RCBD with Sampling – Pooling Experimental and Sampling Error

- As we had with the CRD with sampling, we will have a source of variation for sampling error.
- Calculation of the Experimental Error df is done the same way as if there was no sampling.
- Calculation of the Sampling Error df is done the same way as was done for the CRD with sampling.
- We will test the homogeneity of variance between the Experimental Error MS and the Sampling Error MS. If they are homogeneous a Pooled Error MS can be calculated and used as the denominator of the *F*-test on treatments.

ANOVA Table

SOV	Df	F
Rep	r-1	Rep MS/Pooled Error MS
Trt	t-1	Trt MS/Pooled Error MS
Experimental Error	(r-1)(t-1)	
Sampling Error	(rts-1)-(tr-1)	
Total	trs-1	
Pooled Error MS	Exp Error df + Sampling Error df	

Example

		Treatment			
Rep	Sample	A	B	C	
1	1	78	68	89	
1	2	82	64	87	
		$Y_{11.}$=160	$Y_{21.}$=132	$Y_{31.}$=176	$Y_{.1.}$=468
2	1	74	62	88	
2	2	78	66	92	
		$Y_{12.}$=152	$Y_{22.}$=128	$Y_{32.}$=180	$Y_{.2.}$=460
3	1	80	70	90	
3	2	84	60	96	
		$Y_{13.}$=164	$Y_{23.}$=130	$Y_{33.}$=186	$Y_{.3.}$=480
$Y_{i..}$		476	390	542	$Y_{...}$= 1408

Step 1. Calculate the Correction Factor (CF).

$$\frac{Y_{...}^2}{rts} = \frac{1408^2}{3(3)(2)} = 110{,}136.889$$

Step 2. Calculate the Total SS:

$$Total\ SS = \sum Y_{ijk}^2 - CF$$

$$= \left(78^2 + 82^2 + 74^2 + ... + 96^2\right) - CF$$

$$= 2121.111$$

Step 3. Calculate the Replicate SS.

$$\text{Rep } SS = \sum \frac{Y_{.j.}^2}{ts} - CF$$

$$= \left(\frac{468^2}{3(2)} + \frac{460^2}{3(2)} + \frac{480^2}{3(2)}\right) - CF$$

$$= 33.778$$

Step 4. Calculate the Treatment SS:

$$Treatment\ SS = \sum \frac{Y_{i..}^2}{rs} - CF$$

$$= \left(\frac{476^2}{3(2)} + \frac{390^2}{3(2)} + \frac{542^2}{3(2)}\right) - CF$$

$$= 1936.444$$

Step 5. Calculate the SS Among Experimental Units Total (SSAEUT)

$$SS\ AEUT = \sum \frac{Y_{ij.}^2}{s} - CF$$

$$= \left(\frac{160^2}{2} + \frac{152^2}{2} + \frac{164^2}{2} + \ldots + \frac{186^2}{2} \right) - CF$$

= 2003.111

Step 6. Calculate the Experimental Error SS:

Experimental Error SS = SAEUT – SS TRT – SS REP

= 2003.111 – 1936.444 – 33.778

= 32.889

Step 7. Calculate the Sampling Error SS:

Sampling Error SS = Total SS – SSAEUT

= 2121.111 – 2003.111

= 118.0

Step 8. Complete the ANOVA Table:

SOV	Df	SS	MS	*F*
Rep	r-1=2	33.778	16.889	2.054^{ns}
Trt	t-1 = 2	1936.444	968.222	117.76^{**}
Experimental Error	(r-1)(t-1) = 4	32.889	8.222	
Sampling Error	(trs-1) - (tr-1) = 9	118.0	13.111	
Total	trs-1 = 17	2121.111		

Step 9. Test the homogeneity of variance between the Experimental and Sampling Error MS using the Folded *F*-test.

Step 9.1 Calculate the *F*-value using the Folded *F*-test

Folded *F* = Sampling Error MS / Experimental Error MS

$$F = \frac{8.222}{13.111}$$

$$= 0.627$$

Step 9.2 Look up the table *F*-value

-
- This F-test is a **<u>one-tail</u>** test because there is the expectation that the Experimental Error MS $\left(\sigma_S^2 + s\sigma_E^2\right)$ is going to be larger than the Sampling Error MS $\left(\sigma_S^2\right)$.

- Thus, if you are testing $\alpha = 0.01$, then you need to use the *F*-table for α = 0.01 (Appendix Table IV, page 612).

- $F_{0.01,(ExptErrdf)(SampErrdf)} = F_{0.01;4,9} = 6.42$

Step 9.3 Make conclusions:

- Since the calculated value of *F* (0.627) is less than the Table-*F* value (6.42), we fail to reject H_o: Sampling Error MS = Experimental Error MS at the 99% level of confidence.

