2009

Ratio and proportion: mapping the conceptual field

Christina Vincent
Louisiana State University and Agricultural and Mechanical College

RATIO AND PROPORTION: MAPPING THE CONCEPTUAL FIELD

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Natural Sciences

in

The Interdepartmental Program of Natural Sciences

By
Christina Vincent
B.S., Louisiana State University, 2002
August 2009

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Ratio and proportion are central to the middle school mathematics curriculum, but the full scope and extent of this topic is not described in detail in most state curriculum standards. In this thesis, numerous textbooks from the past one hundred years are sampled, along with several state's standards and the Louisiana state comprehensive curriculum. These sources are used to develop a more defined map of ratio and proportion as a conceptual field and a structured collection of problems. Proportional reasoning involves three phases: 1) the comparison of two magnitudes, expressed as ratio or rate, 2) the comparison of two ratios, called a proportion, and 3) the expression of proportional relationships as functions. As we follow this progression, proportional reasoning tasks change accordingly, through ratios, rates, missing-value proportions, similarity situations, and ultimately functions that express proportionality.

INTRODUCTION

Over the years, proportional reasoning has increasingly been accepted by mathematicians and mathematics educators as a driving skill underlying the use and understanding of algebra. “Proportional Reasoning has been described as the capstone of elementary school arithmetic and the gateway to higher mathematics, including algebra, geometry, probability, statistics, and certain aspects of discrete mathematics,” (Kilpatrick, Swafford & Findell, 2001, pp. 241-254). Proportional reasoning connects with all strands of mathematics in the middle school curriculum: number sense, patterns and sequencing, geometry, measurement, probability and algebra. *The National Council of Teachers of Mathematics Curriculum and Evaluation Standards (1989)* describes proportionality to be “of such great importance that it merits whatever time and effort must be expended to assure its careful development (p. 82).” Current state and national standards, including the Louisiana Grade Level Expectations for middle school students, focus on developing a deep understanding of measurement, ratio, rates, and proportions and the ability to apply them. Scale drawings, comparing similar figures, probability, percents, and slope in linear equations have direct connection to proportional reasoning. Yet, while deep understandings of ratio and proportion are assumed and used as building blocks for all higher mathematics, teachers and students are left wondering how to frame this knowledge. How can we map the conceptual field that defines this topic?

Gerard Vergnaud defines a conceptual field as “a set of situations, the mastering of which requires mastery of several concepts of different natures,” (1988, pp. 141-161). He gives three reasons why such a framework is necessary. First, Vergnaud points out that it is difficult if not impossible to separate mathematical concepts that have similar or overlapping conceptual and practical applications. Second, a framework enables one to study a wide range of information and situations and look at these over time. Third, Vergnaud says that, “…there are usually different procedures and conceptions and also different symbolic representations involved in the mastery by students of the same class of problems." By looking at a conceptual field, a stronger framework can be made with more universal symbols, wording, and solutions.

While a conceptual field, as described by Vergnaud, outlines the concepts and skills that are necessary in the study of ratio and proportion and gives us a better mapping for the teaching and learning of proportional reasoning, there is an essential part of learning that is left out through his model. Students are expected to learn and apply conceptual understanding to tasks. Therefore a "task field" is an important part of the general map. The tasks that are assigned by teachers through class work, textbooks, supplementary materials and other types of assessments help to define the scope and extent of this field. A conceptual framework helps to link historical views and definitions of this topic while a task field outlines specific types of problems that are actually presented to students. Though it may be difficult to find a definition that a majority of mathematicians or teachers will agree on when it comes to ratio and proportion, the tasks related to proportional reasoning remain the same.

A ratio is a comparison of two things with respect to a shared quantitative aspect. If this aspect has been measured by a single unit, then the ratio can be represented by a fractional expression with numbers in the numerator and denominator. A proportion involves a comparison of two ratios and is represented by setting two ratios equal to each other. A family of proportions can be represented as a function. Given many proportions $x_i/y_i = a/b$, all involving a single ratio a/b, there is a relationship between x and y that is the same in all cases (of x and y). It can be expressed by displaying y as a function of x: $y=kx$. Such a function is called a direct proportion. The graph of a proportional function is a line that passes through the origin, (0,0).

