9 ACCEPTANCE SAMPLING

Objectives

After studying this chapter you should

- understand the operation of acceptance sampling schemes;
- be able to draw an operating characteristic for single sampling plans using attributes, double sampling plans using attributes, and single sampling plans for variables;
- be able to select appropriate plans to meet particular conditions.

9.0 Introduction

A large supermarket sells prepacked sandwiches in its food department. The sandwiches are bought in large batches from a catering firm. The supermarket manager wishes to test the sandwiches to make sure they are fresh and of good quality. She can test them only by unwrapping them and tasting them. After the test it will no longer be possible to sell them. She must therefore make a decision as to whether or not the batch is acceptable based on testing a relatively small sample of sandwiches. This is know as **acceptance sampling**.

Acceptance sampling may be applied where large quantities of similar items or large batches of material are being bought or are being transferred from one part of an organisation to another. Unlike statistical process control where the purpose is to check production as it proceeds, acceptance sampling is applied to large batches of goods which have already been produced.

The test on the sandwiches is called a **destructive test** because after the test has been carried out the sandwich is no longer saleable. Other reasons for applying acceptance sampling are that when buying large batches of components it may be too expensive or too time consuming to test them all. In other cases when dealing with a well established supplier the customer may be quite confident that the batch will be satisfactory but will still wish to test a small sample to make sure.

Activity 1

Think of three examples where testing would be destructive.
(Hint: tests involving measuring the lifetime of items are usually destructive.)

9.1 Acceptance sampling attributes

In acceptance sampling by attributes each item tested is classified as **conforming** or **non-conforming**. (Items used to be classified as defective or non-defective but these days no self respecting manufacturing firm will admit to making defective items.)

A sample is taken and if it contains too many non-conforming items the batch is rejected, otherwise it is accepted.

For this method to be effective, batches containing some non-conforming items must be acceptable. If the only acceptable percentage of non-conforming items is zero this can only be achieved by examing every item and removing any which are non-conforming. This is known as **100% inspection** and is not acceptance sampling. However the definition of non-conforming may be chosen as required. For example, if the contents of jars of jam are required to be between 453 g and 461 g, it would be possible to define a jar with contents outside the range 455 g and 459 g as non-conforming. Batches containing up to, say 5% non-conforming items, could then be accepted in the knowledge that, unless there was something very unusual about the distribution, this would ensure that virtually all jars in the batch contained between 453 g and 461 g.

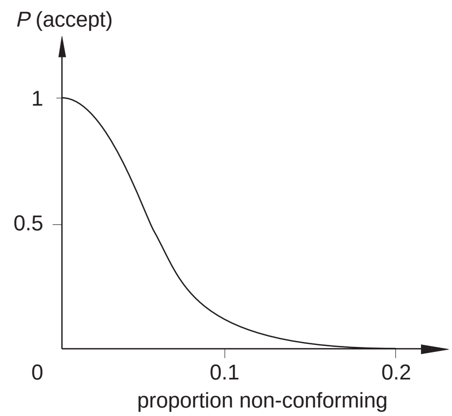
Operating characteristics

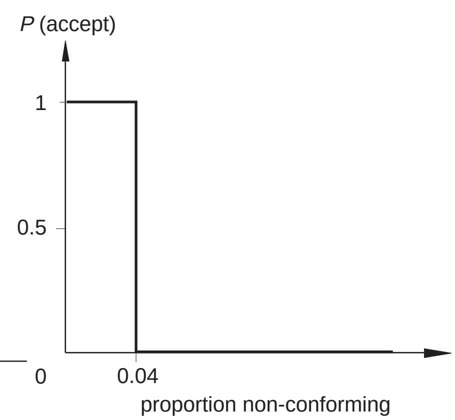
For any particular plan the **operating characteristic** is a graph of the probability of accepting a batch against the proportion non-conforming in the batch. Provided the sample is small compared to the size of the batch and the sampling is random, the probability of each member of the sample being non-conforming may be taken to be constant. In this case the number of non-conforming items in a batch will follow a binomial distribution.

One possible acceptance sampling plan is to take a sample of size 50 and to reject the batch if 3 or more non-conforming items are found. If two or less non-conforming items are found the batch will be accepted. This plan is often denoted by $n = 50$, $r = 3$. For a batch containing a given proportion of non-conforming items the probability of the sample containing two or less non-conforming items may be read directly from tables of the binomial distribution (or may be calculated). For example, if the batch contained 4% non-conforming items, the probability of any particular item in the sample being classified non-conforming is 0.04 and the probability of the batch containing two or less non-conforming items and therefore being accepted is 0.6767. The table below shows the probability of acceptance for a range of other cases.

Operating characteristics for $n = 50, r = 3$

Proportion non-conforming in batch	**Probability of accepting**
0.00	1.000
0.01	0.986
0.02	0.922
0.04	0.677
0.06	0.416
0.08	0.226
0.10	0.112
0.15	0.014
0.20	0.001





Ideally, if up to 4% non-conforming is accceptable, the probability of accepting a batch containing less than 4% non-conforming should be one and the probability of accepting a batch containing more than 4% non-conforming should be zero. If this were the case, the shape of the operating characteristic would be as shown opposite.

Activity 2

Draw the operating characteristic for $n = 50, r = 2$ (i.e. take a sample of åe axes the operating characteristic for $n = 20, r = 1$. Show on your graph the ideal shape of the operating characteristic if up to 5% non-conforming items are acceptable.

What do you notice about the graphs?

