

Standard Deviation as a Measure of Risk for a Mutual Fund

Floyd Vest, Summer 2011 (Preliminary Version)

There are millions of mutual fund investors, many of whom are investing for long term objectives such as retirement and college education. For them, to make decisions, they must understand the risks and rewards of their investments. There are many sources of risk. (See the Side Bar Notes and Exercises.) We will discuss risk as measured by standard deviation. Most mutual funds will disclose the standard deviation for their fund. See the example below for Fund A, from an actual report.

Fund A: March 3, 2011, The average of the annual returns for the last five years was 6.39%. Standard deviation was given as 14.53. Expense ratio .34%. A balanced fund, $\frac{2}{3}$ stocks, $\frac{1}{3}$ bonds. (See the Side Bar Notes for the average annual total return.)

Mutual funds give formal statements of objectives and types of investments which are an indication of their risk posture. McDonald did a study of risk versus fund objectives, 1960-1969, using standard deviations of monthly excess returns (percent per month) over ten years for funds with similar objectives. See Figure 1 below (from Sharpe, p. 653). As an estimate, there may be about 20 funds in each category, and for each fund about 120 monthly returns.

Fund Objectives	Mean of Standard Deviation	Approximate Range of Standard Deviations
Maximum Capital Gain	5.90	4.6 – 7
Growth	4.57	3 - 8
Growth-Income	3.93	2.5 – 4.8
Income-Growth	3.80	2.6 – 4.3
Balanced	3.05	2 – 3.8
Income	2.67	.8 – 3.8

Figure 1 – Risk versus fund objectives: 123 mutual funds, 1960 – 1969

Source: John G. McDonald, “Objectives and Performance of Mutual Funds, 1960-1969,” *Journal of Financial and Quantitative Analysis* 9, no. 3 (June 1974): p. 316.

In Figure 1, the average standard deviation is shown for each type of mutual fund. The overlapping of some of the ranges indicates that some funds with conservative objectives took on more risk than others with less conservative objectives. It is possible that past standard deviations may be a better guide to future risk than statements of objectives. (See the Side Bar Notes for “excess returns.”)

In Figure 1, McDonald used the monthly rate of return which was calculate

$$r_t = \frac{NAV_t - NAV_{t-1} + D}{NAV_{t-1}}$$
 where NAV (Net Asset Value) is Share Price, and D is dividend

paid per share. Example, $r_t = \frac{\$10.03 - \$10.00 + \$0.09}{\$10.00} = 1.20\%$ (Sharpe, p. 652).

Notice that standard deviations of monthly excess returns (in percent per month) are small relative to the standard deviation for annual returns reported for Fund A above.

A review of standard deviation. For the application of standard deviation to monthly rates of return for mutual funds, Sharpe (See the References.) used the

(1) sample standard deviation $S_x = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N-1}}$, where \bar{x} is the mean of the N values and

(2) $\bar{x} = \frac{\sum_{i=1}^N x_i}{N}$. On the TI83/84, the sample standard deviation is denoted S_x and is found in VARS 5 (for Statistical Variables) and the mean is denoted \bar{x} .

Example 1. The following is a simple example from wikipedia. Consider a sample consisting of eight values:

2, 4, 4, 4, 5, 5, 7, 9.

$\bar{x} = \frac{2+4+4+4+5+5+7+9}{8} = 5$. With the values numbered 1 through 8, we could

use Formula 2. For a sample standard deviation, the numerator of Formula 1 is
 $(2-5)^2 + (4-5)^2 + (4-5)^2 + (4-5)^2 + (5-5)^2 + (5-5)^2 + (7-5)^2 + (9-5)^2 = 32$. This gives

$S_x = \sqrt{\frac{32}{7}} = 2.138$. (See the Exercises for calculating \bar{x} and S_x on the TI83/84.)

