

## Chapter 642

# Confidence Intervals for One Standard Deviation using Relative Error

## Introduction

This routine calculates the necessary sample size such that a sample standard deviation estimate will achieve a specified relative distance from the true population standard deviation at a stated confidence level when the underlying data distribution is normal.

Caution: This procedure controls the relative width of the interval as a proportion of the true population standard deviation. For controlling the absolute width of the interval see the procedures Confidence Intervals for One Standard Deviation using Standard Deviation and Confidence Intervals for One Standard Deviation with Tolerance Probability.

## Technical Details

Following the results of Desu and Raghavarao (1990) and Greenwood and Sandomire (1950), let  $s$  be the standard deviation estimate based on a sample from a normal distribution with unknown  $\mu$  and unknown  $\sigma$ . Let  $r$  be the proportion of  $\sigma$  such that  $s$  is within  $r\sigma$  of  $\sigma$  with desired confidence. If

$$p_1 = \Pr(s > \sigma + r\sigma) = \Pr(s > \sigma(1 + r))$$

and

$$p_2 = \Pr(s < \sigma - r\sigma) = \Pr(s < \sigma(1 - r))$$

The confidence level for estimating  $\sigma$  within proportion  $r$  is  $1 - p_1 - p_2$ .

Since  $\frac{(n-1)s^2}{\sigma^2} \sim \chi_{n-1}^2$ , it is useful to rewrite  $p_1$  and  $p_2$  as

$$p_1 = \Pr\left(\frac{(n-1)s^2}{\sigma^2} > (n-1)(1+r)^2\right)$$

and

$$p_2 = \Pr\left(\frac{(n-1)s^2}{\sigma^2} < (n-1)(1-r)^2\right)$$

Using the chi-square distribution, these equations can be solved for any of the unknown quantities ( $n$ ,  $r$ ,  $p_1 + p_2$ ) in terms of the others.

## Confidence Level

The confidence level,  $1 - \alpha$ , has the following interpretation. If thousands of samples of  $n$  items are drawn from a population using simple random sampling and the standard deviation estimates are obtained from each sample, the proportion of those estimates that are within  $r\sigma$  of  $\sigma$  is  $1 - \alpha$ .

## Example 1 – Calculating Sample Size

Suppose a study is planned in which the researcher wishes to be 95% confident that estimated standard deviation is within 10% of the true population standard deviation. In addition to 10% relative error, 5%, 15%, 20% and 25% will also be considered.

The goal is to determine the necessary sample size.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For ..... **N (Sample Size)**  
 Interval Type ..... **Two-Sided**  
 Confidence Level ..... **0.95**  
 Relative Error ..... **0.05 to 0.25 by 0.05**

### Output

Click the Calculate button to perform the calculations and generate the following output.

### Numeric Reports

#### Numeric Results for Two-Sided Relative Error Confidence Intervals

Solve For: **Sample Size**

Target Confidence Level	Actual Confidence Level	Sample Size N	Relative Error
0.95	0.950	769	0.05
0.95	0.950	193	0.10
0.95	0.951	87	0.15
0.95	0.951	49	0.20
0.95	0.952	32	0.25

Confidence Level	The proportion of standard deviation estimates that will be within the relative error of the true standard deviation.
Target Confidence Level	The value of the confidence level that is entered into the procedure.
Actual Confidence Level	The value of the confidence level that is obtained from the procedure.
N	The size of the sample drawn from the population.
Relative Error	The distance from the true standard deviation as a proportion of the true standard deviation.

#### Summary Statements

With a sample size of 769, the probability is 0.95 (95% confidence) that the estimate of the standard deviation will be within 5% of the true population standard deviation.

## Confidence Intervals for One Standard Deviation using Relative Error

**Dropout-Inflated Sample Size**

Dropout Rate	Sample Size N	Dropout- Inflated Enrollment Sample Size N'	Expected Number of Dropouts D
20%	769	962	193
20%	193	242	49
20%	87	109	22
20%	49	62	13
20%	32	40	8

Dropout Rate	The percentage of subjects (or items) that are expected to be lost at random during the course of the study and for whom no response data will be collected (i.e., will be treated as "missing"). Abbreviated as DR.
N	The evaluable sample size at which the confidence interval is computed. If N subjects are evaluated out of the N' subjects that are enrolled in the study, the design will achieve the stated confidence interval.
N'	The total number of subjects that should be enrolled in the study in order to obtain N evaluable subjects, based on the assumed dropout rate. After solving for N, N' is calculated by inflating N using the formula $N' = N / (1 - DR)$ , with N' always rounded up. (See Julious, S.A. (2010) pages 52-53, or Chow, S.C., Shao, J., Wang, H., and Lokhnygina, Y. (2018) pages 32-33.)
D	The expected number of dropouts. $D = N' - N$ .

