

RR 340



Research Report 340

ANALYSIS OF HYDROLOGIC RESPONSE TO RAIN-ON-SNOW

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July 1975

PREPARED FOR
OFFICE, CHIEF OF ENGINEERS
BY
CORPS OF ENGINEERS, U.S. ARMY
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Research Report 340	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ANALYSIS OF HYDROLOGIC RESPONSE TO RAIN-ON-SNOW		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) S.C. Colbeck		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Cold Regions Research and Engineering Laboratory Hanover, New Hampshire 03755		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DA Project 4A161102B52E Task 02, Work Unit 011
11. CONTROLLING OFFICE NAME AND ADDRESS Office, Chief of Engineers Washington, D.C. 20314		12. REPORT DATE July 1975
		13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Hydrology Snow Snowfields		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The equations describing water movement in a dry snow cover are derived and examples of flow through ripe, refrozen and fresh snows are given. The grain size of snow has a large effect on the timing of water discharge. Water is retained by dry snow to raise its temperature and satisfy the irreducible water saturation. These requirements delay and reduce runoff following rain on dry snow.		

PREFACE

This report was prepared by Dr. Samuel C. Colbeck, Jr., Geophysicist, Snow and Ice Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. The work was done under DA Project 4A161102B52E, *Research in Military Engineering and Construction*, Task 02, *Research in Snow, Ice and Frozen Soil*, Work Unit 011, *Properties of Snow and Ice Influencing Winter Mobility and Denial*.

Dr. George Ashton and Dr. Anthony Gow reviewed this manuscript and made many useful suggestions. Dr. Eugene Peck, Director of the Hydrologic Research Laboratory of the National Weather Service first suggested the importance of this topic. Dr. Peck and Dr. Ashton, Chief of the Snow and Ice Branch at CRREL, provided the necessary encouragement during the period of work.

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NOMENCLATURE

Symbols

d	Grain diameter
h	Snowpack thickness
k	Intrinsic permeability
R	Ratio of thermal to capillary water requirements
S_w	Percentage of pore volume occupied by water
S_{wi}	Irreducible water saturation, about equal to 0.07
S^*	Effective water saturation, $(S_w - S_{wi})/(1 - S_{wi})$
S_+	Value of S^* overtaking a front
S_-	Value of S^* overtaken by a front
t	Time
T	Temperature
u_w	Volume flux of water
u_+	Value of u_w overtaking a front
u_-	Value of u_w overtaken by a front
z	Depth coordinate
α	Constant
ξ	Position of wetting front beneath surface
ρ_i	Density of ice
ρ_s	Dry density of snow
ρ_w	Density of water
ϕ	Porosity
ϕ_e	Effective porosity, $\phi(1 - S_{wi})$

ANALYSIS OF HYDROLOGIC RESPONSE TO RAIN-ON-SNOW

by

S.C. Colbeck

Introduction

The movement of water through "ripe" snow covers has received much attention and the physical principles are reasonably well understood (e.g. Colbeck 1974a). Although the mode of flow is complicated by the diverting effects of individual layers, the study of ripe snow covers is simplified by the quasi-stable nature of uniform ice grains of 2 to 3 mm diameter at 0°C . Gerdel (1945) described the ripening phenomenon and noted differences in character of runoff between fresh and ripe snow covers. The flooding potential of rain-on-snow has motivated many studies (e.g. Snyder 1951, Sulahria 1972) but the basic principles of water movement into dry snow have yet to be examined. Further, no analytical or experimental relations describing water flow through dry snow are available. Rain-on-snow events, which are of particular interest to hydrologists concerned with runoff from snow, can produce everything from serious floods to no runoff depending on many variable factors. The movement of liquid into dry snow is addressed here in hopes of providing a better basis for understanding this complex phenomenon. Only by understanding the principles involved can hydrologists deal rationally with the flooding potential of rainfall on dry snow.

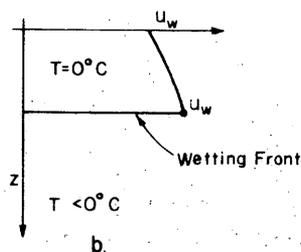


Figure 1. Typical profile of water flux for dry snow. For a dry snow at 0°C above the wetting front and less than 0°C below the wetting front, the wetting front is defined by the value of flux (u_w) feeding the front.