The larger the sample size the steeper the graph. That is, the larger the sample size, the better the plan discriminates between good batches (i.e. batches with a small proportion of non-conforming items) and bad batches (i.e. batches with a large proportion of non-conforming items). Note that, provided the batch is large enough for the binomial distribution to give a good approximation to the probabilities, it is the number of items inspected which determines how good the sampling plan is. The proportion of the batch inspected is not important. Provided the sampling is random it will be better to test say 100 items from a batch of 5000 than to test 10 items from a batch of 500.

Example

A manufacturer receives large batches of components daily and decides to institute an acceptance sampling scheme. Three possible plans are considered, each of which requires a sample of 30 components to be tested:

Plan A: Accept the batch if no non-conforming components are found, otherwise reject.

Plan B: Accept the batch if not more than one non-conforming component is found, otherwise reject.

Plan C: Accept the batch if two or fewer non-conforming components are found, otherwise reject.

(a) For each plan, calculate the probability of accepting a batch containing

(i) 2% non-conforming

(ii) 8% non-conforming.

(b) Without further calculation sketch on the same axes the operating characteristic of each plan.

(c) Which plan would be most appropriate in each of the circumstances listed below?

(i) There should be a high probability of accepting batches containing 2% non-conforming.

(ii) There should be a high probability of rejecting batches containing 8% non-conforming.

(iii) A balance is required between the risk of accepting batches containing 8% defective and the risk of rejecting batches containing 2% non-conforming.

Solution

(a) The probability may be calculated or be obtained directly from tables of the binomial distribution.

For a batch containing 2% non-conforming, the probability of any member of the sample being a non-conforming component is 0.02. (Remember the batch is large so the fact that the sample will normally be drawn without replacement will have a negligible effect on the probabilities of the later members of the sample.) The probability of any member of the sample not being a non-conforming component is

$$1 - 0.02 = 0.98 .$$

The probability of no non-conforming components in the sample is

$$0.98^{30} = 0.545$$

and this is the probability of the batch being accepted if **Plan A** is used.

If **Plan B** is used the batch will be accepted if the sample contains 0 or 1 non-conforming items and the probability of this is

$$0.98^{30} + 30 \times 0.02 \times 0.98^{29} = 0.879 .$$

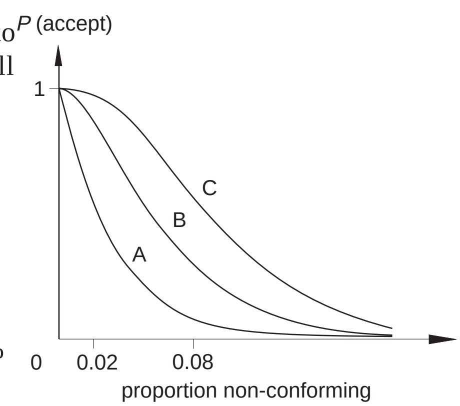
If **Plan C** is used the batch will be acepted if the sample contains 0, 1 or 2 non-conforming components. The probability of this is

$$0.98^{30} + 30 \times 0.02 \times 0.98^{29} + 435 \times 0.02^2 \times 0.98^{28} = 0.978 .$$

Similar calculations may be carried out when the batch contains 8% non-conforming components, or the probabilities may be read directly from tables of the binomial distribution with $n = 30,\ p = 0.08$. This gives the following results for the probability of acceptance

Plan A: 0.082 **Plan B:** 0.296 **Plan C:** 0.565

(b) From part (a) we have two points on the operating characteristic for each plan. In addition, all operating characteristics go through the point (0, 1) because if the batch contains no non-conforming components, every sample will contain no non-conforming components and this must lead to the batch being accepted. Every operating characteristic will also pass through the point (1, 0). However this part of the curve is of no interest. It corresponds to batches which contain only non-conforming items. Acceptance sampling would not be used if there was any possibility of this occurring. The graphs may now be sketched as shown opposite.



(c) (i) Plan C would be the most suitable as it has the highest probability (0.978) of accepting a batch containing 2% non- conforming.

(ii) Plan A has the lowest probability (0.082) of accepting a batch containing 8% non-conforming. Plan A is therefore the most suitable as the probability of rejecting a batch containing 8% non-conforming is $1 - 0.082 = 0.918$, and this is highest of the three plans.

(iii) Plan B would be the most suitable in this case. It can be seen from the graph that it has a lower probability than A of rejecting a batch containing 2% non-conforming and a lower probability than C of accepting a batch containing 8% non-conforming.

Example

(a) An acceptance sampling scheme consists of inspecting 25 items and rejecting the batch if two or more non-conforming items are found. Find the probability of accepting a batch containing 15% non-conforming. Find also the probability of accepting batches containing 2, 4, 6, 8, 10 and 20% non-conforming.

(b) The manufacturer requires a plan with a probability of not more than 0.05 of rejecting a batch containing 3% non-conforming. If the sample size remains 25, what should the criterion be for rejecting the batch if the manufacturer's risk is to be just met?

(c) It is decided to increase the number of items inspected to 50. What should the criterion be for accepting a batch if the consumer's risk of accepting a batch containing 15% non-conforming is to be as near as possible to 10%? Plot the operating characteristic for this plan on the same axes as the first. Does this plan satisfy the manufacturer's risk specified in (b)?

