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Standard Deviation



The standard deviation σ of a probability distribution is defined as the [square root](#) of the [variance](#) σ^2 ,

$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \quad (1)$$

$$= \sqrt{\mu'_2 - \mu^2}, \quad (2)$$

where $\mu = \bar{x} = \langle x \rangle$ is the [mean](#), $\mu'_2 = \langle x^2 \rangle$ is the second [raw moment](#), and $\langle f \rangle$ denotes an [expectation value](#). The [variance](#) σ^2 is therefore equal to the second [central moment](#) (i.e., moment about the [mean](#)),

$$\sigma^2 = \mu_2. \quad (3)$$

The square root of the [sample variance](#) of a set of N values is the sample standard deviation

$$s_N = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}. \quad (4)$$

The sample [standard deviation distribution](#) is a slightly complicated, though well-studied and well-understood, function.

However, consistent with widespread inconsistent and ambiguous terminology, the square root of the bias-corrected variance is sometimes also known as the standard deviation,

$$s_{N-1} = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}. \quad (5)$$

The standard deviation s_{N-1} of a list of data is implemented as [StandardDeviation\[list\]](#).

Physical scientists often use the term [root-mean-square](#) as a synonym for standard deviation when they refer to the [square root](#) of the mean squared deviation of a quantity from a given baseline.

The standard deviation arises naturally in mathematical statistics through its definition in terms of the second [central moment](#). However, a more natural but much less frequently encountered measure of average deviation from the [mean](#) that is used in descriptive statistics is the so-called [mean deviation](#).

The variate value producing a [confidence interval](#) CI is often denoted x_{CI} , and

$$x_{CI} = \sqrt{2} \operatorname{erf}^{-1}(CI). \quad (6)$$

The following table lists the [confidence intervals](#) corresponding to the first few multiples of the standard deviation.

range	CI
σ	0.6826895
2σ	0.9544997
3σ	0.9973002

4σ	0.9999366
5σ	0.9999994

To find the standard deviation range corresponding to a given [confidence interval](#), solve (5) for n , giving

$$n = \sqrt{2} \operatorname{erf}^{-1}(\text{CI}). \tag{7}$$

CI	range
0.800	$\pm 1.28155\sigma$
0.900	$\pm 1.64485\sigma$
0.950	$\pm 1.95996\sigma$
0.990	$\pm 2.57583\sigma$
0.995	$\pm 2.80703\sigma$
0.999	$\pm 3.29053\sigma$

SEE ALSO: [Central Moment](#), [Confidence Interval](#), [Mean](#), [Mean Deviation](#), [Moment](#), [Root-Mean-Square](#), [Standard Deviation Distribution](#), [Sample Variance](#), [Sample Variance Distribution](#), [Standard Error](#), [Variance](#)

REFERENCES:

Kenney, J. F. and Keeping, E. S. "The Standard Deviation" and "Calculation of the Standard Deviation." §6.5-6.6 in [Mathematics of Statistics, Pt. 1, 3rd ed.](#) Princeton, NJ: Van Nostrand, pp. 77-80, 1962.

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