According to Morningstar, the standard deviation for a mutual fund is calculated with three to five years of monthly returns. (See www.morningstar.com in Morningstar Investing Classroom.) Then, monthly values are annualized. Vanguard claims to have has demonstrated that this short term standard deviation does not represent long term risk. (See www.vanguard.com for Vanguard Investment Counseling and Research.)

Uses and limits of standard deviation in investing. Standard deviation is probably used more often as a measure of risk than other measures of risk. It simply gives a form of average variation of fund returns about the mean, both for downside risk and up side volatility. The greater the standard deviation, the greater the risk expressed as volatility about the mean rate of return. Standard deviation is not a measure of rate of return on the investment, but only a measure of dispersion about the mean.

Note that if the underlying data is percent, the standard deviation is in percent (percentage points). For example, for a fund with a standard deviation of 14, 14%, and

an average return of 10% per year, most of the time you can expect future returns to range between $10\% - 14\% = -4\%$ to $10\% + 14\% = 24\%$. This is $\bar{x} \pm 1S_x$. (See www.morningstar.com for Morningstar Investing Classroom for this example.) A standard deviation of 14 (14 percentage points) represents a volatile three years, 2008, 2009, 2010.

Chebyshev's Inequality. Chebyshev's inequality ensures that, for all distributions for which standard deviation S is defined, the percentage of data points within k standard deviations on either side of the mean is greater than or equal to $1 - \frac{1}{k^2}$. Thus for

example, 75% or more of the data is between $\bar{x} - 2S$ and $\bar{x} + 2S$ since $1 - \frac{1}{2^2} = .75$.

(See the Exercises.)

The normal distribution and standard deviation. For mutual funds, it is considered reasonable to assume that returns are distributed normally about their mean. The usual test for this is an examination of a histogram to be a bell shaped curve similar to the normal distribution. Historical data has demonstrated that in the long run, this is true.

The Standard Normal Distribution has a mean μ of zero and a standard deviation σ of one. It is informative to know key areas under the standard normal distribution. See the following You Try It which is an exercise calculating key areas using ShadeNorm(on the TI83/84.

You Try It #1: Set Window at $X_{\min} = -4$, $X_{\max} = 4$, $X_{\text{scl}} = 1$, $Y_{\min} = 0$, $Y_{\max} = .5$, $Y_{\text{scl}} = .1$. To get a shaded graph giving area from 0 to 1, TI83/84 code and commentary: 2nd Draw 1 Enter 2nd Quit 2nd Distr > Draw Enter You see ShadeNorm(. Enter the largest in the range which is 1, the smallest 0, 0 for μ , and 1 for σ) Enter. You see the graph of the standard normal distribution with shaded area from 0 to 1 with at the bottom Area = .34134. Write down the result. Repeat to get the area from 0 to 2. Repeat to get the area from 0 to 3. Do a hand drawing of the distribution curve with appropriate units on the axes and mark the regions from -1 to 1, -2 to 2, and -3 to 3. Give the area for each. Keep this drawing for reference. You should get the area from -1 to 1 is 68%, from -2 to 2 is 95%, and from -3 to 3 is 99.7%.

For the standard normal distribution, $z = \frac{\bar{x} - \sigma}{\sigma}$ is distributed normally with a mean of zero and a standard deviation of one. (See the Exercises for part of the proof.) We try another example on ShadeNorm(. Do WINDOW: $X_{\min} = -4$, $X_{\max} = 4$, $X_{\text{scl}} = 1$, $Y_{\min} = -.05$, $Y_{\max} = .5$, $Y_{\text{scl}} = .1$. Then do ClrDraw. Then for example, find the area above one standard deviation above the mean. Code and commentary: 2nd Dist > Enter You see ShadeNorm(Write 4, 1, 0, 1) Enter You see the shaded curve with the answer that the percentage above 1 is .15862 \approx 16% as it should be.