(d) Discuss the factors to be considered when deciding which of the plans to use. (AEB)

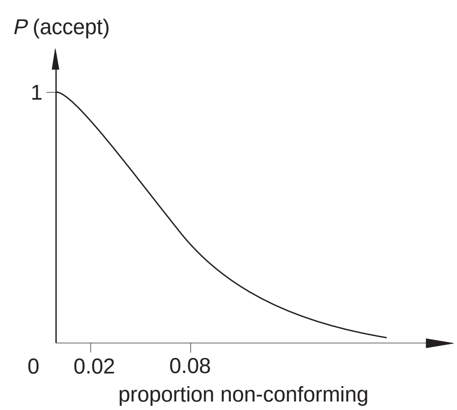
Solution

(a) The batch will be accepted if 0 or 1 non-conforming items are found in a sample of 25 from a batch containing 15%. This may be calculated using the binomial distribution $n = 25,\ p = 0.15$ or read from tables. The probability is

$$0.85^{25} + 25 \times 0.15 \times 0.85^{24} = 0.0931$$

You may wish to check the following figures

Proportion non-conforming	*P*(accept)
0.02	0.911
0.04	0.736
0.06	0.553
0.08	0.395
0.10	0.271
0.20	0.027



(b) For a batch containing 3% non-conforming the probability of r or less non-conforming items in a sample of 25 is given below.

r	P(r or less)
0	0.467
1	0.828
2	0.962
3	0.994

You may check these figures using the binomial distribution. The manufacturer requires a plan with a probability of not more than 0.05 of rejecting a batch containing 3% non-conforming. That is, a probability of at least 0.95 of accepting the batch. The table shows that the probability of the sample containing 2 or less is 0.962, thus $n = 25, r = 3$ will just meet this requirement. (Note accepting if 2 or less are found implies rejecting if 3 or more are found.)

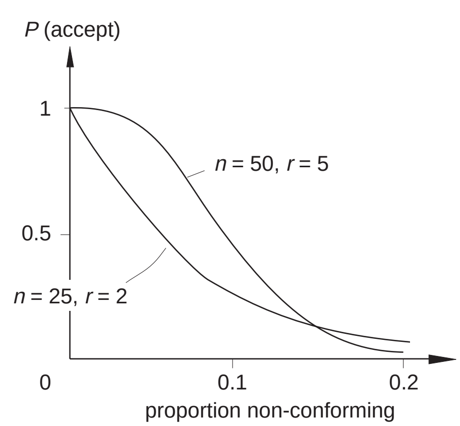
(c) Binomial distribution $n = 50,\ p = 0.15$

r	P(r or less)
1	0.003
2	0.014
3	0.046
4	0.112
5	0.219

A consumer's risk of about 10% or 0.10 of accepting a batch containing 15% non-conforming is given by accepting batches if 4 or less non-conforming items are found. (As can be seen from the table above, the probability of finding 4 or less is 0.112.) This gives the plan $n = 50, r = 5$.

For this plan

proportion non-conforming	P(accept)
0.02	0.997
0.04	0.951
0.07	0.729
0.10	0.431
0.15	0.112
0.20	0.018



From the operating characteristic it can be seen that the probability of accepting a batch containing 3% or 0.03

non-conforming is about 0.98. Thus the probability of rejecting it is about 0.02 which is well below the 0.05 specified in (b). Hence it does meet the manufacturer's risk.

(d) The plan requiring a sample of 50 will require more testing to be carried out and will thus be more expensive. As can be seen from the operating characteristics, it discriminates better between good and bad batches, giving a higher probability of accepting good (small proportion non-conforming) batches and a higher probability of rejecting bad (large proportion non-conforming) batches.

The cost of the extra sampling should be balanced against the cost of making wrong decisions, i.e. the waste involved in rejecting a good batch and the problems and frustrations caused by accepting a bad batch.

Note: This question was phrased in terms of manufacturer's risk and consumer's risk, the idea being that only the manufacturer was concerned if a good batch was rejected and only the consumer was concerned if a bad batch was accepted. These terms are rarely used these days as it is recognised that it is in no one's interest for mistakes to be made. If bad batches are accepted the manufacturer will be faced with customer complaints which are expensive to deal with and, in the long run, business will suffer. If good batches are rejected, the cost of unnecessarily replacing them - or at the least the cost of extensive extra testing - will eventually be borne by the consumer.

Exercise 9A

1. An acceptance sampling scheme consists of taking a sample of 20 from a large batch of items and accepting the batch if the sample contains 2 or less non-conforming items. Draw the operating characteristic for this scheme.

2. An engine component is defined to be defective if its length (in 0.001mm) is outside the range 19950 to 20050.

   (a) An acceptance sampling scheme consists of taking a sample of size 50 from each batch and accepting the batch if the sample contains 2 or fewer defectives. If the sample contains 3 or more defectives the batch is rejected.

   Find the probability of accepting batches containing 2%,5%,10% and 15% defective and draw the operating characteristic.

   (b) The customer complains that the plan in (a) has far too high a risk of accepting batches containing a large proportion of defectives. As far as she is concerned a batch containing 1 in 1000 defectives is bad but she will agree that a batch containing 1 in 10000 defectives is good.

   (i) If lengths of components are normally distributed with mean 20000 and standard deviation 12.8, what proportion are defective?

   (ii) It is decided to define components outside the range $20000 \pm k$ as non-conforming. Find the value of k to two significant figures which will give 5% non-conforming items for this distribution.

   (iii) If the distribution of lengths in a batch is normal with mean 20010 and standard deviation 12.8 about 1 component in 1000 will be defective. What proportion will be non-conforming? If the plan in (a) is applied to non-conforming instead of defective components find from your operating characteristic the probability of accepting this batch.

   (c) Explain why the plan in (b) (iii) should satisfy the customer. (AEB)

9.2 Double sampling plans

The following is an example of a **double sampling plan**.