The effects of buried ice layers are neglected although these semi-permeable boundaries are most significant during the ripening period. Once the snow is thoroughly soaked and is quasi-stable thermodynamically, the ice layers are much more permeable (Gerdel 1954) and their effect on the flow field can be described (Colbeck 1975). However, where water enters dry snow for the first time, a complicated routing of the water around the less permeable ice layers can take place. In this report the effects of temperature, flow rate and grain size are examined; layering effects are a subject for further investigations.

We first described the propagation of the leading edge of a wave of liquid water into the interior of dry snow (a "wetting front"). As a wetting front advances into subfreezing snow (Fig. 1), the water is the principal source of the thermal energy needed to raise the snow to its melting temperature since the advection of water generally transports heat energy faster than heat conduction. Therefore, we consider the dry snow ahead of the advancing front to be stable and deal with the dynamics of the front itself. We also consider the coupling between the metamorphism of the wetted snow and the movement of the liquid water.

Theory

Three requirements must be met before the wetting front can propagate past any level. First, a volume of water per unit volume of snow equal to

$$\frac{T\rho_i}{160\rho_w} (\phi - 1)$$

must be frozen onto the snow grains in order to supply enough latent heat to raise the snow from a temperature T to 0°C . Second, a volume of water per unit volume of snow equal to ϕS_w must be supplied to fill the residual water requirement, i.e. the water retained as an adsorbed film and the water held immobile in the menisci between the snow grains. Third, any additional water supplied will be mobile and, in principle, the wetting front will propagate downward. In practice, the front will propagate at a rate determined by the rate at which water can be supplied to raise the liquid saturation to the level which corresponds to the value of water flux just above the wetting front. The flux and saturation above the front are related by

$$u_w = \alpha k S^3 \quad (1)$$

where the effects of tension gradients have been neglected. Tension gradients are of minor consequence since their only serious effect is to steepen the slope of the advancing front (Noblanc and Morel-Seytoux 1972, Colbeck 1974b) which, in the case considered here, is already sharply distorted because of the lack of liquid water in the subfreezing snow below the wetting front.

The total requirement for liquid water per unit volume of snow at the wetting front is

$$\phi S_w + (\phi - 1) \frac{T\rho_i}{160\rho_w}$$

Hence the wetting front propagates at a speed given by (see Appendix)

$$\frac{d\xi}{dt} = \frac{u_w}{\phi S_w + (\phi - 1) T\rho_i / 160\rho_w} \quad (2)$$

where u_w is the volume flux just above the wetting front and S_w is the water saturation necessary to sustain that value of flux. If the quantity of water passing the snow surface is small, the thermal and capillary requirements may not be satisfied; hence no runoff results. This condition of no runoff occurs when

$$\left[\phi S_{wi} + (\phi - 1) \frac{\rho_i T}{160\rho_w} \right] h \leq \text{total surface flow per unit area.}$$

The relative significance of the capillary and thermal terms in this equation must be evaluated for each case where the equation is to be applied. When a serious flooding potential exists snow temperatures are generally close to freezing; hence the thermal requirement is likely to be small and can be neglected. The relative significance of the thermal and water saturation requirements is assessed by taking the ratio R of the two terms in the denominator of eq 2, or

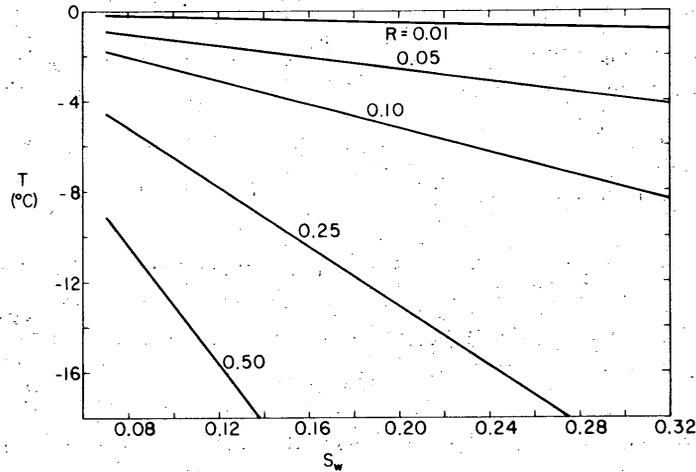


Figure 2. Values of the ratio of thermal to water saturation requirements R are given for various temperatures ahead of the wetting front and water saturations behind the front for a snow of porosity $2/3$. For normal values of temperature and water flux, R is small and the water saturation requirement is about ten times greater than the thermal requirement.