In the Louisiana state comprehensive curriculum, proportional reasoning is first referred to in the sixth grade standards. It reads, "Use models and pictures to explain concepts or solve problems involving ratio, proportion, and percent with whole numbers," (Louisiana State Department of Education, 2009). This standard assumes that the meaning of ratios and fractions is understood. But, to an educator or a mathematician, this standard may seem quite ambiguous. There is no clear or universal definition of proportional reasoning held by all.

This thesis maps the conceptual and task fields of ratio and proportion by a sampling of tasks taken from several sources: textbooks of the past one hundred years, current standards from multiple states, and Louisiana's comprehensive curriculum. The goal of this sampling is "to delineate distinct domains" of proportional reasoning (Vergnaud, 1983, pp. 128-174). Existing classifications are explored as well as contemporary opinions. Opinions that we gathered from mathematicians and current mathematics educators by having them complete a classification task are compared to existing classifications to give a more in-depth look at the types of problems and skills that are assumed to be part of what it means to reason proportionally.

This thesis begins by outlining the historical framework of ratio and proportion and comparing the Euclidean definitions to more contemporary definitions. This shows an historical progression in the conceptual understanding of ratio, rational numbers and related ideas. Analysis of current state standards from around the country and of Louisiana's comprehensive curriculum reflect this progression. The task set that we compiled by sampling text books is examined in light of this progression and used to define a task field for proportionality problems.

The empirical input to this thesis included information that we obtained about how experts classify proportional reasoning problems. A task set was given to seven different groups that consisted of mathematicians and/or mathematics educators. They were asked to categorize the problems that were included in the task set. Their responses were analyzed, and incorporated into a proposed conceptual field map.

CHAPTER 1 PROPORTIONAL REASONING

1.1 Euclid's Definitions and Comparison to Modern Conceptual Structures

To begin describing a conceptual framework for proportional reasoning, it is reasonable to start by looking at the history of ratio and proportion. The earliest definition recorded is from Euclid's *Elements Book V: Theory of Abstract Proportions.* Arguably the best mathematics book ever written, "no other book except the *Bible* has been so widely translated and circulated" (Allen, 1997).

Euclid's theory of ratio and proportion concerns 'magnitudes". His magnitudes were not measured by numbers but were the direct objects of experience such as segments, areas or volumes. His definitions of ratio and proportion concerned comparison of such things. He meant for these comparisons to be made directly, without a third object to compare or measure by (Gratten-Guinness, pp.56-64).

There is a difference between the modern sense of the word "measure" and Euclid's. The term "measure" is used by Euclid with reference to the idea of dividing something into equal parts. Euclid would say that "A measures B" if B is composed by adding A to itself repeatedly. A more modern term that is closely related to this meaning of measure is "an aliquot part of." An aliquot part of a magnitude is a part that is contained an exact number of times in that magnitude. We say A is an aliquot part of B if B is a whole number multiple of A.

The Greeks did not have rational numbers at the time of Euclid, and certainly did not understand the real number system (in any sense similar to the way we do). Euclid did not think of numbers when making his comparisons. Definition 5 from Book V of the *Elements*, which we look at below, shows that ratios defined in the Euclidean manner (without taking real-number measures of the things being compared) can nonetheless enter into mathematical arguments. In particular, such ratios can be compared with one another. Definition 5 is the key that makes this possible, and allows ratios to enter proportions.

Euclid begins *Book V* with a few words about measure and then gives the following definitions:

> Definition 3. A ratio is a sort of relation in respect of size between two magnitudes of the same kind.
>
> Definition 4. Magnitudes are said to have a ratio to one another which can, when multiplied, exceed one another.
>
> Definition 5. Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever are taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.
>
> Definition 6. Let magnitudes which have the same ratio be called proportional.

The requirement that ratios compare things "of the same kind" persists in modern curricula although, as we later see, this is far less important for us than for Euclid. Definitions 4 and 5 have little apparent connection with school book treatments of ratio and proportion. Definition 6 carries over in the way proportion is treated in modern middle school curricula.

