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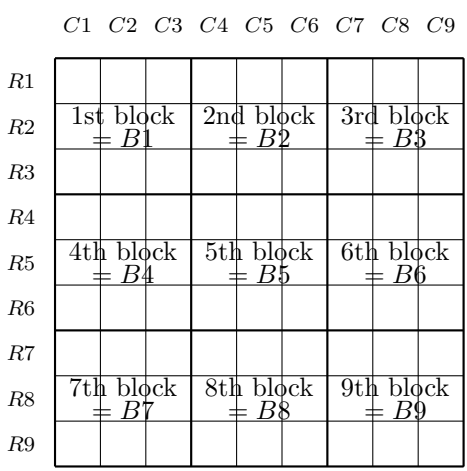
Chapter 1

The Classic Sudoku

A grid with 9×9 cells is divided into nine blocks with 3×3 cells. We will call this grid as a **Sudoku diagram**. In the following, the symbol "Ra" means the ath row in the diagram, the symbol "Cb" means the bth column in the diagram, and the symbol "Bd" means the dth block in the diagram. The cell shared by Ra and Cb is denoted by $\overline{ab}$.

Sudoku diagram



Sudoku is a number placement puzzle game which begins from a Sudoku diagram where n cells are respectively filled with a digit from 1 to 9 and the remaining $(81-n)$ cells are empty. This Sudoku diagram is called a **problem of Sudoku** (Sudoku puzzle). The

player of Sudoku is asked to **place a digit from 1 to 9 into each of the empty cells in the problem of Sudoku such that each digit appears exactly once in each row, in each column, and in each block.** We call a row, a column and a block all as **a unit**. The **basic requirement** of the Sudoku game is that **no digit may appear more than once in each unit**. The completed Sudoku diagram that meets this basic requirement of the Sudoku game is referred to as the **answer of Sudoku** (solution of Sudoku). The filled cells in a problem of Sudoku are the **clues for solving** the problem of Sudoku to the answer of Sudoku. The number n of the filled cells in a problem of Sudoku is called its **clue number**.

Each cell in a Sudoku diagram belongs to one row, one column and one block. Briefly, we say that **each cell belongs to three units**, or **three units share a common cell**. One block and one row share three common cells, so as one block and one column. Here we list a problem of Sudoku and its answer as an example. This example is equivalent to **the most difficult Sudoku puzzle** designed by the Finnish mathematician Arto Inkala.

The problem of Sudoku with $n = 23$ (see p. 182)

1					**7**	**9**		
	3			**2**			**8**	
		9	**6**					**5**
		5	**3**					**9**
	1			**8**			**2**	
6					**4**			
3						**1**		
	4						**7**	
		7						**3**

The answer of Sudoku (see p. 133)

1	6	2	8	5	**7**	**9**	3	4
5	**3**	4	1	**2**	9	7	**8**	6
7	8	**9**	**6**	4	3	2	1	**5**
4	7	**5**	**3**	1	2	8	6	**9**
9	**1**	3	5	**8**	6	4	**2**	7
6	2	8	7	9	**4**	3	5	1
3	5	6	4	7	8	**1**	9	2
2	**4**	1	9	3	5	6	**7**	8
8	9	**7**	2	6	1	5	4	**3**

A problem of Sudoku is proper if there is one and only

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one answer of Sudoku solved from the problem. In the present book we are interested only in explaining **how to solve a proper problem of Sudoku to its answer of Sudoku**. Up to now, no proper problem of Sudoku with clue number n less than 17 has been found.

In order to give readers an intuitive feeling and concept of how to solve a problem of Sudoku, we introduce a very easy problem of Sudoku with clue number $n = 34$, and solve it by the usual **local solution**. In the next chapter the problem will be solved again by the **global solution** for comparison. In the calculation process, some empty cells are filled with digits, respectively. Throughout the present book, a cell is called a **fixed cell with x if the cell is filled with only one digit x**. Be reminded that the only **basic requirement** of Sudoku is that **for a given fixed cell with x, no other cell can be filled with x** in the unit which the fixed cell belongs to.