- Therefore, we can calculate a Pooled Error MS

Step 10: Calculate the Pooled Error df and the Pooled Error MS

Pooled Error df = Sampling Error df + Experimental Error df = (9+4) = 13

Pooled Error MS = <u>Sampling Error SS + Experimental Error SS</u>
Sampling Error df + Experimental Error df

$$= \frac{118.0 + 32.889}{4 + 9} = 11.607$$

Step 11: Complete the ANOVA using the Pooled Error MS as the denominator of the *F*-test

SOV	Df	SS	MS	*F*
Rep	r-1=2	33.778	16.889	1.455^{ns}
Trt	t-1 = 2	1936.444	968.222	83.42^{**}
Experimental Error	(r-1)(t-1) = 4	32.889	8.222	
Sampling Error	(trs-1) - (tr-1) = 9	118.0	13.111	
Total	trs-1 = 17	2121.111		
Pooled Error	Expt Error df + Samp Error df=13	150.889	11.607	

Step 12. Calculate LSD.

$$LSD_{TRT} = t_{.05/2}\sqrt{\frac{2PooledErrorMS}{rs}}$$

$$= 2.16\sqrt{\frac{2(11.607)}{3*2}}$$

$$= 4.24$$

Step 13. Compare treatment means

Treatment	Mean
B	65.0 a
A	79.3 b
C	90.3 c

SAS for the RCBD with Sampling

Commands

```
options pageno=1;
data rcbdsamp;
input TRT $ Rep   Sample      Yield;
datalines;
A     1     1     78
A     1     2     82
A     2     1     74
A     2     2     78
A     3     1     80
A     3     2     84
B     1     1     68
B     1     2     64
B     2     1     62
B     2     2     66
B     3     1     70
B     3     2     60
C     1     1     89
C     1     2     87
C     2     1     88
C     2     2     92
C     3     1     90
C     3     2     96
;;
proc anova;
class rep trt;
model yield=rep trt rep*trt;
*comment rep*trt is the experimental error;
test h=rep trt e=rep*trt;
means trt/lsd e=rep*trt;
title 'RCBD with Sampling - Using the Experimental Error as the
Denominator of the F-test';
run;
proc anova;
class rep trt;
model yield=rep trt;
*comment by leaving out the rep*trt term, you are allowing SAS to
calculate the pooled error;
means trt/lsd;
title 'RCBD with Sampling - Using the Pooled Error as the Denominator
of the F-test';
run;
```

RCBD with Sampling - Using the Pooled Error as the Denominator of the F-test

Output

Obs	TRT	Rep	Sample	Yield
1	A	1	1	78
2	A	1	2	82
3	A	2	1	74
4	A	2	2	78
5	A	3	1	80
6	A	3	2	84
7	B	1	1	68
8	B	1	2	64
9	B	2	1	62
10	B	2	2	66
11	B	3	1	70
12	B	3	2	60
13	C	1	1	89
14	C	1	2	87
15	C	2	1	88
16	C	2	2	92
17	C	3	1	90
18	C	3	2	96

RCBD with Sampling - Using the Experimental Error as the Denominator of the F-test

The ANOVA Procedure

Class Level Information		
Class	**Levels**	**Values**
Rep	3	1 2 3
TRT	3	A B C

Number of Observations Read	18
Number of Observations Used	18

RCBD with Sampling - Using the Experimental Error as the Denominator of the F-test

The ANOVA Procedure

Dependent Variable: Yield

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	2003.111111	250.388889	19.10	<.0001
Error	9	118.000000	13.111111		
Corrected Total	17	2121.111111			

R-Square	Coeff Var	Root MSE	Yield Mean
0.944369	4.629026	3.620927	78.22222

Source	DF	Anova SS	Mean Square	F Value	Pr > F
Rep	2	33.777778	16.888889	1.29	0.3221
TRT	2	1936.444444	968.222222	73.85	<.0001
Rep*TRT	4	32.888889	8.222222	0.63	0.6552

Tests of Hypotheses Using the Anova MS for Rep*TRT as an Error Term					
Source	**DF**	**Anova SS**	**Mean Square**	**F Value**	**Pr > F**
Rep	2	33.777778	16.888889	2.05	0.2434
TRT	2	1936.444444	968.222222	117.76	0.0003

RCBD with Sampling - Using the Experimental Error as the Denominator of the F-test

The ANOVA Procedure

t Tests (LSD) for Yield

Note: This test controls the Type I comparisonwise error rate, not the experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	4
Error Mean Square	8.222222
Critical Value of t	2.77645
Least Significant Difference	4.5965

Means with the same letter are not significantly different.			
t Grouping	**Mean**	**N**	**TRT**
A	90.333	6	C
B	79.333	6	A
C	65.000	6	B

RCBD with Sampling - Using the Pooled Error as the Denominator of the F-test

The ANOVA Procedure

Class Level Information		
Class	**Levels**	**Values**
Rep	3	1 2 3
TRT	3	A B C

Number of Observations Read	18
Number of Observations Used	18

RCBD with Sampling - Using the Pooled Error as the Denominator of the F-test

The ANOVA Procedure

Dependent Variable: Yield

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	1970.222222	492.555556	42.44	<.0001
Error	13	150.888889	11.606838		
Corrected Total	17	2121.111111			

R-Square	Coeff Var	Root MSE	Yield Mean
0.928863	4.355388	3.406881	78.22222

Source	DF	Anova SS	Mean Square	F Value	Pr > F
Rep	2	33.777778	16.888889	1.46	0.2690
TRT	2	1936.444444	968.222222	83.42	<.0001

Dependent Variable: Yield

Note: This test controls the Type I comparisonwise error rate, not the experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	13
Error Mean Square	11.60684
Critical Value of t	2.16037
Least Significant Difference	4.2494

Means with the same letter are not significantly different.			
t Grouping	**Mean**	**N**	**TRT**
A	90.333	6	C
B	79.333	6	A
C	65.000	6	B