In modern times, we measure things by associating numbers with them in a systematic way according to accepted rules. Most adults think of things as having measures attached without thinking of the process by which that measure is obtained. The automaticity by which we do this is recent historically, beginning perhaps in the Renaissance (Porter, 1995, p. 57).

Contemporary times have also seen the creation of new, more versatile representations of numbers: the rational and real number system. Ratio and proportion, today, assume measurement by real numbers. Euclid's definitions have given a foundation from which to build the concepts of ratio and proportion but they are not up-to-date. When ratio and proportion are used today, measurement in terms of a real number is automatically assumed. In this manner, ratio and proportion in Euclid's terms are just not enough to go on when looking for a conceptual map of proportional thinking.

This suggests an observation relevant to teaching. There is no reason to believe children make an easy transition to understanding the numerical measures assigned to things. In fact, the weakest portion of the Louisiana state high-stakes tests is often the measurement section. Measurement concepts are an integral part of the school curriculum. Perhaps, before instruction, children view the world in a way similar to Euclid and compare things in nature directly to one another.

1.2 Ratio and Proportion in Textbooks for Teachers

In this section, we review four textbooks that are designed to provide teachers with orientation in the conceptual field of ratio and proportion. Three recent books written by mathematicians that included ratio and proportion were chosen: *Elementary Mathematics for Teachers* by S. Baldridge and T. Parker, *Mathematics for Elementary School Teachers* by S. Beckmann, and *Arithmetic for Teachers: With Applications and Topics from Geometry* by G. Jensen. *Mathematics for Elementary School Teachers* written by P. O'Daffer was chosen because it is a classic, widely used text for the education of teachers. All of these books emphasize a modern, measurement-based view of the conceptual field.

While Baldridge and Parker recognize the ancient perspective, they do not develop their ideas based on it. Looking at their explanation of ratios and proportions, they say, "One can compare measurements without specifying a unit," (Baldridge & Parker, 2005, pp. 167-168). Baldridge and Parker use the example that Carla is twice as tall as her brother, emphasizing that no unit was mentioned and that the children could be measured in many alternate units such as inches, meters, feet, etc., or compared directly with no measurement at all. The children's height can be compared as a ratio using measurement, provided that the same unit is used for both measurements (pp. 167-168). Parker and Baldridge's definition of ratio states:

> Definition 1.1. We say that the *ratio* between two quantities is *A:B* if there is a unit so that the first quantity measures *A* units and the second measures *B* units. (In writing the ratio one does not specify the unit.) (Baldridge & Parker, 2005, pg. 167).

Parker and Baldridge point out that a ratio can be stated unambiguously without mentioning the (common) unit used for measuring. Of course, this definition doesn't cover the idea of rates, where different measures are used, speed, density or cost per unit, for example.

The definition of proportion given by Parker and Baldridge is very contemporary. It shows the modern tendency to put numerical assignments to things, especially when comparing them. In their definition they state:

> Definiton 1.4. Two ratios are *equivalent* (are "equal ratios") if one is obtained from the other by multiplying or dividing all of the measurements by the same (nonzero) number. (Baldridge & Parker, 2005, pg. 168).

Essentially, Euclid's definition says that ratios compare magnitudes of the same kind. In contrast, Parker and Baldridge define ratios as number pairs that originate from measuring two things with the same unit. This distinction shows the difference in Euclidean thinking to modern day.

Rational numbers are frequently used as the basis for defining ratio and proportion. Beckmann asserts that, "ratios are essentially just fractions, and understanding and working with ratios and proportions really just involves understanding and working with multiplication, division, and fractions" (pp. 299-300). She states, "A proportion is a statement that two ratios are equal" (pp. 299-300). She goes on to define a rate as "a ratio between two quantities that are measured in different units" (2005, pp. 299-300). Beckmann hints at a conceptual map of ratio and proportion by progressing from the relationship of two like things being compared (ratio), to two unlike things having a relationship (rate), and finally to the idea of two relationships having a relationship (proportion).

