a less reliable basis for hunting for trouble than points outside of control limits. This can be understood from the control charts given below – please look into the points between the parallel lines

X-bar Chart with points normally distributed

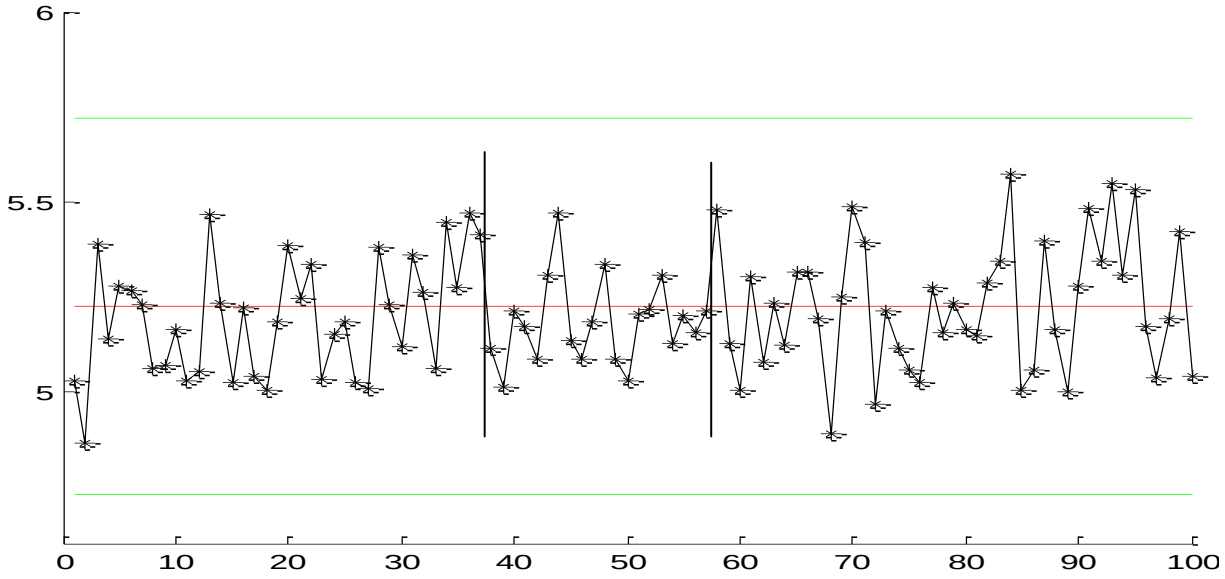


Fig. 4 X-bar Chart with points normally distributed (Showing lack control as per the rule: In 20 successive points on the control chart, at least 16 are on the same side of the central line, but the variation is within the chance cause variation)

Patterns of variation of X-bar and R charts

- Recurring cycles
- Trends
- Jumps in process level
- High proportion of points near or outside limits
- Stratification or lack of variability