Geometric interpretation of standard deviation. For example, we will start with a population of two values x and y . This defines a point $P = (x, y)$. Consider line L through $M\left(\frac{x+y}{2}, \frac{x+y}{2}\right)$ and the origin. The perpendicular distance from P to line L is

$$d = \sqrt{\left(x - \frac{x+y}{2}\right)^2 + \left(y - \frac{x+y}{2}\right)^2} = \sqrt{2\left(\frac{1}{2}\right)^2 (x-y)^2} . \text{ Using } x \text{ and } y, \text{ we calculate}$$

$$\text{standard deviation} = S = \sqrt{\frac{\left(x - \frac{x+y}{2}\right)^2 + \left(y - \frac{x+y}{2}\right)^2}{2}} = \sqrt{\frac{2\left(\frac{1}{2}\right)^2 (x-y)^2}{2}} .$$

So $d = \sqrt{2} S$.

This interpretation of standard deviation could be used when regression lines are calculated between the rates of return of mutual funds and of a benchmark (surrogate) to compare the volatility of the mutual fund to the general market volatility represented by a benchmark, often the version of the S&P 500 Index of Stocks with dividends reinvested.

Example 2: Calculating mean and standard deviation on the TI83/84. We will use the data set in Example 1 of 2,4,4,4,5,5,7,9. First enter the data in a List. To display the Stat list editor, press STAT 1. You see vertical columns into which you enter your data as a list, with a list name displayed at the head of the column. Code and commentary: 2nd Quit 2nd LIST The bottom line is the entry line. Choose your column and enter the data: 2 Enter 4 Enter and so on. Then to calculate the mean, 2nd Quit 2nd List < to Math menu 3 You see mean(on the home screen. 2nd L2) Enter You see 5 as the mean. To calculate the standard deviation, 2nd List < to MATH 7 You see on the home screen StdDev(Then 2nd L2) Enter You see 2.138 as the standard deviation. (It appears the StdDev gives the sample standard deviation.)

Application of standard deviation. Is a standard deviation of 7 high or low for a mutual fund? How would you apply it? We suggest that a person start by comparing similar funds. If two funds have a similar history of returns, then the one with the smaller standard deviation gives the same return with less risk.

Fund B: Value at Risk (VaR) “is an estimate of the maximum loss that can be expected over a short period of time. It is can be calculated at the 97.5% confidence level.” (www.vanguard.com, Vanguard Investment Counseling and Research). VaR is the losing rate of return for below which there is 2.5% probability. We will illustrate this with Fund B which is a large value mutual fund, for which for 19 years from 1990 to 2009, the historical arithmetic mean annual return was 9.47% and the standard deviation was 14.63% . Fund B is fabricated from actual historical data in Table 8, p. 16 of Xion. See the References. We will assume for sake of an example that annual returns in Fund B are distributed normally. For Fund B, with $\mu = 9.47\%$ and $\sigma = 14.63$, with $N = 19$ years, what is an estimate for VaR? For the 95% interval about the mean, there is $2\frac{1}{2}\%$ below $\mu - 2\sigma$. So we will use this as an estimate for VaR, $\mu - 2\sigma = 9.47 - 2(14.63) =$

-19.79 with 97.5% confidence of a return greater than -19.79% per year in the time frame of 19 years. (See the article in this course “Investment Portfolio Design to Optimize Performance and Minimize Risk” for a comparison of VaR to CVaR which does not assume normality of the distribution.)

Example 3: Using ShadeNorm on the TI84 to calculate probabilities.

