

## Chapter 11. Sampling Distributions

**Note.** In this chapter we consider what happens if we take a sample from a population over and over again. We will see that the means of the samples are normally distributed, regardless of the distribution of the original population. This is called the *Central Limit Theorem* and is the backbone of most of the statistical analysis we will perform in the future.

### Parameters and Statistics

**Definition.** A **parameter** is a number that describes the population. In statistical practice, the value of a parameter is not known because we cannot examine the entire population. A **statistic** is a number that can be computed from the sample data without making use of any unknown parameters. In practice, we often use a statistic to estimate an unknown parameter.

**Note.** The text emphasizes this with the comment: “**S**tatistics come from **S**amples, and **P**arameters come from **P**opulations.” For example, the population mean (a parameter) is denoted  $\mu$  and a sample mean (a statistic) is denoted  $\bar{x}$ .

**Example S.11.1. Stooge Statistics and Stooge Parameters.**

A sample of size 30 is taken from the population of 190 Three Stooges films. In the sample, 13 of the films have Curly as the third stooge, 13 of the films have Shemp as the third stooge, and 4 of the films have Joe as the third stooge. For this sample, the percentage of Curly films is  $100\% \times 13/30 = 43.33\%$ .

(1) Is this a parameter or a statistic? (2) What are the three statistics this sample yields and what are the three corresponding parameters of the population?

**Statistical Estimation and the Law of Large Numbers**

**Theorem.** Draw observations at random from any population with finite mean  $\mu$ . As the number of observations drawn increases, the mean  $\bar{x}$  of the observed values *is expected to* get closer and closer to the mean  $\mu$  of the population.

**Example.** Example 11.3 page 274. This example illustrates the Law of Large Numbers. It describes a situation where there is a population with mean  $\mu = 25$ . Samples of size 1 are taken and a running average from the samples is computed. Notice that the average of the samples gets close to the population mean of 25 as the number of samples  $n$  gets large:

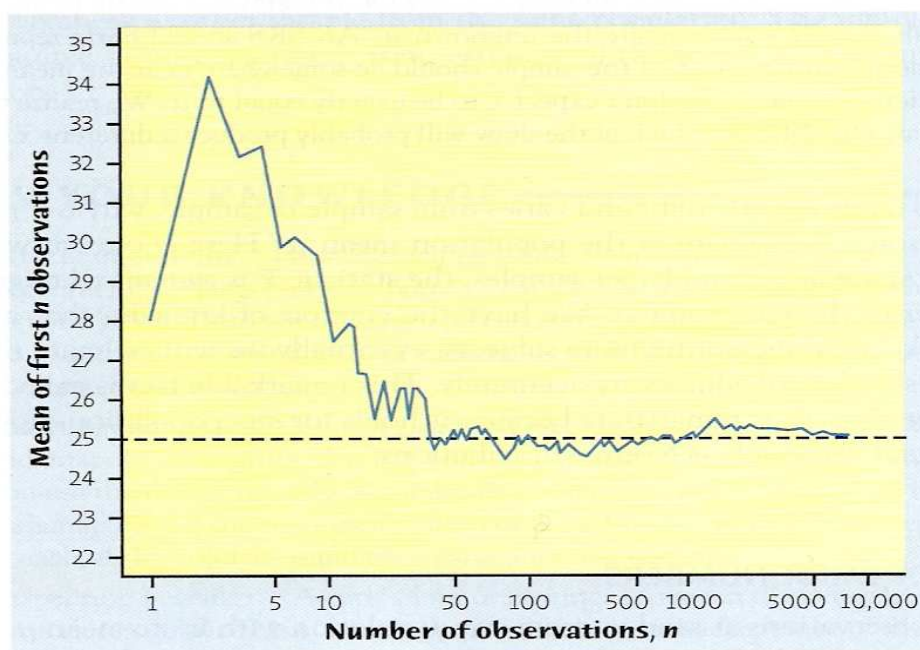


Figure 11.1 Page 274

**Example.** The BPS Applet *Law of Large Numbers* is a simulation of an experiment similar to that in the previous example. It is based on rolling a die a repeated number of times. Input 100 rolls and watch the running average approach 3.5

## Sampling Distributions

**Note.** Now for a real subtlety. We are ready to consider two populations. One is a population from which we will sample and then use the statistics from these samples to estimate parameters of this population. The second population is the *population of samples* from the original population. We will see that the population of samples is normally distributed, regardless of the distribution of the original population.

**Example.** Consider a population with a mean of 25 for some parameter (this is based on the sulfur content parameter of wine mentioned in Example 11.2 on page 273). We follow this protocol:

- Take a large number of samples of size 10 from the population.
- Calculate the sample mean  $\bar{x}$  for each sample.
- Make a histogram of the values of  $\bar{x}$ . Notice this is a histogram of the *population of samples*.
- Examine the distribution displayed in the histogram for shape, center, spread, as well as outliers or other deviations.