Take a sample of size 30. Accept the batch if 0 or 1 non-conforming items are found and reject the batch if 3 or more non-conforming items are found. If exactly 2 non-conforming items are found take a further sample of size 30. Accept the batch if a total of 4 or fewer (out of 60) are found, otherwise reject the batch. This plan is denoted

$$n = 30; \quad a = 1, r = 3,$$

$$n = 30; \quad a = 4, r = 5.$$

*T*he acceptance number is a,i.e. the batch will be accepted if up to a non-conforming items are found. The rejection number is r, i.e. the batch will be rejected if r or more non-conforming items are found.

Note that the acceptance and rejection numbers refer to all items that have been inspected, not just to the most recent sample. There is no reason why the first and second sample need be of the same size, but in practice this is nearly always the case.

The idea behind double sampling plans is that a very good batch or a very bad batch may be detected with a relatively small sample but for an intermediate batch it is desirable to take a larger sample before deciding whether to accept or reject.

Example

A firm is to introduce an acceptance sampling scheme. Three alternative plans are considered.

Plan A Take a sample of 50 and accept the batch if no non-conforming items are found, otherwise reject.

Plan B Take a sample of 50 and accept the batch if 2 or fewer non-conforming items are found.

Plan C Take a sample of 40 and accept the batch if no non-conforming items are found. Reject the batch if 2 or more are found. If one is found, then take a further sample of size 40. If a total of 2 or fewer (out of 80) is found, accept the batch, otherwise reject.

(a) Find the probability of acceptance for each of the plans A, B and C if batches are submitted containing

(i) 1% non-conforming (ii) 10% non-conforming.

(b) Without further calculation, sketch on the same axes the operating characteristic for plans A, B and C.

(c) Show that, for batches containing 1% non-conforming, the average number of items inspected when using plan C is similar to the number inspected when using plans A or B.

(AEB)

Solution

(a) **Plan A:** accept 0.

$$P(\text{accept}) = (1-p)^{50}.$$

For $p = 0.01$, $P(\text{accept}) = 0.99^{50} = 0.605$;

for $p = 0.1$, $P(\text{accept}) = 0.9^{50} = 0.005$.

Plan B: accept 0, 1 or 2.

$P(\text{accept})$

$$= (1-p)^{50} + 50 \times p \times (1-p)^{49} + \left(50 \times \frac{49}{2}\right) \times p^2 \times (1-p)^{48}$$

for $p = 0.01$, P (accept) $= 0.986$;

for $p = 0.1$, P (accept) $= 0.112$

Plan C: accept 0 in first sample (in which case no second sample will be taken) or 1 in first sample and 0 in second sample or 1 in first sample and 1 in second sample.

There are no other ways of accepting the batch - if 2 or more are found in the first sample the batch is immediately rejected and if 1 is found in the first sample and 2 or more in the second (giving a total of 3 or more) the batch is rejected.

The samples are of equal size and the batch is large so the probability of acceptance may be expressed as

$$P(0) + P(1) \times P(0) + P(1) \times P(1)$$

$$P(0) = (1-p)^{40} \qquad P(1) = 40 \times p \times (1-p)^{39}.$$

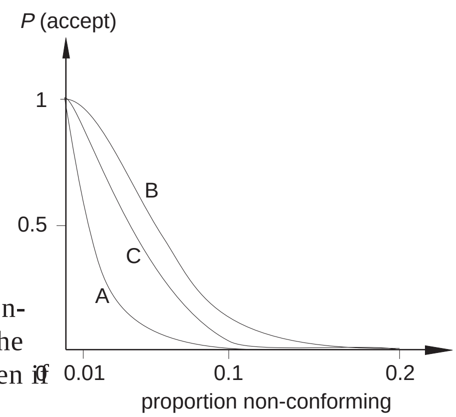
For $p = 0.01$ $P(0) = 0.669$ and $P(1) = 0.270$

$P(\text{accept}) = 0.669 + 0.270 \times 0.669 + 0.270^2 = 0.923$.

For $p = 0.1$ $P(0) = 0.0148$, $P(1) = 0.0.0657$.

$P(\text{accept}) = 0.0148 + 0.0657 \times 0.0148 + 0.0657^2 = 0.020$.

(b) The operating characteristics are shown opposite.



(c) For **Plan C**, if the first sample contains 0 or 2 or more non-conforming, a decision as to whether to accept or reject the batch is made immediately. A second sample is only taken if the first sample contains exactly 1 non-conforming item. The average number of items inspected is

$$40 + 40 \times P(1)$$

For batches containing 1% non-conforming the average number of items inspected is

$$40 + 40 \times 0.270 = 50.8.$$

Thus the average number inspected is similar to the 50 inspected in the single sample plans.

Note: This calculation only applies when $p = 0.01$. For other values of *p* you would have to make a further calculation. However it can be stated that if a single and a double sampling plan have similar operating characteristics (not the case here), the double sampling plan will, on average, require less items to be inspected than the single sampling plan. This will be true for any value of *p*. Against this, the double sampling plan is more complex to operate.

Activity 3

The three plans in the previous example are to be considered for use in a situation where it is expected that most batches submitted will contain about 1% non-conforming but that occasionally batches will contain about 10% non-conforming. Decide which of the three plans would be most suitable in each of the following cases:

(i) it is important that batches containing 1% non-conforming should be accepted as frequently as possible;

(ii) it is important that batches containing 10% non-conforming should be rejected as frequently as possible;

(iii) a balance should be struck between the risk of accepting batches containing 10% non-conforming and the risk of rejecting batches containing 1% non-conforming.

Example

The following acceptance sampling plans have similar operating characteristics.

Plan 1 Take a sample of size 80 and reject the batch if 6 or more non-conforming items are found.