X-bar Chart - Jumps in process level

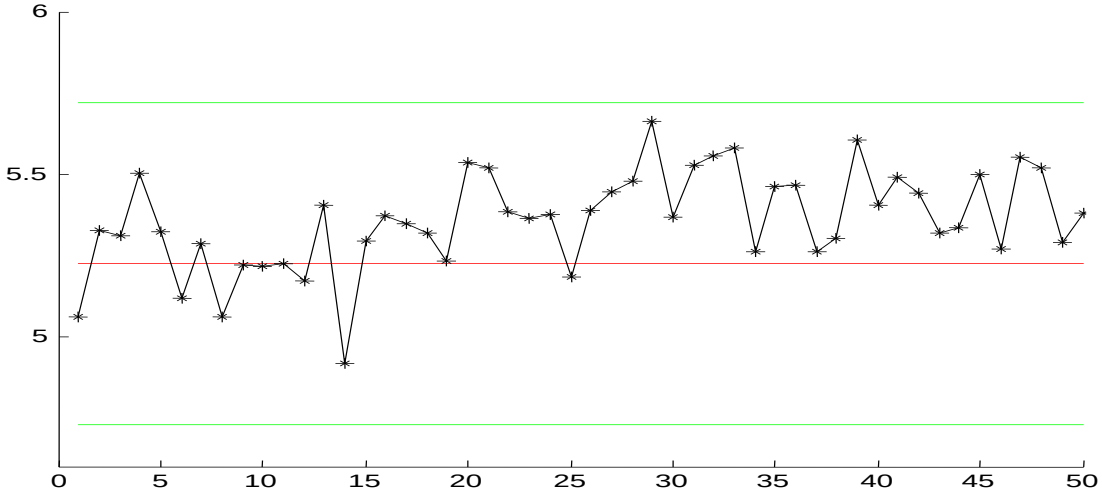


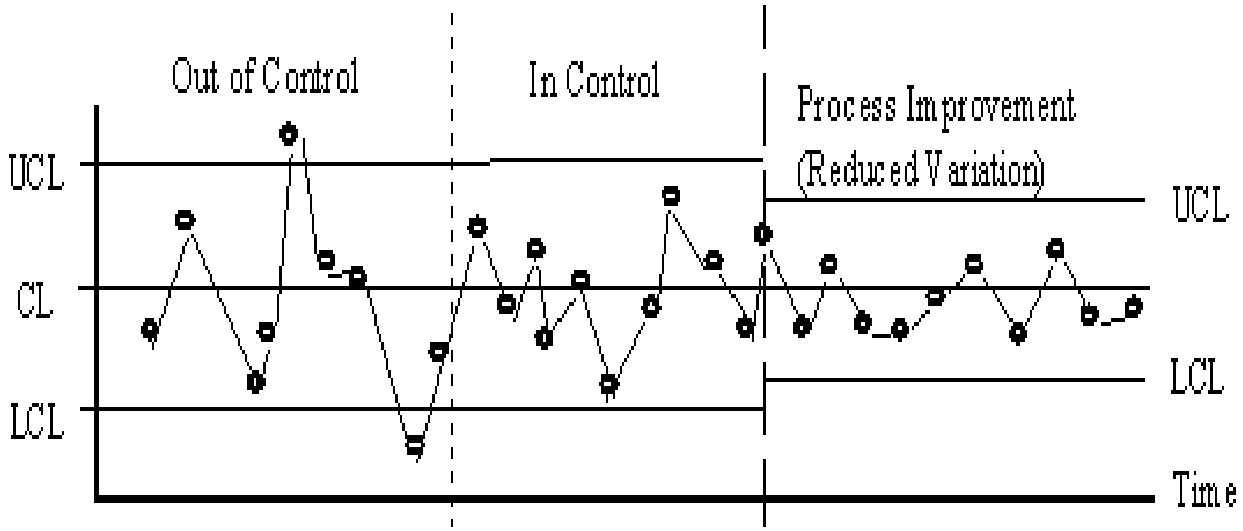
Fig. 5. X-bar chart to demonstrate jump in process average
(CL=5.225, UCL=5.7198, LCL=4.7302
Points are generated from normal distribution
After 20 points a shift is provided by 0.2)

A classification of ways in which lack of control may occur

- Process average may change
- Process dispersion may change
- Both process average and dispersion may change

Process Improvement

The following control chart shows the improvement of a process. The standard deviation decreases as the process becomes more capable.



Average Chart Vs Range Chart

- Between-sample variation is shown by a average chart
  - Sample averages, rather than individual values, are more sensitive to detecting process changes
- Within-sample variation is shown by a range chart

Specification Limits Vs Control Limits

- Specification limits apply to individual values
- Control limits in a chart of averages represent three standard deviation of sample averages
- Control limits can not be compared to specification limits
- Specification limits should not be placed on a control chart for averages
- Averages inherently vary less than the individual measurements going into the averages

Process Capability

- Process capability provides a quantified prediction of process adequacy
- It is the measured, inherent variation of the product turned out by a process
- Inherent variation results from a process, which is in a state of statistical control

Process capability information serves multiple purposes:

1. Predicting the extent of variability that process will exhibit. Such capability information, when provided to designers, provide important information in setting realistic specification limits.
2. Choosing from among competing processes that which is most appropriate for the tolerances to be met.
3. Providing a quantified basis for establishing a scheduled of periodic process control checks and readjustments
4. Assigning machines to classes of work for which they are best suited

Standardized Formula

- Process capability = $\pm 3\sigma$ $(a\,total\ of\ 6\sigma)$

  Where σ = the standard deviation of the process under a state of statistical control.

- If the process is centered at the nominal specification and follows a normal probability distribution, 99.73 percent of production will fall within $\pm 3\sigma$ of the nominal specification.

Relationship to Product Tolerance

- A major reason for quantifying process capability is to be able to compute the ability of the process to hold product tolerance.
- Planners try to select process with the 6σ process capability well within the tolerance width.
- A measure of this relationship is the capability ratio:

$$C_p = Capability\ ratio = \frac{USL - LSL}{6\sigma}$$

where USL = upper specification limit

LSL = lower specification limit

- The C_p index measures the potential capability, assuming that the process average is equal to the midpoint of the specification limits and the processing is operating in statistical control
- Some industries now express defect rates in terms of parts per million
- A defect rate of one part per million requires a capability ratio of about 1.63 (see fig. 7)

C_{pk} Capability Index

- C_p is a suitable measure for process capability as long as the process average is equal to the midpoint of the specification limits