$$R = 0.00573 (1 - \phi^{-1}) T S_w^{-1} \quad (3)$$

Values of R are given in Figure 2 for various values of temperature and water saturation for a representative porosity of $2/3$. In the normal range of water saturations ($0.07 < S_w < 0.25$) and temperatures, R assumes low values, indicating the relative insignificance of the temperature effect on the rate of propagation of the wetting front. Accordingly, eq 2 is approximated by

$$\frac{d\xi}{dt} = \frac{u_w}{\phi S_w} \quad (4)$$

which is sufficiently accurate for most cases in which infiltrating water eventually reaches the bottom of the snow cover and presents a serious flooding potential. The rate of movement of the wetting front is then given by

$$\frac{d\xi}{dt} = \frac{\alpha k S_w^3}{\phi S_w} \quad (5)$$

where S_w is the water saturation and k is the intrinsic permeability just behind the front. Given the permeability of the snow above the wetting front we can calculate the spatial and temporal distribution of water flux and saturation using the theory developed previously for a ripe snowpack. The most significant problem remaining is to quantify the rate of increase of permeability of the wetted snow which results from the grain growth through melt metamorphism.

Shimizu (1970) related the intrinsic permeability of snow to two basic parameters of dry snow — grain size d and density ρ_s :

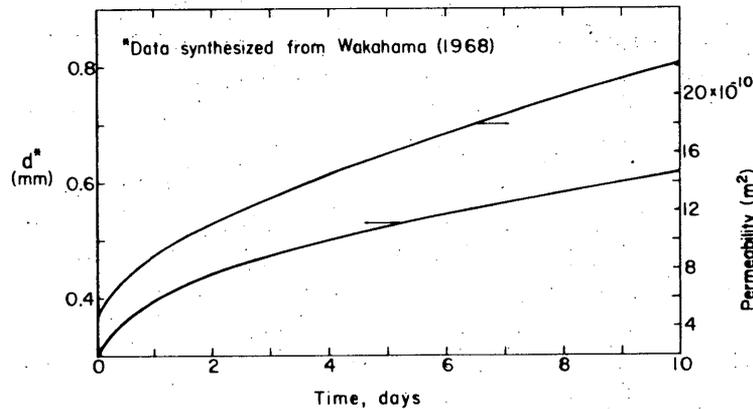


Figure 3. Grain size d and intrinsic permeability k as a function of time following the introduction of 10% water saturation. The grain growth is deduced from Wakahama's (1968) data and permeability is calculated using Shimizu's (1970) equation for a snow of ice density 0.35 Mg/m^3 .

$$k = 7.7d^2 \exp(-7.8\rho_s) \quad (\text{mm}^2). \quad (6)$$

This expression does not account for the effect of grain shape of freshly fallen snow because these effects disappear within a few hours even in subfreezing snow (Shimizu 1970). The possibility of increasing density is neglected here although some densification usually occurs when water first infiltrates into dry snow. The effects of densification and ice layer decomposition tend to be offsetting but clearly both of these areas need further investigation.