Plan 2 Take a sample of size 50 and accept the batch if 2 or fewer non-conforming items are found. Reject the batch if 5 or more non-conforming items are found. If 3 or 4 non-conforming items are found take a further sample of size 50 and reject the batch if a total of 7 or more non-conforming items (out of 100) are found. Otherwise accept.

The following table gives the probability of obtaining *r* or less successes in *n* independent trials when the probability of success in a single trial is 0.04.

r	$n=50$	$n=80$
0	0.1299	0.0382
1	0.4005	0.1654
2	0.6767	0.3748
3	0.8609	0.6016
4	0.9510	0.7836
5	0.9856	0.8988
6	0.9964	0.9588
7	0.9992	0.9852

(a) Verify that both plans have similar probabilities of accepting batches containing 4% non-conforming.

(b) The cost of the sampling inspection is made up of the cost of obtaining the sample plus the cost of carrying out the inspection. A firm estimates that for a sample of size n the cost, in pence, of obtaining the sample is $400+4n$ and the cost of inspection is $24n$. For batches containing 4% non-conforming, compare the expected cost of the following three inspection procedures:

(i) Use **Plan 1**;

(ii) Use **Plan 2**, obtaining the second sample of 50 only if required to do so by the plan;

(iii) Use **Plan 2**, but obtain a sample of 100. Inspect the first 50, but only inspect the second 50 if required to do so by the plan. (AEB)

Solution

(a) For **Plan 1**, $n=80, p=0.04$; accept if 5 or less found

From table $P(\text{accept})=0.8988$.

For **Plan 2**, the batch can be accepted in the following ways

1st sample	2nd sample
0	
1	
2	
3	0
3	1
3	2
3	3
4	0
4	1
4	2

$$P(\text{accept}) = P(0)+P(1)+P(2)+P(3)P(0)+P(3)P(1)+$$
$$P(3)P(2)+P(3)P(3)+P(4)P(0)+P(4)P(1)+P(4)P(2)$$

This can be evaluated using the table given, noting that

$$P(r) = P(r \text{ or less}) - P(r-1 \text{ or less}).$$

Thus for example

$$P(4) = 0.9510 - 0.8609 = 0.0901$$

However, the evaluation can be speeded up by writing

$P(\text{accept})$

$= P(2 \text{ or less}) + P(3)P(3 \text{ or less}) + P(4)P(2 \text{ or less})$

$= 0.6767 + (0.8609 - 0.6767)0.8609 + 0.0901 \times 0.6767$

$= 0.896.$

This probability is similar to the 0.899 obtained for **Plan 1**.

(b) (i) The cost for **Plan 1** is $400 + 4\times 80 + 24 \times 80 = £26.40$.

(ii) In this case the second sample of 50 will be obtained only if the first sample contains 3 or 4 defectives. The probability of this occurring is

$$0.9510 - 0.6767 = 0.2743$$

The expected cost is the cost of obtaining and testing the first sample plus $0.2743\times$(the cost of obtaining and testing the second sample)

$$= 400 + 4\times 50 + 24\times 50 + 0.2743(400 + 4\times 50 + 24 \times 50)$$

$$= £22.94$$

(iii) The expected cost is now the cost of obtaining a sample of 100 and testing 50 of these plus $0.2743\times$(the cost of testing a further 50)

$$= 400 + 4\times 100 + 24\times 50 + 0.2743 \times 24 \times 50$$
$$= £23.29$$

Hence the expected cost of the double sampling plan is less than that of the single sampling plan no matter whether two separate samples of 50 are taken as required, or a single sample of 100 is taken. This calculation, of course, applies only to the case where batches containing 4% non-conforming are submitted. However, the conclusion is probably true for all other possible batches. The double sampling plan is, however, more complex to operate.

Activity 4

For Plan 2 in the Example above, calculate the expected number of items inspected if the proportion non-conforming in the submitted batch is 0.00, 0.02, 0.04, 0.06, 0.08, 0.10 and 0.15. Draw a graph of this expected number against the proportion non-conforming.

Is this graph consistent with the statement that, for plans with similar operating characteristics, the expected number inspected will be less for a double sampling plan than for a single sampling plan?

Would it be possible to construct a triple sampling plan?

Exercise 9B

1. (i) An acceptance sampling scheme consists of taking a sample of size 20 and accepting the batch if no non-conforming items are found. If 2 or more non-conforming items are found the batch is rejected. If 1 non-conforming item is found a further sample of 20 is taken and the batch is accepted if a total of 2 or fewer (out of 40) non-conforming items are found. Otherwise it is rejected. This plan is denoted

$$n = 20,\ \ a = 0,\ \ r = 2$$

$$n = 20,\ \ a = 2,\ \ r = 3\,.$$

Find the probability of accepting a batch containing 4% non-conforming.

(ii) Find the probability of accepting a batch containing 3% non-conforming for the plan

$$n = 40,\ \ a = 0,\ \ r = 3$$

$$n = 40,\ \ a = 2,\ \ r = 3$$

(iii) Find the probability of accepting a batch containing 5% non-conforming for the plan

$$n = 30,\ \ a = 0,\ \ r = 3$$

$$n = 30,\ \ a = 3,\ \ r = 4$$

2. When checking large batches of goods the following acceptance sampling plans have similar operating characteristics.

Plan 1: Take a sample of size 50 and accept the batch if 3 or fewer non-conforming items are found, otherwise reject it.

Plan 2: Take a sample of size 30, accept the batch if zero or one non-conforming items are found and reject the batch if 3 or more are found. If exactly 2 are found, take a further sample of size 30. Accept the batch if a total of 4 or fewer (out of 60) are found, otherwise reject it.