- Often process average is not at the midpoint; then a capability index that shows the variation as well as the location of process average is required
- C_{pk} shows the proximity of process average to either upper specification limit (*USL*) or lower specification limit (*LSL*)
- C_{pk} is estimated by

$$\hat{C}_{pk} = \min\left[\frac{\hat{X} - LSL}{3\sigma}, \frac{USL - \hat{X}}{3\sigma}\right]$$

- This estimate shows the process performance

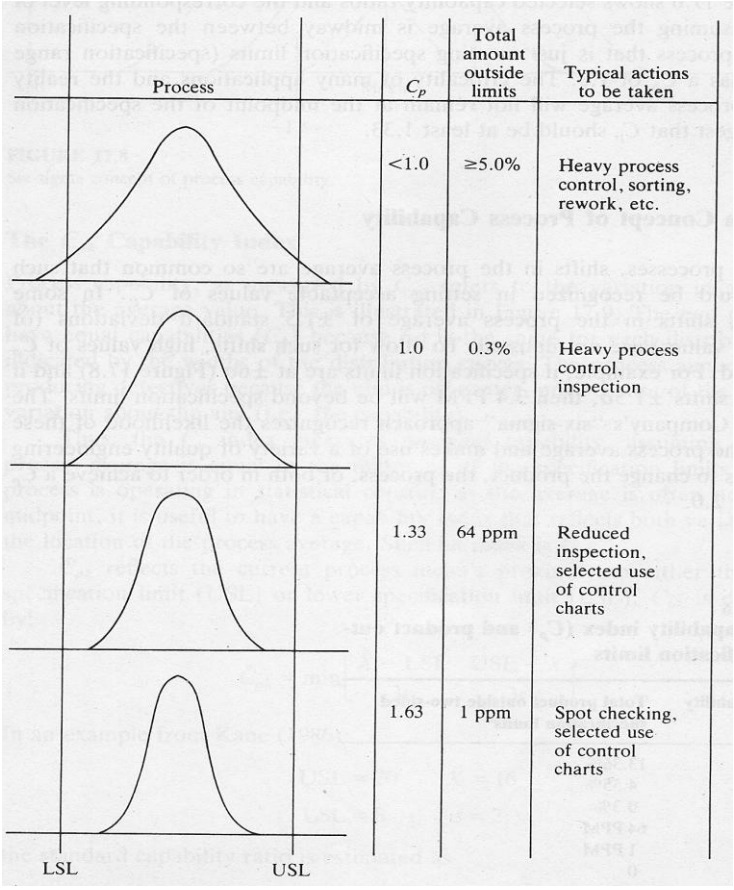


Fig.7 Process variability

Six-sigma Concept of Process capability

- Shifts in process average are so common that such shifts should be recognized in setting values of C_p.

- In some industries, shifts in the process average of ± 1.5 standard deviation (of individual values) are not unusual.
- To allow such shifts, high values of C_p are needed
- For example, if specification limits are at $\pm 6\sigma$ (see fig. 8) and if the mean shifts $\pm 1.5\sigma$, then 3.4 PPM will be beyond specification limits
- Six-sigma approach recognizes the likelihood of these shifts in the process average and makes use of a variety of quality engineering techniques to change the product, the process, or both in order to achieve a C_p of at least 2.

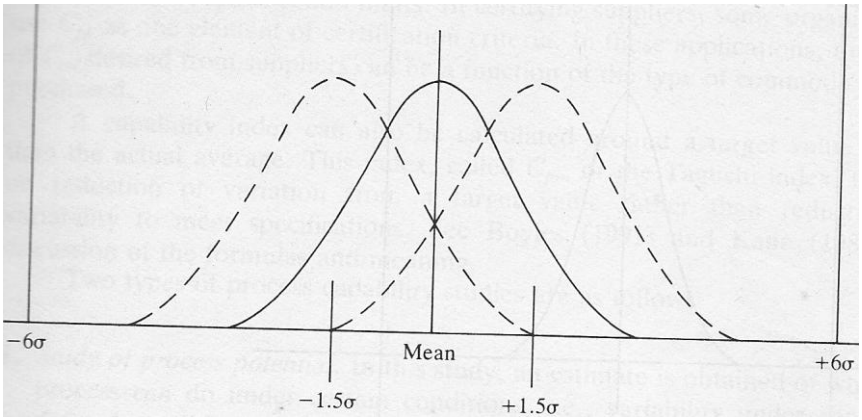


Fig. 8 Six-sigma concept of process capability

Table 1 Process capability index (C_p) and product outside specification limits

Process capability index (C_p)	Total product outside two-sided specification limits (Assume that the process is centered midway between the specification limits)
0.5	13.36%
0.67	4.55%
1.00	0.3%
1.33	64PPM
1.63	1PPM
2.00	0

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