The increase of grain size in the presence of liquid water recorded by Wakahama (1968) was explained thermodynamically by Colbeck (1973). These results show an increased growth rate at higher water saturations and decelerating grain growth with time as the average grain size increases. For the expected water saturations following a heavy rainfall on cold snow, the grain size d as synthesized here from Wakahama's data is given by

$$d = d_0 + 0.10t^{1/2} \quad (7)$$

where d_0 (mm) is the initial grain size and t (days) is the elapsed time during which the snow is wetted. One example of the increase of grain size with time is shown in Figure 3. The rapid increase of both grain size and permeability following the introduction of water into dry snow decelerates with time although the growth continues indefinitely. That is, the "ripe" snowpack is only quasi-stable thermodynamically and grain growth continues at an ever decreasing rate throughout the lifetime of the snow cover. Although eq 7 is suitable for our purposes here, it must be remembered that Wakahama's laboratory experiments do not duplicate the outside environment. Further observations of natural snow covers are needed to quantify the grain growth phenomenon. There is, for example, some indication that natural grain growth occurs more rapidly than the laboratory observations indicate, possibly because of the penetration of solar radiation into the snow cover.

If the grain size and density of the snow cover are known, eq 6 is used to calculate permeability and eq 5 is used to calculate the rate of propagation of the wetting front. Equation 1 relates water

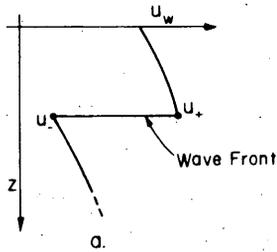


Figure 4. Typical profile of water flux for ripe snow. For a ripe snow at 0°C , the wave front at any depth is defined by the maximum (u_+) and minimum (u_-) fluxes at that depth.

flux to water saturation above the wetting front. Given a surface boundary condition in terms of saturation (or flux), the profile of saturation (or flux) is determined from

$$\left. \frac{dz}{dt} \right|_{S^*} = 3\alpha k \phi_e^{-1} S^{*2} \quad (8)$$

where the permeability k may be time- and space-dependent. For a given situation the profile of saturation above the front is calculated and the saturation immediately above the front determines the rate of propagation of the front. Combining eq 5 and 8,

$$\frac{d\xi}{dt} = \frac{1}{3} \left(1 - \frac{S_{wi}}{S_w} \right) \left. \frac{dz}{dt} \right|_{S^*} \quad (9)$$

describes the important relation between the rate of propagation of the wetting front and the rate of movement of values of saturation into that front. The rate of movement of the front increases with water saturation both because of the increased speed of S^* and because of the increased ratio of mobile to immobile water (S_w/S_{wi}). In a ripe snow cover experiencing daily melting and/or rain, the first mechanism gives a faster runoff response to heavier surface melting and/or rain. For dry snow, the second mechanism must also be considered. The significance of this second mechanism decreases with increasing saturation and, since ($S_w > S_{wi}$),

$$\frac{d\xi}{dt} < \frac{1}{3} \left. \frac{dz}{dt} \right|_{S^*} \quad (10)$$

Thus the rate of propagation of the wetting front in dry snow is always less than one third the speed of values of S^* feeding the front. In contrast to dry snow, in a ripe snow cover the values of S^* feeding a propagating wave front (S_+) and the values of S^* overtaken by the wave front (S_-) combine to determine the rate of movement of the front according to (see Fig. 4; eq 11 is derived in the Appendix)

$$\frac{d\xi}{dt} = \alpha k \phi_e^{-1} (S_+^2 + S_+ S_- + S_-^2) \quad (11)$$

where S_- represents the antecedent conditions resulting from the previous day's flow. Thus for a ripe snow cover the speed of values of water saturation overtaking the wave front and the speed of the front itself are related by

$$\frac{d\xi}{dt} = \frac{1}{3} \left(1 + \frac{S_-}{S_+} + \frac{S_-^2}{S_+^2} \right) \left. \frac{dz}{dt} \right|_{S^*} \quad (12)$$

where $\left. \frac{dz}{dt} \right|_{S^*}$ is the rate of propagation of S_+ . Clearly the speed of the front increases with both the surface flux and the moisture remaining from the previous day's flow. For a ripe snow cover

$$\frac{d\xi}{dt} > \frac{1}{3} \frac{dz}{dt} \Big|_{s^*} \quad (13)$$

so for equal surface fluxes, depth, permeability, etc., the wave front moving through ripe snow will always travel faster than a wetting front moving through dry snow.