Jensen makes the idea of ratio a little more concrete. Avoiding the word “unit,” he defines ratio as, “a fraction formed by the number of one group of objects divided by the number of another group” (2003, pp. 241-242). He is in agreement that ratios are fractions and carry with them all the conceptual features of fractions, but in his definition, Jensen defines ratios more like Euclid. He sets up a visual representation of ratios in words when using “group of objects” and conveys the sense of a concrete comparison. However, his definition does link to the concepts of rational numbers and of measurements. Like the other authors, Jensen defines proportions as equivalent ratios, but he explains this using the most widely accepted schema for proportion in contemporary textbooks; $a:b :: c:d$ or $a:b = c:d$ (or in fraction form, $a/b = c/d$). Proportion is a multiplicative structure, wherein both ratios simplify to the same number or “unit rate”.

O’Daffer defines ratio by stating, “a ratio is an ordered pair of numbers used to show a comparison between like or unlike quantities written *x to y, x/y, x divided by y, or x:y where y cannot equal zero*” (p. 350). He mentions other ways of expressing ratios and does not limit ratios to fractions. He also points out two very important characteristics of ratios, that they show a comparison and the quantities can be like or unlike. Thus, this definition includes rates (in contrast to Beckmann, who demanded quantities in ratios) without calling them such. He identifies ratios as fractions, and defines proportion in the following words; “two ratios are equivalent ratios if their respective fractions are equivalent or if the quotients of the respective terms are the same” (O'Daffer, p. 350).

1.3 Other Descriptions of the Conceptual Field

As we suggested in the introduction, the conceptual field of proportion extends to include functions and variables. The excerpts from the textbooks above hint at this but do not address the connection in depth. Previously, proportion was referred to as a relationship between ratios. Looking at more modern explanations, proportions are seen as relations between variable quantities. This conceptual shift is in line with the modern tendency to put numerical values on things.

Wikipedia says, "two quantities are said to be proportional if they vary in such a way that one of the quantities is a constant multiple of the other, or equivalently if they have a constant ratio" (Proportionality, 2009). Viewing ratios as fractions turns them into numbers that are gotten by division. When a is divided by b, it will yield a number: $a/b = e$. When c is divided by d, it will also yield a number: $c/d = f$. If the two numbers are equal ($e=f$), then the four terms a, b, c, d are said to be proportional. This is the basis for expressing proportions as functions, as we stated earlier.

Cramer, Post and Currier have recently researched methods for teaching and assessing proportional reasoning in elementary and middle schools. Cramer et al. suggest that, along with the other accepted expressions, proportions can be written as a function. Given that there are two (varying) quantities, x and y, being compared and a constant ratio, k, between them, a function can be set up $y = k x$ (Cramer, Post & Currier, 1993, pp. 159-178). This function expresses a proportionality, but it also helps us conceptualize the difference between proportionality and other relationships that occur commonly but are expressed by more complicated functions. Only when the graph of a function passes through the origin and is a line does it represent a direct proportion. A contrasting example is given by the following: "A taxicab charges $1.00 flat fee plus 50 cents per kilometer. The cost for one kilometer is $1.50; the cost for 2 kilometers is $2.00. Though a relationship between cost and kilometers exists and can be written using a function rule (y = .50 x + 1.00, y = cost, x = kilometers), it is not proportional because both addition and multiplication define the relationship. The graph of this function does not go through the origin, [though it is a line]." (Cramer, Post & Currier, pp. 159-178). The same is true for functions that pass through the origin but are not linear. They do not represent proportions.

1.4 The Functional Approach

In the introduction we pointed out that a family of proportions is associated with a function. We explain that here. Suppose $y_1/x_1 = a/b$, $y_2/x_2 = a/b$, *and* $y_3/x_3 = a/b$, *etc*. The relationship between x and y in all cases remains the same. It can be expressed $y = (a/b)x$. If we rename a/b to be the constant k, then we can state this relationship as $y = kx$. Note that now, y appears as the output of a function of a very simple kind. As above, we call this a "proportional function." Now we state a simple proposition that gives us another way of recognizing proportional functions.