For Fund B what is the probability that $x < -10\%$? The area under the normal curve below $x = -10\% = -.10$ gives the probability that $x < -10\%$. First set WINDOW: $Xmin = 9.47 - 4(14.63)$, $Xmax = 9.47 + 4(14.63)$, $Xscl = 10$, $Ymin = -.05$, $Ymax = .05$, $Yscl = .01$. (We use four standard deviations below the mean and above the mean for $Xmin$ and $Xmax$.) For ShadeNorm(there are four blanks, the first is the largest value on the range. The next is the point to shade above or below. The next is μ , and the last is σ . For ShadeNorm(Code and commentary: clear any drawings, 2nd Draw Select 1:ClrDraw Enter Enter. Then 2nd Quit 2nd Distr > Draw Enter You see on the home screen ShadeNorm(write $-.10, -49.05, 9.47, 14.63$) Enter You see the graph of the shaded normal curve, with shading in the left tail up to $-.10$. You read at the bottom: Area = .0916. Conclusion: The probability of $x < -10\%$ is 9.16%.

A small sample and Student's t-distribution, Fund B from Value at Risk VaR above: Choose a random sample of 5 years for Fund B. We evaluate the probability that

the sample mean \bar{x} is less than 0, $P(\bar{x} < 0)$. The variable $t_v = \frac{\bar{x} - \mu}{\frac{S_x}{\sqrt{n}}}$ is distributed by

the t-distribution with n degrees of freedom. We use $t_v = \frac{0 - 9.47}{\frac{14.63}{\sqrt{5}}} = -1.45$ where

$P(t < -1.45) = P(\bar{x} < 0)$. On the TI83/84, we use Window: $Xmin = -8$, $Xmax = 8$, $Xscl = 1$, $Ymin = -.05$, $Ymax = .1$. We will use for shade draw: Shade_t(lower bound, upper bound, df). TI code and commentary: 2nd Draw Enter Enter to clear graph screen, 2nd Distr > 2. You see Shade_t(Write in $-8, -1.45, 5$) Enter. You see the graph of the t-distribution with the left tail shaded with area of .103. Thus $P(t < -1.45) = P(\bar{x} < 0) = .103 = 10.3\%$. (See other articles in this course for more on the Student's t-distribution which is used for small samples and other applications.)

Why invest in a stock mutual fund? Because it gives you professional management, immediate diversification, and a small expense ratio. In the long term, historically stocks have averaged about 9.5% with dividends reinvested (depending on the time frame and the index used), outpacing inflation by about 6.2%. For example $\$10,000(1 + .095)^{48} = \$779,611$, from age 22 to age 70. Is this enough to fund retirement? See articles in this course such as “Taking the Long View of Life,” for long term financial planning and investing for retirement.

For a history of the annual earning of the S&P 500 Index of stocks, see http://en.wikipedia.org/wiki/5%26P_500. See <https://personal.Vanguard.com/us/insights/investingtruths/investing-truth-about-risk> for a yearly earnings history of 70% stocks, 25% bonds, 5% cash, back to 1928.

Exercises. Show all your work. Put on units. Label answers. Give formulas. Refer to TI83/84 code. Summarize. Give proofs in general terms. Several exercises refer to Fund B under Value at Risk VaR above, which is a large value stock mutual fund, which for 19 years of annual returns from 1990 to 2009, the mean $\mu = 9.47\%$, and the standard deviation $\sigma = 14.63\%$. Fund B is fabricated from actual historical data. For sake of an example, we will assume that the annual returns in Fund B are distributed normally.

1. For the set of annual returns of 5%, 10%, and a loss of 7%, (a) calculate the mean \bar{x} . (b) Calculate the standard deviation. (c) Calculate the total rate of return TR . (d) Discuss the difference between \bar{x} and TR , their applications, and illustrate the effect of misapplying them.

2. The variance $V = \frac{\sum_{i=1}^N (x - \bar{x})^2}{N}$ and the standard deviation $S = \sqrt{V}$. (a) Prove that

$$V = \frac{\sum_{i=1}^N x^2}{N} - \bar{x}^2 \text{ and } S = \sqrt{\frac{\sum_{i=1}^N x^2}{N} - \bar{x}^2} \quad \text{(b) Prove that } \underline{V(cx)} = c^2 V(x) \text{ for a constant } c,$$

and that $S(cx) = |c|S(x)$. (c) Prove that $V(x+b) = V(x)$ for a constant b .