To separate the effects of grain size and water saturation on the rate of frontal movement, we examine the case of equal depth, density and surface flux and formulate the equations in terms of fluxes instead of saturations. The ratio of front speed in ripe snow to front speed in dry snow for equal surface fluxes is

$$\frac{(d\xi/dt)_{\text{ripe}}}{(d\xi/dt)_{\text{dry}}} = \left(\frac{S_w}{S_w - S_{wi}} \right) \left(\frac{d_r}{d} \right)^{2/3} \left[1 + \left(\frac{u_-}{u_+} \right)^{1/3} + \left(\frac{u_-}{u_+} \right)^{2/3} \right] \quad (14)$$

where d_r is the grain size in ripe snow, d is the grain size in dry snow, u_+ is the flux immediately above the front in both ripe and dry snow and u_- is the flux immediately below the front in ripe snow. The first parenthetical expression is due to the necessity of increasing the water saturation of dry snow, the second expression is the permeability effect of grain size, and the third expression accounts for the antecedent moisture in a ripe cover. All three expressions increase the ratio of the wave speeds. The ratio of grain sizes often assumes values around ten, thereby increasing the ratio of wave speeds by about five. The effect of antecedent conditions is fairly small for shallow snow with large flow rates but, for normal conditions, the first expression could increase the ratio of speeds by an order of magnitude. Temperature and ice layer effects, which have been largely ignored in this treatment, also tend to increase the ratio describing the relative rate of propagation in ripe snow.

Application

In hydrologic terms, the "lag time" between the introduction of water on the surface and its arrival at the base is equal to snow thickness divided by the speed of frontal movement. This lag can be calculated directly for either ripe or dry snows using the analysis given here. For snow covers of equal thickness and equally heavy surface fluxes, eq. 14 shows how differences in grain sizes and antecedent liquid content between fresh and ripe snow covers control their relative lag times, the lag time being many times larger for fresh snow than for ripe snow. To illustrate we consider rain-on-snow for ripe (large grains, wet), fresh (small grains, dry) and refrozen (large grains, dry) snow covers. The stored water and lag time are calculated for the assumed values of ρ_s , d , h , u_w , T and duration shown in Table I. For a ripe snow cover, even neglecting any antecedent liquid, the lag time before the water reaches the bottom of the ripe snow is only 0.15 hr (530 s), whereas lag times for the fresh

Table I. Assumed values of ρ_s , d , h , u_w , T and duration.
Calculated values of storage and lag.

	ρ_s	d	h	u_w at surface	T	Duration	Stored	Lag
Fresh	0.3 Mg/m ³	0.2 mm	1 m	10 ⁻² mm/s	-5°C	10,800 s	56.5 mm ³	7200 s
Ripe	0.3 Mg/m ³	2 mm	1 m	10 ⁻² mm/s	0°C	10,800 s	0	530 s
Refrozen	0.3 Mg/m ³	2 mm	1 m	10 ⁻² mm/s	-5°C	10,800 s	56.5 mm ³	5250 s