Proposition. Suppose f is a function from the real numbers to the real numbers. If $xf(a) = f(xa)$ for all real numbers a and x, then there is a constant k so that $f(x) = kx$ for all x.

Proof. Let $k := f(1)$. Then, $f(x) = f(x \cdot 1) = x \cdot f(1) = k \cdot x$. This is true for all x. Q.E.D.

The proposition shows that a function f is proportional if it is such that a multiplicative increase/decrease in the input causes the same multiplicative increase/decrease in the output. Here are three examples that illustrate proportion as a function.

1) Circles. Consider many circles. We create a separate name for each one by using an index i. If C_i is a circle in this family, let d_i be its diameter and let c_i be its circumference. Then $c_1/d_1 = c_2/d_2 = \ldots etc.$, for all i. This common ratio is called pi (π), and the general property of circles that we have observed is described by the proportional function, $c = \pi d$.

2) 30 °- 60° - 90° Triangles. Consider many such triangles. Call them $\Delta A_iB_iC_i$. For any

two, $|A_iC_i| / |A_iB_i| = |A_jC_j| / |A_jB_j|$. This common ratio is ½. In all 30°- 60°- 90° triangles, if x is the length of the short side adjacent to the 60° angle and z is the length of the hypotenuse. Here, we get the proportional function: $x = ½ z$.

3) Lumps of pure gold. Consider a family of gold nuggets, indexed by i. Let v_i be the volume of the i^{th} lump of gold, measured in cubic centimeters (cc). Let m_i be mass of the i^{th} lump, in grams (g). Then $m_1/v_1 = m_2/v_2 =$…etc. The common ratio is about 19.32, the density of gold in *g/cc*. Here, we get the proportional function: $m=(19.32)v$.

1.5 Standards Around the Country

To get a feel of how the United States teaches ratio and proportion, we looked into the content standards for mathematics in the middle grades in multiple states. We examined states representative of different geographic locations. One state was chosen from the west coast, California, one from the east coast, Georgia, and one from the northern part of the country, Connecticut. Texas and Mississippi were chosen because they border Louisiana. The following standards were copied directly from documentation provided by the state at the official site of the state's department of education. The standard was included if it directly mentioned ratio, proportion, or proportional reasoning detected by word search of the document. Some additional excerpts were included based on personal judgment of relevance.

The California standards contain references to scale drawings, dimensional analysis, and rates and direct variation.

California (Mathematics Framework for California Public Schools, 2009)
Seventh Grade
Number Sense

- Use proportions to solve problems (e.g., determine the value of *n* if 4/7 = *n*/21, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

Measurement and Geometry

- Construct and read drawings and models made to scale.
- Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.
- Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

Algebra and Functions

- Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).
- Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.
- Solve problems involving rates, average speed, distance, and time.

The Connecticut standards show how ratio and proportion are taught using numerical, statistical, and spatial problem solving. It seems emphasis is put on rates containing different units and conversion between units. The standards also show how proportional reasoning taught in Connecticut includes the functional approach.

Connecticut (Connecticut State Department of Education)

Sixth Grade

Numerical and Proportional Reasoning

- Use ratios and rates (involving different units) to compare quantities.
    a. Solve problems involving simple ratios.
    b. Solve extended numerical, statistical and spatial problems.

Geometry and Measurement

- Use measurements to examine the ratios between corresponding side lengths of scale models and similar figures.
    a. Identify congruent and similar figures.
    b. Solve extended numerical, statistical and spatial problems.
- Use ratios and powers of 10 to convert between metric units.

Seventh Grade

Numerical and Proportional Reasoning

- Write ratios and proportions to solve problems in context involving rates, scale factors and percentages.
  - a. Solve problems involving ratios.
  - b. Solve one-step problems involving proportions in context.

Eighth Grade

Algebraic Reasoning

- Write and solve problems involving proportional relationships (direct variation) using linear equations ($y = mx$).
  - a. Solve multistep problems involving ratio or proportion, and explain how the solution was determined.