(a) Using the following table, verify that the two plans have similar probabilities of accepting a batch containing 5% non-conforming. The table gives the probability of obtaining r or more successes in n independent trials when the probability of a success in a single trial is 0.05.

r	$n = 30$	$n = 50$
0	1.0000	1.0000
1	0.7854	0.9231
2	0.4465	0.7206
3	0.1878	0.4595
4	0.0608	0.2396

(b) For the second plan, evaluate the expected number of items inspected each time the plan is used when the proportion non-conforming in the batch is 0, 0.02, 0.05, 0.10 and 1.00. Sketch a graph of the expected number of items inspected against the proportion non-conforming in the batch.

(c) What factors should be considered when deciding which of the two plans is to be used?

9.3 Acceptance sampling by variable

Acceptance sampling can be carried out by measuring a variable rather than classifying an item as conforming or non-conforming. Variables such as thickness, strength or weight might be measured. A typical plan would be to take a sample of size *n* and reject the batch if the mean measurement, $\bar{x}$, is less than *k*. This would be appropriate for, say, the strength of a batch of climbing ropes where a large value is desirable. If the variable was, say, percentage of impurity in raw material,where a small value *w*as desirable, the plan would be of the form - take a sample of size *n* and reject the batch if the mean measurement, $\bar{x}$, is greater than *k*.

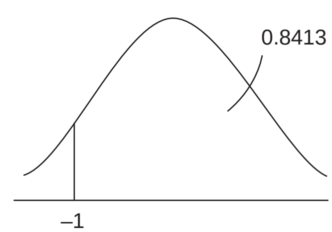
Usually it is easier and quicker to classify an item as conforming or non-conforming than to make an exact measurement. However, the information gained from an exact measurement is greater and so smaller sample sizes are required. A decision as to whether to use attributes or variables will depend on the particular circumstances of each case.

Operating characteristic

A component for use in the manufacture of office machinery will fail to function if the temperature becomes too high. A batch of these components has a mean failure temperature of 95.6˚C. The standard deviation is 2.4˚C. The company receiving this batch operates the following acceptance sampling scheme - test a sample of size 16 and reject the batch if the mean failure temperature is less than 95.0˚C.

It is reasonable to assume normal distribution since we are concerned with the mean of a reasonably large sample. The batch will be accepted if the sample mean exceeds 95.0˚C.

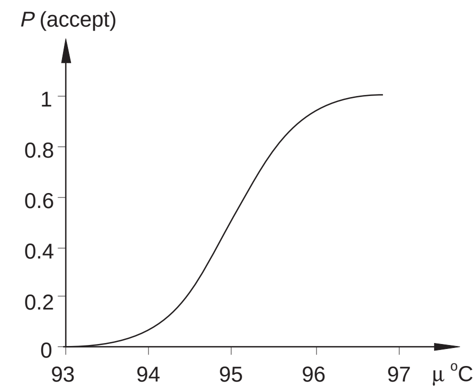
$$z = \frac{95 - 95.6}{\left(\frac{2.4}{\sqrt{16}}\right)} = -1$$



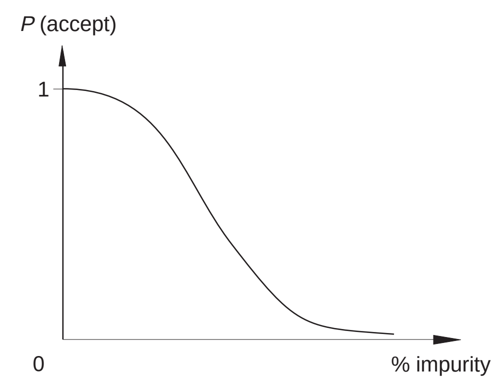
The probability of the batch being accepted is 0.841.

The operating characteristic can be constructed by carrying out this calculation for batches with different means (assuming the standard deviation remains at 2.4˚C). The calculations can be put in a table as shown on the next page. (Be careful to use the correct tail of the normal distribution, this will depend on the sign of *z* and will change when this changes).

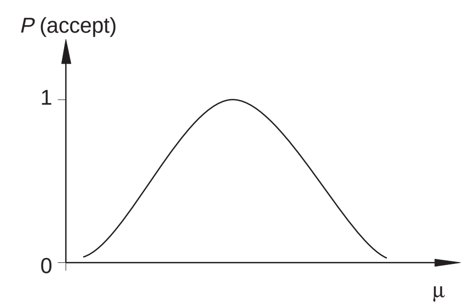
μ	$(k-\mu)/\left(\frac{\sigma}{\sqrt{n}}\right)$	P(accept)
93.2	3.0	0.001
93.8	2.0	0.023
94.4	1.0	0.159
94.7	0.5	0.308
95.0	0.0	0.500
95.3	–0.5	0.691
95.6	–1.0	0.841
96.2	–2.0	0.977
96.8	–3.0	0.999



Note that the shape of the operating characteristic is a reflection in a vertical line of the typical shape for an attributes scheme. This is because, in this case, the good batches have large mean values whereas for attributes good batches have small proportions of non- conforming items.



An operating characteristic for percentage impurity, where a good batch has a low mean, would have shape shown opposite.



In other cases, such as the diameter of screw caps for bottles of vinegar, the mean of a good batch must be neither too big nor too small and the shape of the operating characteristic would be as shown in the diagram on the right.

Is it possible for P(accept) to equal one in an acceptance sampling by variables scheme?