A very easy problem of Sudoku with $n = 34$

	5	9				7	3	
			6			1		9
3			4				5	
	3	6	1					5
	1	4	7		6	2	8	
2					4	6	7	
	4				3			8
5		3			7			
	9	2				3	6	

1. There are six fixed cells in $R5$, so the remaining three empty cells can be filled with 3, 5 or 9. However, the cell $\overline{59}$ has to be filled with 3 because in $C9$ there are a fixed cells with 5 and a fixed cells with 9. The cell $\overline{51}$ has to be filled with 9 because there is a fixed cell with 5 in $C1$. Then, the cell $\overline{55}$ has to be filled with 5.

2. The cell $\overline{37}$ has to be filled with 8 because the digits from 1 to 9 except for 8 all appear in the fixed cells at three units sharing this cell $\overline{37}$. For the same reason the cell $\overline{62}$ has to be filled with 8.

3. There are seven fixed cells in $B4$, so the remaining two empty cells can be filled with 5 or 7. Since there is a fixed cell with 7 in $R6$, the digit 7 has to be filled in the cell $\overline{41}$, and then, the digit 5 has to be filled in the cell $\overline{63}$.

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4. There are five fixed cells in $B7$, so the remaining four empty cells can be filled with 1, 6, 7 or 8. Checking the four empty cells, we find that the digit 8 can be filled only in the cell $\overline{91}$, because each of three other cells belongs to at least one unit containing a fixed cell with 8. In the same reason the digit 7 can be filled only in the cell $\overline{73}$. Then, the digit 1 can be filled only in the cell $\overline{71}$. The last empty cell $\overline{82}$ has to be filled with 6.

5. There are seven fixed cells in $C1$, so the remaining two empty cells can be filled with 4 or 6. Since there is a fixed cell with 6 in $R2$, the digit 6 has to be filled in the cell $\overline{11}$, and then, the digit 4 has to be filled in the cell $\overline{21}$.

6. There are seven fixed cells in $C3$, so the remaining two empty cells can be filled with 1 or 8. Since there is a fixed cell with 1 in $R2$, the digit 1 has to be filled in the cell $\overline{33}$, and then, the digit 8 has to be filled in the cell $\overline{23}$.

Now the diagram with 9 blocks becomes as follows.

6	**5**	**9**				**7**	**3**	
4		8	**6**			**1**		**9**
3		1	**4**			8	**5**	
7	**3**	**6**	**1**					**5**
9	**1**	**4**	**7**	5	**6**	**2**	**8**	3
2	8	5			**4**	**6**	**7**	
1	**4**	7			**3**			**8**
5	6	**3**			**7**			
8	**9**	**2**				**3**	**6**	

7. There are six fixed cells in $B3$, so the remaining three empty cells can be filled with 2, 4 or 6. The digit 4 has to be filled in the cell $\overline{19}$ because there is a fixed cell with 4 both in $R2$ and $R3$. The digit 6 has to be filled in the cell $\overline{39}$ because there is a fixed cell with 6 in $R2$. Then, the digit 2 has to be filled in the cell $\overline{28}$.

8. There are six fixed cells in $C8$, so the remaining three empty cells can be filled with 1, 4 or 9. The digit 9 has to be filled in the cell $\overline{78}$ because there are two fixed cells filled with 1 and 4 in $R7$, respectively. The digit 4 has to be filled in the cell $\overline{48}$ because there is a fixed cell with 1 in $R4$. Then, the digit 1 has to be filled in the cell $\overline{88}$.

9. There are six fixed cells in $C7$, so the remaining three empty cells can be filled with 4, 5 or 9. The digit 9 has to be filled in

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the cell $\overline{47}$ because there are two fixed cells filled with 4 and 5 in $R4$, respectively. The digit 4 has to be filled in the cell $\overline{87}$ because there is a fixed cell with 4 in $R7$. Then, the digit 5 has to be filled in the cell $\overline{77}$.

10. There are six fixed cells in $C9$, so the remaining three empty cells can be filled with 1, 2 or 7. The digit 1 has to be filled in the cell $\overline{69}$ because there are two fixed cells filled with 2 and 7 in $R6$, respectively. The digit 2 has to be filled in the cell $\overline{89}$ because there is a fixed cell with 2 in $R9$. Then, the digit 7 has to be filled in the cell $\overline{99}$.