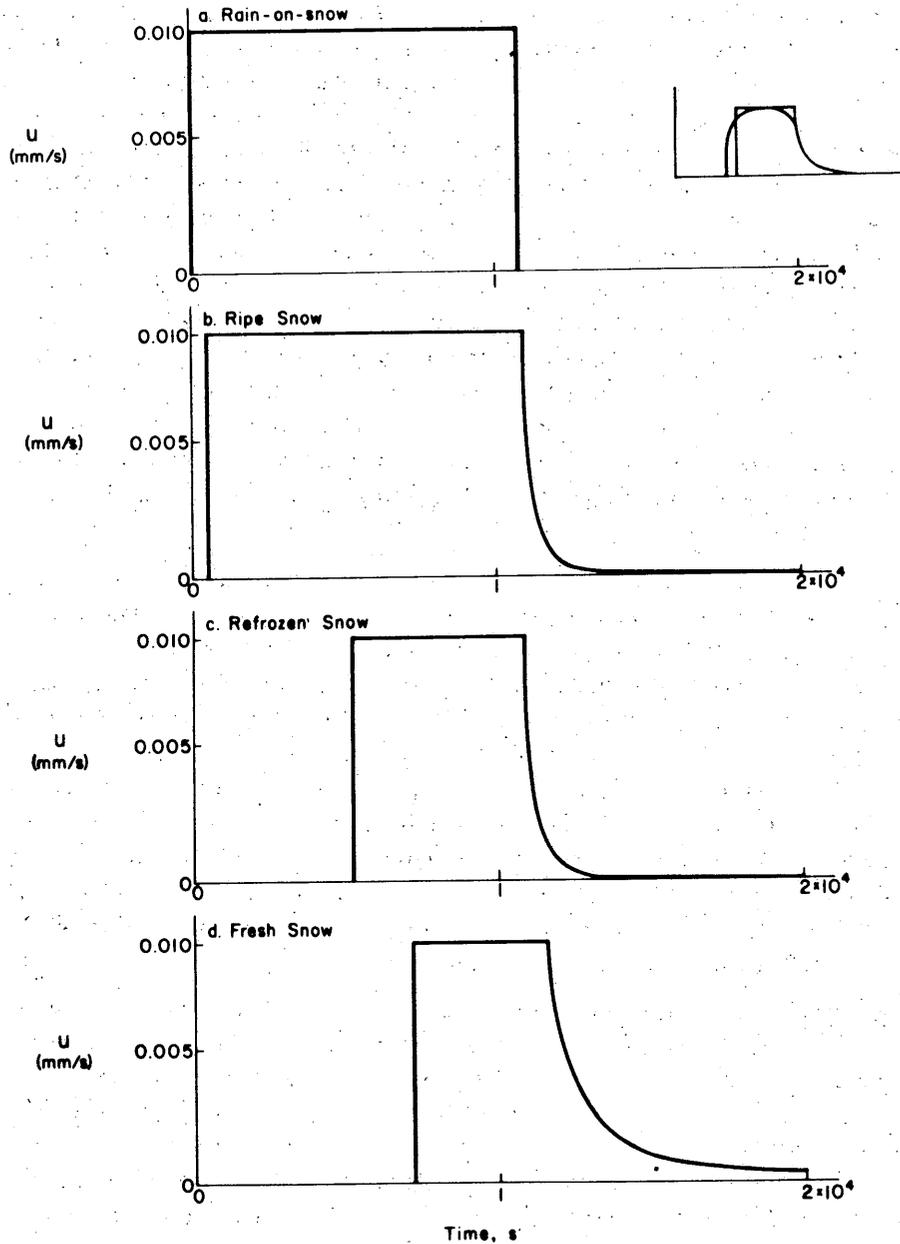


Figure 5. The flux of water as a function of time at the surface (a) and at the bottom of ripe (b), refrozen (c) and fresh (d) snows. As shown in the insert, the discharge from a real snow cover would have corners rounded by tension gradients, inhomogeneous texture of snow, and lateral variations in the flow.

and refrozen snows are 2.00 hr (7200 s) and 1.46 hr (5250 s) respectively. As shown in Figure 5, these surface fluxes represent heavy rainfalls (about 1.4 in./hr) which move quickly through the porous, ripe snow cover but are significantly delayed by refrozen and fresh snow covers. If the rain stops after 3 hours (10,800 s), 108 mm^3 of liquid has entered each mm^2 of surface area. Of this amount 102.6 mm^3 of transient water will have drained through the ripe snow cover and 5.4 mm^3 of water

will still be in transition at the cessation of the rain. The 5.4 mm^3 of water will drain at an ever decreasing rate unless more water passes the upper surface or the snow cover refreezes. By contrast, the fresh snow cover will have drained 26.8 mm^3 , 24.7 mm^3 will be in transition and 56.5 mm^3 will have been used to raise the temperature to 0°C (9.4 mm^3) and the water saturation to its residual, or irreducible, value (47.1 mm^3). Thus about one-half of the water is retained by the fresh snow cover, one-quarter is drained during the rainstorm, and one-quarter is drained slowly after the rainstorm. In addition to the retention and increased lag caused by the low temperature and small grain size of the fresh snow, the discharge from the fresh snow after cessation of rain is delayed much longer than the discharge from the ripe or refrozen snows. The lag for drainage decreases as $d^{2/3}$ so the recession limb of the hydrograph lags about 4.6 times more for fresh snow (in this example) than the recession limb for ripe and refrozen snows. During the recession the fresh snow drains about 4.6 times as much water as the other snows. Thus fresh snow greatly delays the discharge of water and stores much of the infiltrating water although the maximum rate of flow may be the same as for ripe snow. In summary, the onset of flow following a rain on fresh snow is significantly delayed, much of the rainfall is stored for release later and, following the cessation of rain, the discharge takes longer.