Numerical and Proportional Reasoning

- Use proportional reasoning to write and solve problems in context.
  - a. Solve problems involving ratios.
  - b. Solve problems involving proportions in context.
  - c. Solve multistep problems involving ratio or proportion, and explain how the solution was determined.

Georgia's state mathematics standards are detailed. They show a progression from rate to proportion to scaling and functions. These standards mention the schema by which the proportion is being used, showing the formula for both direct proportions and functions. They are also repetitive in the use of the word "relationship" to describe proportional reasoning and the skills necessary to solve proportional reasoning tasks.

Georgia (Georgia State Department of Education, 2009)

Sixth Grade

Geometry

- Use the concepts of ratio, proportion and scale factor to demonstrate the relationships between similar plane figures.
- Interpret and sketch simple scale drawings.
- Solve problems involving scale drawings.

Measurement

- Students will convert from one unit to another within one system of measurement (customary or metric) by using proportional relationships.

Algebra

- Students will understand the concept of ratio and use it to represent quantitative relationships.
- Students will consider relationships between varying quantities.

  a. Analyze and describe patterns arising from mathematical rules, tables, and graphs.
  b. Use manipulatives or draw pictures to solve problems involving proportional relationships.
  c. Use proportions ($a/b=c/d$) to describe relationships and solve problems, including percent problems.
  d. Describe proportional relationships mathematically using $y = kx$, where k is the constant of proportionality.
  e. Graph proportional relationships in the form $y = kx$ and describe characteristics of the graphs.
  f. In a proportional relationship expressed as $y = kx$, solve for one quantity given values of the other two. Given quantities may be whole numbers, decimals, or fractions. Solve problems using the relationship $y = kx$.
  g. Use proportional reasoning ($a/b=c/d$ *and* $y = kx$) to solve problems.

Seventh Grade

Geometry

- Students will use the properties of similarity and apply these concepts to geometric figures.

  a. Understand the meaning of similarity, visually compare geometric figures for similarity, and describe similarities by listing corresponding parts.
  b. Understand the relationships among scale factors, length ratios, and area ratios between similar figures. Use scale factors, length ratios, and area ratios to determine side lengths and areas of similar geometric figures.
  c. Understand congruence of geometric figures as a special case of similarity: The figures have the same size and shape.

Eighth grade did not directly mention ratios, proportion, or proportional reasoning as a new skill.

The Mississippi standards are the shortest and most ambiguous of those sampled. One standard mentions solving operations using “concepts of ratios and proportions” but no further explanation is given as to the definition of these concepts. These standards seem to list tasks related to these concepts instead, such as finding unit rates, making scale drawings, and finding equivalent ratios.

Mississippi (Mississippi Content Standards for Mathematics, 2009)

Sixth grade does not explicitly express ratio, proportion or proportional reasoning in the content standards.

Seventh Grade

Number and Operations

- Apply concepts of rational numbers and perform basic operations emphasizing the concepts of ratio, proportion, and percent with and without the use of calculators.
    a. Solve real-life problems involving unit price, unit rate, sales price, sales tax, discount, simple interest, commission, and rates of commission.

Measurement

- Apply appropriate techniques, tools, and formulas to determine measurements with a focus on real-world problems. Recognize that formulas in mathematics are generalized statements about rules, equations, principles, or other logical mathematical relationships.
    a. Solve problems involving scale factors using ratios and proportions.

Eighth Grade

Measurement

- Understand measurable attributes of objects and apply various formulas in problem solving situations.
    a. Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

The mathematics standards set forth by Texas are also detailed, comparable to the standards of Georgia. While these standards do not explicitly state the formulas used in proportional reasoning, they do mention skills necessary to reason proportionally. Unlike the Mississippi standard that says students will solve operations using "concepts of ratio and proportion," Texas delineates the skills, such as multiplication and division of whole or rational numbers, that are used in solving problems in proportional situations.