Activity 5

Think of an example where acceptance sampling by variables could be applied and the value of the variable should be

(i) as large as possible,

(ii) as small as possible,

(iii) neither too large nor too small.

Example

(a) Before cement is delivered to a civil engineering site, a number of small bricks are made from it. Five are chosen at random and measured for compressive strength (measured in $\mathrm{N\,m^{-2}} \times 10^9$). This is known to be normally distributed with standard deviation 5.5. The batch of cement is accepted for delivery if the mean compressive strength of the five bricks is greater than 51. Draw the operating characteristic for this plan.

(b) It is decided to redesign the plan. The customer requires that the probability of accepting a batch with a mean strength of 47 or less should be less than 0.1. The manufacturer requires that the probability of rejecting a batch with a mean strength of 52.5 or more should be less than 0.05. By consulting your operating characteristic which, if either, of these criteria are satisfied by the current plan.

(c) If n is the sample size and k is the compressive strength which must be exceeded by the sample mean for the batch to be accepted, find the minimum value of n to satisfy the manufacturer's requirements if k remains at 51.

(d) If k is changed to 49.4, find the minimum value of n to satisfy the customer's requirements. Verify that using this value of n the manufacturer's requirements will also be met. (AEB)

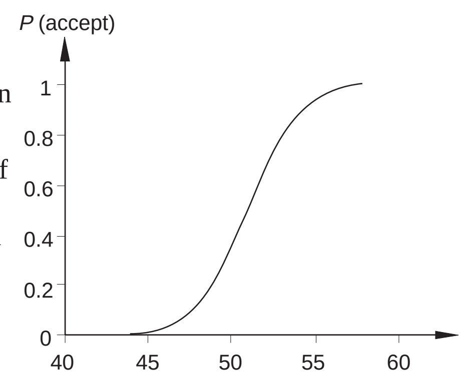
Solution

(a) The operating characteristic is a graph of probability of acceptance against mean strength of bricks from the batch of cement. First, suitable values of this mean strength must be chosen so that the probability of acceptance can be calculated and the graph drawn. The standard deviation is 5.5. Since samples of size five are being taken, the standard error is

$$\frac{5.5}{\sqrt{5}} = 2.46.$$ For most purposes a graph which extends between 2 and 3 standard errors either side of k will be adequate. In this case, say, 44 to 58. Steps of 2 will give 8 points and this will usually be adequate. If a more detailed graph is required, further points can be interpolated and the range can be extended.

μ	$(51-\mu)/\left(\frac{5.5}{\sqrt{5}}\right)$	P(accept)
44	2.846	0.002
46	2.033	0.021
48	1.220	0.111
50	0.407	0.342
52	–0.407	0.658
54	–1.220	0.889
56	–2.033	0.979
58	–2.846	0.998

Note: Interpolation was used in reading from tables of the normal distribution. However to find P(accept) to 3 decimal places this only affected the result for the middle two points and then only by 0.001.



(b) From the graph the probability of accepting a batch with a mean strength of 47 is approximately 0.05. This is less than 0.1 and so satisfies the customer's requirement.

The probability of accepting a batch with a mean strength of 52.5 is approximately 0.73. Hence the probability of rejecting it is approximately 0.27. This is much larger than 0.05 and so does not meet the manufacturer's requirement.

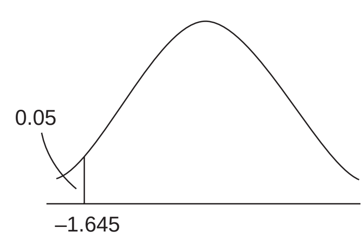
(c) To satisfy the manufacturer's requirement

$$z = \frac{(51-52.5)}{\left(\frac{5.5}{\sqrt{n}}\right)} < -1.645$$

$$-0.2727\sqrt{n} < -1.645$$

$$\sqrt{n} > 6.032$$

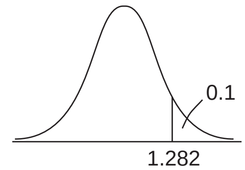
$$n > 36.4$$



The minimum value of n to satisfy the manufacturer's requirement is 37.

(d) To satisfy the customer's requirement

$$z = \frac{(49.4 - 47)}{\left(\frac{5.5}{\sqrt{n}}\right)} > 1.282$$



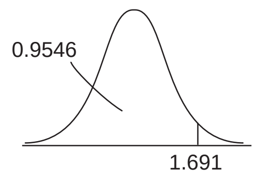
$$0.4364\sqrt{n} > 1.282$$

$$\sqrt{n} > 2.94$$

$$n > 8.63$$

The minimum value of n to satisfy the customer's requirement is 9.

To calculate the manufacturer's risk if $n = 9$



$$z = \frac{(52.5 - 49.4)}{\left(\frac{5.5}{\sqrt{9}}\right)} = 1.691$$

Probability of accepting the batch is 0.955. Probability of rejecting is $1 - 0.955 = 0.045$. This is less than 0.05 and so satisfies the manufacturer's risk.

Exercise 9C

1. An acceptance sampling plan consists of weighing a sample of 6 loaves of bread and accepting the batch if the sample mean is greater than 900g. Draw the operating characteristic if the standard deviation is known from past experience to be 12g.

2. An acceptance sampling plan consists of measuring the percentage of fat in a sample of 8 prepackaged portions of boiled ham. The batch is rejected if the mean proportion exceeds 42%. If the standard deviation is estimated to be 3%, draw the operating characteristic.

3. The quality of a certain chemical is measured by the time it takes to react. (The shorter the time, the better the quality). This time is known to be normally distributed with a standard deviation of 8 seconds. Nine samples are taken from each batch and the batch accepted if the mean reaction time is less than 33.5 seconds.