3. (a) For Chebyshev's inequality, find a k for which 50% or more of the data are within $\bar{x} \pm kS$. (b) What is true of the Probability P that $(\bar{x} - kS < x < \bar{x} + kS)$? (c) What is $P(\bar{x} - 3S < x < \bar{x} + 3S)$?

4. (a) For Fund B under Value at Risk VaR above, find the value for which the probability of a rate of return being below this value is 16%. (b) Give the interval about the mean for which we have confidence that 95% of returns are in the interval.

5. For the geometric interpretation of standard deviation, prove that line L is perpendicular to the line from $P(x,y)$ to $M\left(\frac{x+y}{2}, \frac{x+y}{2}\right)$ and thus d is the perpendicular distance from point P to line L at point M .

6. For the Balanced funds in Figure 1, for the fund with greatest and the fund with the least standard deviation, give for each the interval about their mean for which we expect 68% of the monthly returns x to fall. Assume a sample size of $N = 120$ and a normal distribution. Give probability statements. (It seems that McDonald studied monthly returns for these funds for about ten years from 1960 to 1969.)

7. Use the TI83/84 statistical functions to calculate the mean and standard deviation of (a) the data set: 5%, 10%, and -7%.
8. For an ultra short term bond fund, the yearly standard deviation is .64% (Morningstar, Investing Classroom). (a) For a mean rate of return of 3%, assuming a normal distribution, give the interval about the mean in which 95% of the annual returns fall. (b) Bond duration would be another indicator of risk. For a certain short term bond fund (9/5/11), duration is 2.2 years. What does this tell about the change in share prices from a current \$10.60, if interest rates rise by one percentage point? (See the article in this course "Bond Duration.") (c) Compare the effect on rate of return of an interest rate rise of one percentage point in terms of duration, and in terms of the probabilities and returns estimated from the normal distribution.
9. List and discuss at least twenty different kinds of risk for different kinds of mutual funds. Don't worry about overlap of your kinds of risk because they may vary in generality or context.
10. Under the above heading of "Value at Risk (VaR)", the downside risk was emphasized. Look up a stock mutual fund on the internet and evaluate it based on all the information they give you. For a rough estimate, use the standard deviation to examine the upside volatility, for example, $9.47 + 2(14.63) = 38.73\%$. Have stock funds ever earned anywhere near this in a year? Assuming the normal distribution, what is probability that a return would be 38.73% or greater?
11. Give a general proof that the mean of $z = \frac{\bar{x} - \mu}{\sigma} = 0$, and the standard deviation is 1.
12. Use ShadeNorm to calculate for Fund B under Value at Risk VaR above the probability of an annual return $x < 2\%$. (a) Do it using the normal distribution, about the given mean with the given standard deviation, and (b) using the standard normal z distribution. Draw the shaded graphs with units and explain your calculations.
13. From Fund B under Value at Risk VaR above, choose a small random sample of 10 years. What is $P(\bar{x} < -10\%)$? What is $P(\bar{x} > 10\%)$? If you want to sample consecutive years, you can use stratified sampling.

Answers to Exercises.

1. (a) $\bar{x} = 2.67\%$ (b) $S_x = 8.74\%$ (c) $(1+.05)(1+.10)(1-.07) = (1+TR)^3$,
 $TR = 2.41\%$. (d) Example $\$1000(1+.0241)^3 = \1074.06 . Using \bar{x} will be off by \$8.20.