Texas (Texas Essential Knowledge and Skills for Mathematics, 2009)

Sixth Grade

- Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions. The student is expected to:
    - a. use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates;
- Patterns, relationships, and algebraic thinking. The student solves problems involving direct proportional relationships. The student is expected to:
    - a. use ratios to describe proportional situations;
    - b. represent ratios and percents with concrete models, fractions, and decimals;
    - c. use ratios to make predictions in proportional situations.
- Patterns, relationships, and algebraic thinking. The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes. The student is expected to:

  a. use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, arithmetic sequences (with a constant rate of change), perimeter and area;

Seventh Grade

- Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:
    - a. use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio;
- Patterns, relationships, and algebraic thinking. The student solves problems involving direct proportional relationships. The student is expected to:
    - a. estimate and find solutions to application problems involving percent; and
    - b. estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

Eighth Grade

- Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations. The student is expected to:

  a. compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals;
  b. select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships;

- Patterns, relationships, and algebraic thinking. The student identifies proportional or non-proportional linear relationships in problem situations and solves problems. The student is expected to:

  a. compare and contrast proportional and non-proportional linear relationships; and
  b. estimate and find solutions to application problems involving percents and other proportional relationships such as similarity and rates.

A look at these standards as a representation of standards of our country shows a wide view of what defines proportional thinking, the way it's identified and taught in mathematics classes and the importance (or lack thereof) that is put on it. With such visible differences in the content standards, it is hard to understand what the word "standard" is supposed to express, with relation to the concept ratio and proportion in our country.

CHAPTER 2 TAKING A SAMPLE

In this chapter we develop an empirically-based classification of the tasks that define the field of proportional reasoning. We systematically sampled the ratio and proportion problems in a selection of school textbooks to create a list of 50 problems representative of the work that school children have been expected to perform, with the intent of using this sample to develop a map of the task field.

The sample was drawn from mathematics textbooks from the past one hundred years that were accessed online or available from local libraries. After the problems were collected, seven expert subjects (three mathematicians and four groups of mathematics educators) were asked to classify them according to their own perception of similarity of the problems. In doing this, we assumed that different people involved in teaching mathematics would provide different perspectives on the sample set. The classifications obtained from different subjects would provide information from which we could infer structural features of the task field for ratio and proportion. To examine this, we prepared a graph with vertices corresponding to the problems. We drew an edge between two vertices if the problems were grouped together by at least four of the expert subjects.

In this chapter, we review existing descriptions of the conceptual and task fields of proportional reasoning. Then we describe our empirical study and present this graph. We discuss what it shows about the task field.

2.1 Existing Classifications of Proportional Reasoning Tasks

Since we are describing the criteria for classifications of proportional reasoning problems, we looked at several existing classifications. In the following excerpts, the criterion for each classification was extracted directly from the publication.

Following the ideas of Freudenthal (1978, 1983), Ben-Chaim et al. describe proportional reasoning problems in three broad categories.

1. "Ratios: Comparing two parts of a single whole, as in the 'ratio of girls to boys in a class is 15 to 10', or 'a segment is divided in the golden ratio'."

2. "Rates: Comparing magnitudes of different quantities with a connection, as in 'miles per gallon', or 'people per square kilometer', or 'kilograms per cubic meter', or 'unit price'."

3. "Scalar or Similar: Comparing magnitudes of two quantities that are conceptually related, but not naturally thought of as parts of a common whole, as in 'the ratio of sides of two triangles is 2 to 1'"(Ben-Chaim, Fey, Fitzgerald, Benedetto & J. Miller, 1998, pp. 36,247-273).

Cramer et al. divides proportional reasoning into three categories that appear frequently in the literature about this topic. The explanation accompanying each class includes some direct quotes.

1. "Missing value problems: Three pieces of information are given and the task is to find the fourth or missing piece of information. Example: Mr. Short measures 4 buttons in height or six paper clips and the height of Mr. Tall is six buttons. What is Mr. Tall's height in paper clips? The solution: 4 buttons/6 paper clips = 6 buttons/ x paper clips."

2. "Numerical comparison problems: Two complete rates/ratios are given and a numerical answer is not required, but the rates or ratios are to be compared. Example: Which mixture of orange juice and water will have a stronger orangey taste, 3 parts orange to 4 parts water or 2 parts orange to 3 parts water?"