The refrozen snow cover considered in this example was once ripe but was subsequently refrozen to -5°C . Its response to rain-on-snow is intermediate between the other two cases. For refrozen snow, the lag time to the onset of drainage from the snow cover is more than that for ripe snow but once this drainage begins, the remainder of the drainage is similar to that in ripe snow with a quick recession following the cessation of rain. The wetting front arrives sooner for refrozen snow than for fresh snow because of the larger grain size of the refrozen snow. Compared to ripe snow, however, it is the saturation requirement rather than grain size effect that is responsible for the increased lag for refrozen snow. That is, the arrival in refrozen snow is lagged about ten times more than for ripe snow, strictly because of the water saturation term in eq 14. In contrast, the increased lag for fresh snow as compared to ripe snow is mostly due to grain size effects, although the saturation effects are also important. The total amount of runoff from the refrozen and fresh snows is equal but the runoff is distributed quite differently, simply because of the difference in grain size.

Although Figure 3 shows the rate of increase of permeability with time, this effect has been ignored here because the time constants for runoff from a short, intense rainstorm are small compared to the time constants for grain growth. Once liquid water enters fresh snow, metamorphism accelerates and permeability increases significantly on a daily basis. Hence, for the subsequent days' melt and/or rain, the permeability should be adjusted upward until the quasi-stable grain size of 2 to 3 mm is reached. If the snow is refrozen and rapid grain growth interrupted, more liquid infiltration will be necessary to restart rapid metamorphism. Each time water infiltrates into a subfreezing snow cover, a small but permanent density increase occurs due to the thermal and capillary water requirements, and each time a snow cover refreezes, the residual water content is frozen in place. The porosity decrease for each freeze-thaw cycle is about 5 to 10%, depending on the temperature at the time of liquid infiltration. This porosity decrease was not considered here because it is only a small part of the total freeze-thaw metamorphism that changes the grain size, porosity and texture of a snow cover.

The discharge hydrographs shown in Figure 5 have sharp corners because tension gradients and inhomogeneities in the snow cover were not considered in the calculations. It seems likely that tension gradients would round off the corners, although snow layers and drainage channels would have a greater effect on distorting the shape of the hydrograph. It is difficult to treat these geometrical aspects of the snow cover because of the lack of quantitative information about them. Furthermore, there is some feedback between the development of drainage channels and the flow field because of the coupling between grain growth and liquid water saturation through permeability.

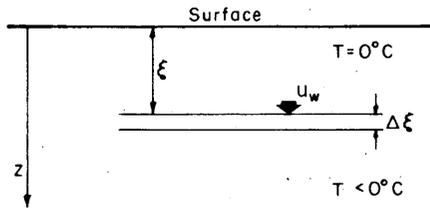
Although these effects are not included in the calculations given here, the discharge hydrographs in Figure 5 do correctly show the relative response of fresh, refrozen and ripe snow covers to a heavy rainfall. These results clearly show the importance of such variables as grain size, temperature and surface flux on the runoff from a snow cover.

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APPENDIX

I. Derivation of wetting front speed



In a time interval Δt , a volume of water $u_w \Delta t$ passes each unit area and enters the interval $\Delta \xi$. This water raises the temperature and water saturation to a level equal to

$$\left[\phi S_w + (\phi - 1) \frac{T \rho_i}{160 \rho_w} \right] \Delta \xi \quad (\text{A1})$$

before the water can pass this level. The water saturation S_w is set by the flux rate according to