$$2. (a) V = \frac{\sum_{i=1}^N (x^2 - 2x\bar{x} + \bar{x}^2)}{N} = \frac{\sum_{i=1}^N x^2}{N} - 2\bar{x} \frac{\sum_{i=1}^N x}{N} + \frac{N\bar{x}^2}{N} = \frac{\sum_{i=1}^N x^2}{N} - \bar{x}^2$$

$$(b) V(cx) = \frac{\sum (cx - \text{mean of } cx)^2}{N} = \frac{(\sum cx - c\bar{x})^2}{N} = \frac{c^2 \sum (x - \bar{x})^2}{N} = c^2 V$$

$$3. (a) P(\bar{x} - \sqrt{2} S < x < \bar{x} + \sqrt{2} S) \geq \frac{1}{2}$$

$$(b) P(\bar{x} - 3S < x < \bar{x} + 3S) \geq 1 - \frac{1}{k^2} = 1 - \frac{1}{3^2} = \frac{8}{9}$$

4. (a) Between $\mu - \sigma$ and $\mu + \sigma$ is 68%. $50\% - 34\% = 16\%$. $P(x < \mu - \sigma) = 16\%$.
 $\mu - \sigma = 9.47 - 14.63 = -5.16\%$. $P(x < -5.16\%) = 16\%$. (b) Have 95% confidence that
 \bar{x} is in the interval $(\mu - 2\sigma, \mu + 2\sigma) = (-19.79, 38.73)$.

5. Slope of PM = -1. Slope of L is 1. They are negative reciprocals so are perpendicular.

6. (a) For monthly returns x , where the fund means μ are not known and
 $N = 120$, for the fund with $S = 2$, $P(\mu - 2 < x < \mu + 2) = .68$. For the fund with
 $S = 3.8$, $P(\mu - 3.8 < x < \mu + 3.8) = .68$. If average monthly returns are roughly around
 $.8\%$, the fund with $S = 2$ has about one-third the downside risk as the one with $S = 3.8$.

8. (a) $P(3 - 2(.64) < x < 3 + 2(.64)) = .95$. (b) With a duration of 2.2, if interest rates
rise by one percentage point, then the Share price of \$10.60 declines by 2.2% to
 $10.60(1 - .022) = \$10.37$.

$$11. \text{ The mean of } z = \sum \frac{\left(\frac{x - \bar{x}}{\sigma}\right)}{N} = \frac{1}{\sigma} \left[\frac{\sum x}{N} - \frac{N\bar{x}}{N} \right] = \frac{\bar{x} - \bar{x}}{\sigma} = 0.$$

$$\text{The standard deviation} = \sum \left[\frac{\frac{x - \bar{x}}{\sigma}}{N} \right]^2 = \frac{1}{\sigma^2} \sum \left(\frac{x - \bar{x}}{N} \right)^2 = \frac{\sigma^2}{\sigma^2} = 1.$$

Side Bar Notes

Arithmetic average and average annual total return TR . For five consecutive annual returns r_1, r_2, r_3, r_4, r_5 , the average total return TR is such that $(1 + r_1)(1 + r_2)(1 + r_3)(1 + r_4)(1 + r_5) = (1 + TR)^5$. The arithmetic average is in general greater than or equal to the average total return TR . See the article in this course “Evaluation of Investments: irr, mirr, AM, TR, and HM.”

Excess rate of return. Excess rate of return $= r_e = r_M - r_f$, where r_M is the rate of return for the mutual fund, and r_f is the risk free rate of return, usually a U. S. Treasury security. It is a measure of the premium the investor earns for the added risk. (See Sharpe.)

Volatility of the S&P 500 Index of Stocks, which is used as a benchmark for comparison of mutual funds. From 2011, for the recent past, the standard deviation has ranged from 18% to 22%. (www.morningstar.com).

Important statistics and measures of risk for Fund A. A balanced fund, 5 year average rate of return 6.39%, A Five Star Fund, Expense ratio .34%, Standard deviation 14.53, Sharpe Ratio .42, Sortino Ratio .58, R-squared 98.37, Beta 1.02, Alpha .04, Standard Index 5.06, Risk measures are over the most recent three years back from March 2011 (www.morningstar.com).