**Dropout Summary Statements**

Anticipating a 20% dropout rate, 962 subjects should be enrolled to obtain a final sample size of 769 subjects.

**References**

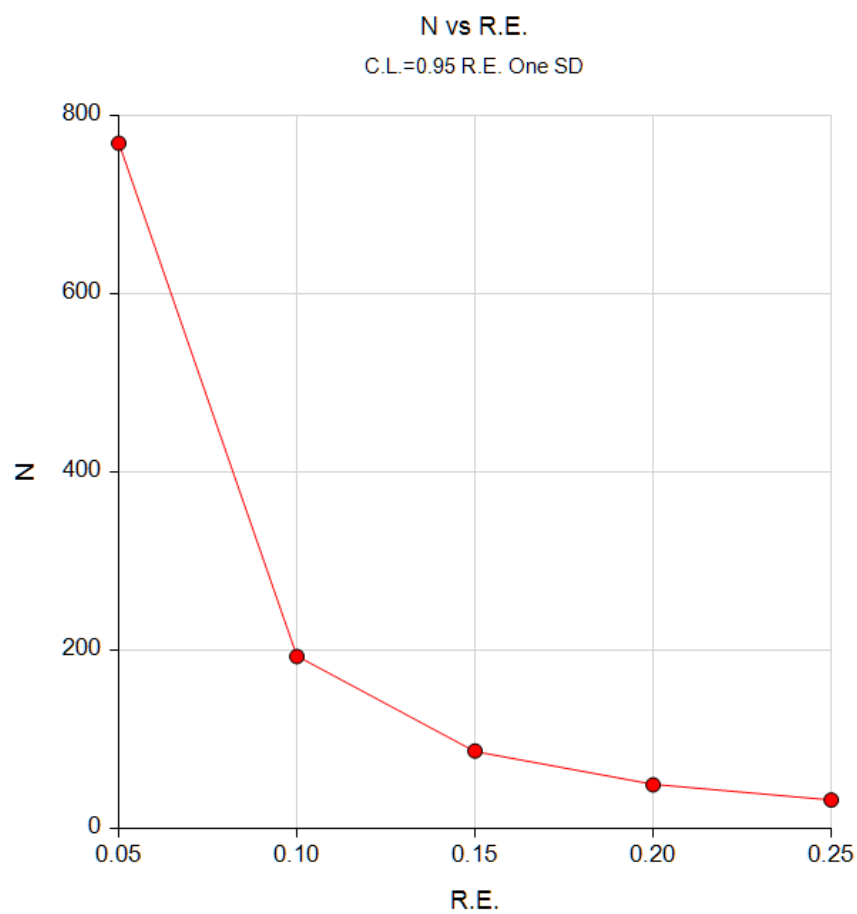
Greenwood, J. A. and Sandomire, M. M. 1950. 'Sample Size Required for Estimating the Standard Deviation as a Per Cent of its True Value', Journal of the American Statistical Association, Vol. 45, No. 250, pp. 257-260.  
Desu, M. M. and Raghavarao, D. 1990. Sample Size Methodology. Academic Press. New York.

This report shows the calculated sample size for each of the scenarios.

## Confidence Intervals for One Standard Deviation using Relative Error

## Plots Section

## Plots



This plot shows the sample size versus the relative error.

## Example 2 – Validation using Greenwood and Sandomire

Greenwood and Sandomire (1950) page 259 give an example of a calculation in which the desired confidence level is 80% and the relative error is 10%. The necessary sample size is 84.

### Setup

If the procedure window is not already open, use the PASS Home window to open it. The parameters for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

#### Design Tab

Solve For ..... **Sample Size**  
Interval Type ..... **Two-Sided**  
Confidence Level ..... **0.80**  
Relative Error ..... **0.1**

### Output

Click the Calculate button to perform the calculations and generate the following output.

#### Numeric Results for Two-Sided Relative Error Confidence Intervals

Solve For: [Sample Size](#)

Target Confidence Level	Actual Confidence Level	Sample Size N	Relative Error
0.8	0.802	84	0.1

**PASS** also calculated the necessary sample size to be 84.