11. There are six fixed cells in $R2$, so the remaining three empty cells can be filled with 3, 5 or 7. The digit 7 has to be filled in the cell $\overline{22}$ because there are two fixed cells filled with 3 and 5 in $C2$, respectively. The digit 3 has to be filled in the cell $\overline{25}$ because there is a fixed cell with 3 in $C6$. Then, the digit 5 has to be filled in the cell $\overline{26}$.

12. There are six fixed cells in $R3$, so the remaining three empty cells can be filled with 2, 7 or 9. The digit 2 has to be filled in the cell $\overline{32}$ because there are two fixed cells filled with 7 and 9 in $C2$, respectively. The digit 7 has to be filled in the cell $\overline{35}$ because there is a fixed cell with 7 in $C6$. Then, the digit 9 has to be filled in the cell $\overline{36}$.

Now the diagram with 9 blocks becomes as follows.

6	**5**	**9**				**7**	**3**	4
4	7	8	**6**	3	5	**1**	2	**9**
3	2	1	**4**	7	9	8	**5**	6
7	**3**	**6**	**1**			9	4	**5**
9	**1**	**4**	**7**	5	**6**	**2**	**8**	3
2	8	5			**4**	**6**	**7**	1
1	**4**	7			**3**	5	9	**8**
5	6	**3**			**7**	4	1	2
8	**9**	**2**				**3**	**6**	7

13. There are seven fixed cells in $R6$, so the remaining two empty cells can be filled with 3 or 9. The digit 3 has to be filled in the cell $\overline{64}$ because there is a fixed cell with 3 in $C5$. Then, the digit 9 has to be filled in the cell $\overline{65}$.

14. There are seven fixed cells in $R7$, so the remaining two empty cells can be filled with 2 or 6. The digit 6 has to be filled

in the cell $\overline{75}$ because there is a fixed cell with 6 in $C4$. Then, the digit 2 has to be filled in the cell $\overline{74}$.

15. There are seven fixed cells in $R8$, so the remaining two empty cells can be filled with 8 or 9. The digit 9 has to be filled in the cell $\overline{84}$ because there is a fixed cell with 9 in $C5$. Then, the digit 8 has to be filled in the cell $\overline{85}$.

16. There are seven fixed cells in $R4$, so the remaining two empty cells can be filled with 2 or 8. The digit 8 has to be filled in the cell $\overline{46}$ because there is a fixed cell with 8 in $C5$. Then, the digit 2 has to be filled in the cell $\overline{45}$.

17. There are six fixed cells in $R1$, so the remaining three empty cells can be filled with 1, 2 or 8. Because there is a fixed cell with 2 both in $C4$ and in $C5$, respectively, the cell $\overline{16}$ has to be filled with 2. Because there is a fixed cell with 1 in $C4$, the cell $\overline{15}$ has to be filled with 1, and then, the cell $\overline{14}$ has to be filled with 8.

18. There are six fixed cells in $R9$, so the remaining three empty cells can be filled with 1, 4 or 5. Because there is a fixed cell with 4 both in $C4$ and in $C6$, respectively, the cell $\overline{95}$ has to be filled with 4. Because there is a fixed cell with 1 in $C4$, the cell $\overline{96}$ has to be filled 1, and then, the cell $\overline{94}$ has to be filled 5.

Now we obtain the answer of Sudoku as follows.

The answer of Sudoku

6	**5**	**9**	8	1	2	**7**	**3**	4
4	7	8	**6**	3	5	**1**	2	**9**
3	2	1	**4**	7	9	8	**5**	6
7	**3**	**6**	**1**	2	8	9	4	**5**
9	**1**	**4**	**7**	5	**6**	**2**	**8**	3
2	8	5	3	9	**4**	**6**	**7**	1
1	**4**	7	2	6	**3**	5	9	**8**
5	6	**3**	9	8	**7**	4	1	2
8	**9**	**2**	5	4	1	**3**	**6**	7

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