   (a) Draw the operating characteristic for this plan.

   (b) The manufacturer requires a plan which has a probability of rejection of less than 0.05 if the mean reaction time of the batch is 30 seconds. The customer requires a plan that has a probability of acceptance of less than 0.10 if the mean reaction time of the batch is 35 seconds. Use your operating characteristic to find which, if either,of these conditions this plan will meet.

   (c) If the criterion for acceptance remains unchanged, find the smallest sample size that would enable the plan to satisfy the customer's requirement.

   (d) If the criterion for acceptance is for the sample to be accepted if the mean is less than 32.8 seconds, find the smallest sample size that would enable the plan to satisfy the manufacturer's requirement. Verify that this plan would also satisfy the customer's requirement.

9.4 Miscellaneous Exercises

1. (a) A manufacturer will accept a risk of not more than 10% of a batch of items containing 2% non-conforming being rejected. If a decision is to be made by examining a sample of 50 items, find the appropriate decision procedure.

   (b) Draw the operating characteristic for the above plan and indicate on the graph the ideal shape of the operating characteristics if batches containing up to 5% non-conforming are acceptable.

   (c) Would this plan satisfy a customer who specified a risk of not more than 5% of a batch containing 11% non-conforming being accepted?

2. (a) Large batches of wrappers for sliced loaves are to be checked by examining a random sample of 50. If the customer will accept a risk of not more than 5% of a batch containing 10% non-conforming wrappers being accepted, what should the criterion be for rejecting the batch?

   (b) A double sampling plan is specified by

   $$n = 20, \quad a = 0, \quad r = 2$$

   $$n = 20, \quad a = 2, \quad r = 3$$

   (i) What is the probability of a batch containing 10% non-conforming being accepted?

   (ii) What is the average number of items inspected when batches containing 10% non-conforming are submitted?

3. A random sample of 20 from a large batch of components is to be tested, and by counting the number non-conforming, a decision is to be made as to whether the batch should be accepted or rejected by the customer. If the producer is willing to accept the risk of not more than 2% of a batch containing 1% or less non-conforming being rejected, what should be the criterion for rejecting batches? Using tables, plot on graph paper the operating characteristic for this scheme. If the sample size is increased to 50 but the producer's risk is unchanged, plot the operating characteristic of this new scheme on the same graph paper.

   Compare the risk of accepting a batch containing 9% non-conforming components for the two schemes. Sketch, on the same graph paper, the ideal shape of the operating characteristic if a batch containing up to 4% non-conforming is mutually acceptable to both the producer and the customer.

4. A wholesaler packs sugar into bags of nominal weight 1000 g with an automatic machine. It is known from previous experience with the machine that the weights of bags are normally distributed with standard deviation 5 g.

   A retailer, considering the purchase of a large batch, does not want too many bags to be noticeably underweight: he states that an acceptable sampling scheme must be such that if the mean weight per bag is 1000 g, the probability of the batch being accepted must be no more than 0.10.

   The wholesaler, who wishes to avoid repacking the bags, states that if the mean weight per bag is 1005 g, the probability of rejection must be no more than 0.05.

   (a) Design a sampling and decision procedure to satisfy both the wholesaler and retailer.

   (b) Plot the operating characteristic for this sampling scheme.

5. (a) An acceptance sampling scheme consists of taking a sample of 25 from a large batch of components and rejecting the batch if 3 or more non-conforming items are found. What is the probability of accepting batches containing 2%, 4%, 6%, 10%, 15% and 20% non-conforming?

   Use your results to draw an operating characteristic.

   From your operating characteristic, estimate
   (i) the probability of accepting a batch containing 11% non- conforming,

   (ii) the proportion non-conforming in a batch that has a probability of 0.6 of being rejected.

   (b) An alternative plan requires a sample of 40 to be taken from the batch and the batch to be rejected if four or more non-conforming are found. Verify that both plans have similar probabilities of of rejecting batches containing 4% non-conforming, and comment on the advantages and disadvantages of the second plan compared to the first.

   (c) If more than one out of eight successive batches from a particular supplier are rejected, a more stringent form of inspection is introduced. What is the probability of more than one out of the next eight batches being rejected if all batches contain 4% non-conforming?

   (d) The more stringent inspection requires samples of 100 from each batch. The original form of inspection is reinstated if a sample contains no non-conforming items. What is the proportion non-conforming in the batch if the probability of no defectives in a sample of 100 is 0.5? (AEB)

6. (a) A hotel group buys large quantities of towels for use by guests. When a batch is received, a sample of 25 towels is subjected to a test of water absorption. If no more than one towel fails the test the batch is accepted. If two or three towels fail, a further sample of 25 towels is tested. The batch is then accepted if a total of no more than three (out of 50) fail the test. Otherwise it is rejected. If a batch of towels, containing 7% which would fail the test, is submitted what is

    (i) the probability of its being rejected,

    (ii) the expected number of towels inspected?

   (b) In another test, the towels are checked for visual defects. If the defects are distributed at random with a mean of 2 defects per towel, how many defects would be exceeded (on a particular towel) with a probability of just over 5%?

   (c) In a final check, the lengths of 25 towels are measured and the batch rejected if the mean length is less than a specified value k. What should be the value of k to give a probability of 0.99 of accepting a batch with mean length 106 mm and standard deviation 6 mm?

   (d) For towels from a particular supplier, the probabilities of a batch failing these tests are p_1, p_2 and p_3, respectively. Write down an expression for the probability of the batch passing all three tests, stating any assumption you have needed to make.

(AEB)