Investment Company Institute’s survey of mutual fund investors. Sixty-nine percent of respondents said they examined the fund’s investment risk. Only fund performance was cited more often. Only 28% of respondents stated that they are very confident in their ability to assess the risk of a single fund based on standard deviation. Only 26% stated they have used standard deviation or other measures of risk (standard deviation being the measure that obtained the highest confidence rating). The survey revealed that the vast majority of investors who used risk measures did not understand them. Only 35% of duration users stated that they used duration to determine a fund’s sensitivity to interest rates. “Although no measure of risk can be used to predict a fund’s returns,” 44% of standard deviation users perceived this measure to be designed for this purpose. The survey indicates that quantitative risk measurement would complicate an evaluation of risk for most investors. For long term investors, the short term volatility measures of risk are not particularly relevant. Membership of the Investment Company Institute includes 5,861 investment companies, which have over 38 million individual investors (http://www.ici.org/policy/comments/96_SEC_RISK_RES_COM).

From wikipedia. In “Investment Management Reflections,” published in 2000, the authors argue “that not only did 75% of actively managed equity mutual funds underperform the Vanguard S&P 500 Index Fund but that after taking into account taxation, 66

out of 71 mutual funds in the sample under performed. A later study that looked at the 1990's found that 322 out of 355 mutual funds in the sample under performed after tax" (wikipedia.org/wiki/Robert_D._Arnott).

Do you want to study something useful? Check www.barnsandnoble.com. Used textbooks are sold for less than shipping charges. Consider Alexander, Fundamentals of Investing, 1993, 18 chapters, 557 pages, \$1.99. Check on used mathematics textbooks such as for precalculus, calculus, and statistics. See Lee, et. al. in the references which was used as a reference for this article.

For a certain Short Term Bond Fund, NAV is \$10.73, Average Coupon is 3.4%, Expense ratio is .22%. The formula for the SEC 30-day annual yield = $2 \left[\frac{(a-b)/cd+1}{6} \right]$, where a = dividends and interest, b = accrued expenses, c = average daily number of outstanding shares that were entitled to distributions, d = the maximum public offering price per share on the last day of the period. (http://en.wikipedia.org/wiki/30-day_yield) (Wikipedia didn't use italics for variables.) (a) Use the above information given for the fund to estimate the SEC 30-day annual yield. (b) Summarize and explain the mathematics used in the formula. The answer to part (a) is about 3.20% per year.

Feds sue banks over risky investments (Denton Record Chronicle, Sept. 3, 2011). The FHFA said the mortgage-backed-securities were sold to Fannie and Freddie (organizations which provide mortgage insurance) based on misstatements and omissions. The regulator said Fannie and Freddie didn't have the needed risk-management capabilities.

Enough retirement assets to meet their dreams? Smart Money magazine says that most people don't have enough financial assets set aside to meet their retirement dreams. Due to a poor stock market, many investors lost out on three years of earnings, said Alecia Munnell, Director of the Center for Retirement Research, Boston College. Vanguard reports that in the three years since December 2007, the median yearly total return for participants in their retirement programs has been 0.1 percent – which means about half have done worse than break even. (Smart Money magazine, July 2011). People need to understand the history of stock market earnings. There have been periods when profit from stocks was marginal. See http://en.wikipedia.org/wiki/S%26P_500_Index.

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Teachers’ Notes

For a free course in financial mathematics, with emphasis on personal finance, for upper high school and undergraduate college, see <http://www.comap.com/FloydVest/Course/index.html>. Just click on an article in the annotated bibliography, download it, and teach it. Unit 1: The Basics of Mathematics of Finance, Unit 2: Managing Your Money, Unit 3: Long-term Financial Planning, Unit 4: Investing in Stocks and Bonds, Unit 5: Investing in Real Estate, Unit 6: Solving Financial Formulas for Interest Rate.