This process leads to the following information:

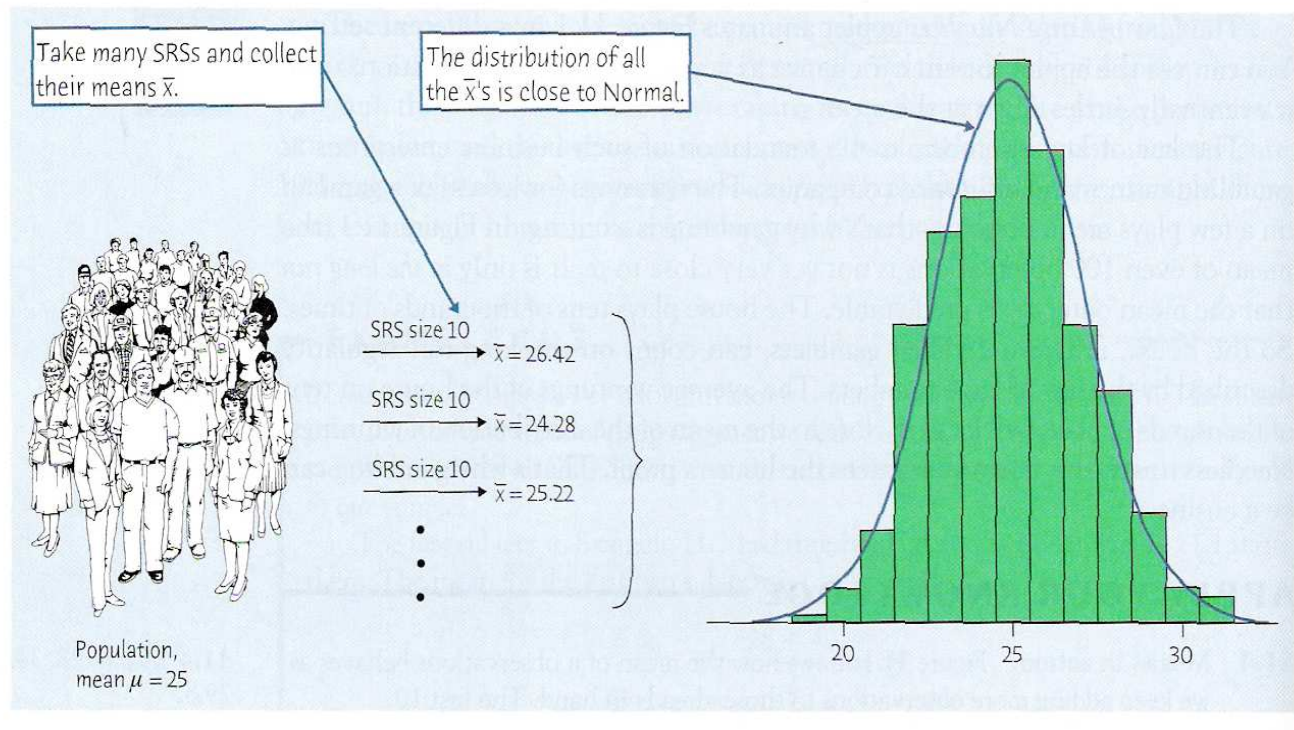


Figure 11.2 Page 276

Notice that the distribution of samples is approximately normal with center near 25 (the mean of the original population). This illustrates the idea of a “sampling distribution.”

**Definition.** The **sampling distribution** of a statistic is the distribution of values taken by the statistic in all possible samples of the same size from the same population.

**Note.** The text's statement about "all possible samples" implies that there is a limiting process here and that the law of large numbers applies.

## The Sampling Distribution of $\bar{x}$

**Theorem.** Suppose that  $\bar{x}$  is the mean of a simple random sample (SRS) of size  $n$  drawn from a large population with mean  $\mu$  and standard deviation  $\sigma$ . Then the sampling distribution of  $\bar{x}$  has **mean**  $\mu$  and **standard deviation**  $\sigma/\sqrt{n}$ .

**Definition.** An **unbiased estimator** of a population parameter is a statistic which is "correct on average" in many samples. As illustrated above,  $\bar{x}$  is an unbiased estimator of  $\mu$ .

**Note.** The fact that the sampling distribution of  $\bar{x}$  has standard deviation  $\sigma/\sqrt{n}$  implies that large samples will give better estimates of population parameters than small samples (since the sampling distributions are less spread out when  $n$  is large). In other words, if the population has a normal distribution  $N(\mu, \sigma)$ , then the sampling distribution will have the normal distribution  $N(\mu, \sigma/\sqrt{n})$ . The next section will liberate us from the assumption of a normal original population.

**Example.** Exercise 11.9 page 280.

## The Central Limit Theorem

**Note.** If the original population is not normally distributed, then it turns out that the sampling distribution will still be normally distributed. **This is why the normal distribution is so important!** The most important result for our use of statistics is the following theorem.

### Theorem. Central Limit Theorem.

Draw an SRS of size  $n$  from any population with mean  $\mu$  and finite standard deviation  $\sigma$ . When  $n$  is large, the sampling distribution of the sample mean  $\bar{x}$  is approximately normal with distribution  $N(\mu, \sigma/\sqrt{n})$ .

**Note.** Again, this is extremely important!!! It justifies the use of the normal distributions when dealing with sample data. Informally, the “when  $n$  is large” comment means that for non-normal populations, we need large samples for the Central Limit Theorem to apply.

**Example.** There is a BPS Central Limit Theorem Applet. Access it and play with different sample sizes  $n$  and probabilities/proportions  $p$  to see how it affects the distribution.

**Example.** Exercise 11.11 page 285.

**Example S.11.2. Stooge Sampling Distributions.**

Suppose the average number of slaps per film in the Three Stooges' films is  $\mu = 12.95$  with a standard deviation of  $\sigma = 4.50$ . You want to estimate  $\mu$  by taking a sample of Stooges' films. You only have time to watch 10 films. (1) What are the mean and standard deviation of the average number of slaps per film  $\bar{x}$  in a sampling distribution for samples of size 10 films? (2) Use the Central Limit Theorem to find the probability that the average number of slaps per film in the sample of size 10 is less than 11 slaps.

**Note.** We do not cover the “process control” topics from the remainder of this chapter.

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