$$u_w = \alpha k S^{*3} \quad (\text{A2})$$

where

$$S^* = \frac{S_w - S_{wi}}{1 - S_{wi}} \quad (\text{A3})$$

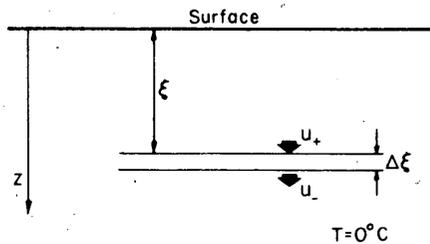
Now balancing the rate at which water enters the layer $\Delta \xi$ with the rate of propagation of the layer,

$$u_w \Delta t = \left[\phi S_w + (\phi - 1) \frac{T \rho_i}{160 \rho_w} \right] \Delta \xi \quad (\text{A4})$$

and allowing $\Delta \xi, \Delta t \rightarrow 0$,

$$\frac{d\xi}{dt} = \frac{u_w}{\phi S_w + (\phi - 1) \frac{T \rho_i}{160 \rho_w}} \quad (\text{A5})$$

II. Derivation of wave front speed



In a time interval Δt , a volume of water $u_+ \Delta t$ per unit area enters the layer $\Delta \xi$ and a volume of water $u_- \Delta t$ per unit area leaves the layer. Therefore the volume $(u_+ - u_-) \Delta t$ accumulates in the layer, raising its effective water saturation from S_- to S_+ . The balance of water is given by

$$(u_+ - u_-) \Delta t = \phi_e (S_+ - S_-) \Delta \xi \quad (\text{A6})$$

But since

$$u_+ = \alpha k S_+^3 \quad (\text{A7})$$

and

$$u_- = \alpha k S_-^3 \quad (\text{A8})$$

when $\Delta\xi, \Delta t \rightarrow 0$,

$$\frac{d\xi}{dt} = \frac{\alpha k}{\phi_e} \frac{S_+^3 - S_-^3}{S_+ - S_-} \quad (\text{A9})$$

or

$$\frac{d\xi}{dt} = \frac{\alpha k}{\phi_e} (S_+^2 + S_+ S_- + S_-^2) \quad (\text{A10})$$

III. Derivation of equation 14

The wave front speed in ripe snow is

$$\frac{d\xi}{dt} = \frac{\alpha k_r}{\phi_e} (S_+^2 + S_+ S_- + S_-^2) \quad (\text{A10})$$

and the wetting front speed in dry snow is

$$\frac{d\xi}{dt} = \frac{u_w}{\phi S_w + (\phi - 1) \frac{T\rho_i}{160\rho_w}} \quad (\text{A5})$$

Neglecting the temperature effect their ratio is

$$\frac{(d\xi/dt)_{\text{ripe}}}{(d\xi/dt)_{\text{dry}}} = \frac{\alpha k_r S_w}{1 - S_{wi}} \frac{S_+^2 + S_+ S_- + S_-^2}{u_w} \quad (\text{A11})$$

where u_w is the value of u_+ at the wetting front. Relating u_+ and u_- to S_+ and S_- through eq A7 and A8,

$$\frac{(d\xi/dt)_{\text{ripe}}}{(d\xi/dt)_{\text{dry}}} = \frac{\alpha k_r S_w}{1 - S_{wi}} \frac{u_+^{2/3} + u_+^{1/3} u_-^{1/3} + u_-^{2/3}}{\alpha^{2/3} k_r^{2/3} u_+} \quad (\text{A12})$$

or

$$\frac{(d\xi/dt)_{\text{ripe}}}{(d\xi/dt)_{\text{dry}}} = \frac{\alpha^{1/3} k_r^{1/3} S_w}{(1 - S_{wi}) u_+^{1/3}} \left[1 + \left(\frac{u_-}{u_+} \right)^{1/3} + \left(\frac{u_-}{u_+} \right)^{2/3} \right] \quad (\text{A13})$$

Now, relating u_+ and S_w through eq A3 and A7 and k to d through eq 6,

$$\frac{(d\xi/dt)_{\text{ripe}}}{(d\xi/dt)_{\text{dry}}} = \left(\frac{S_w}{S_w - S_{wi}} \right) \left(\frac{d_r}{d_d} \right)^{2/3} \left[1 + \left(\frac{u_-}{u_+} \right)^{1/3} + \left(\frac{u_-}{u_+} \right)^{2/3} \right] \quad (\text{A14})$$