3. "Qualitative prediction and comparison problems: A comparison not dependent on specific numerical values is to be made. Example: If Devan ran fewer laps in more time than she did yesterday, would her running speed be a)faster, b)slower, c)exactly the same,'" (Cramer, Post & Currier, 1993, pp. 159-178).

Behr et al. give a finer classification of proportional reasoning tasks. They propose seven categories for ratio and proportion problems. They assert that "…the following types of proportion related problems all arise naturally. Yet, types 3 through 7 have been neglected in textbook-centered instruction and research" (Lesh, Post & Behr, 1988, pp. 93-118). The following classifications were taken directly from their publication.

1. “Missing value problems: A/B=C/D where three values (including one complete ratio pair) are given, and the goal is to find the missing part of the second (and equivalent) rate pair.”

2. “Comparisons problems: *A/B<=?=>C/D* where all four values are given, and the goal is to judge which is true: *A/B < C/D or A/B = C/D* or *A/B > C/D”*

3. “Transformation problems:

   a. Direction of change judgments: an equivalence is given of the form *A/B =C/D* Then, one or two of the four values *A*, *B*, *C*, or *D* is increased or decreased by a certain amount, and the goal is to judge which relation (<,>, or =) is true for the transformed values.

   b. Transformations to produce equality: an inequality is given of the form *A/B < C/D*. Then, for one of the four values *A*, *B*, *C*, or *D*, a value for x must be found so that, for example *(A*+x*)/B = C/D*.”

4. “Mean value problems: Two values are given, and the goal is to find the third.

   a. Geometric means: *A/x = x/B*

   b. Harmonic means: *A/B = (A-x)/(x-B)”*

5. “Proportions involving conversions from ratios, to rates, to fractions: The ratio of boys to girls in a class was 15 to 12. What fraction of the class was boys?”

6. “Proportions involving unit labels as well as numbers: (3 feet)/(2 seconds) = x miles per hour or 5 feet/ second = x miles/ hour.”

7. “Between-mode translation problems: A ratio (or fraction or rate or quotient) is given in one representation system, and the goal is to portray the same relationship using another representation system.”

Karplus, Pulos, and Stage divide proportional problem solving tasks into three categories: comparison problems, missing-value problems and adaptive restructuring. Briefly, comparison problems are those that can be solved by recognizing numerical relationships. Missing-value problems require calculations to get to an answer. Adaptive restructuring involves the transforming representations and using qualitative comparisons (Karplus, Pulos & Stage, 1983, pp. 45-90).

2.2 A Hundred Years of Textbooks

To gain insight into proportionality tasks, a book search was conducted on Google Books. When “ratio and proportion” was entered as the search term, over 17,000 books were returned. To narrow the search, the advanced book search was used with “ratio and proportion” entered for an exact phrase search. That search was still returned more items than we could deal with. The advanced search was restricted to full view texts only, written in English and copyrighted between Jan 1900 to May of 2009. This search yielded more than 700 books, again too large to deal with. The search was narrowed again, with all other elements staying identical except for the copyright dates. This time, we used dates between Jan 1920 to May 2009. About 250 books were found using this search criterion. Ten books were chosen because they had sections or major headings identified by the author as "ratio" and/or "proportion". Most of the results were from the 1920s; presumably, copyright prevents many more recent books from appearing under "full view". Since Google was not showing many books after a copyright date of 1930, we used textbooks that were conveniently available from the Louisiana State University Middleton Library to supplement the sample. We selected textbooks that contained chapters or headings labeled "ratio" or "proportion" or some combination.

We identified the sections in all these books that had headings including "ratio" or "proportion". From each labeled section, every fifth problem was chosen to include in the sample.

The sample included a large number of problems (See Appendix C). Each problem was briefly analyzed and grouped with obviously similar problems. Using this general overview, a task set of 50 problems was selected to represent as best we could the entire field. The particular problems were chosen to give an overview of all the different types of problems that we could discern. The fifty problems are listed below. The numbering was arbitrary.

Sample Task Set of Problems

1. State the ratio of: 8 lb. to 2 lb.
2. State the ratio of: 1 ½ ft. to 8 in.
3. $5/x = 1/3$
4. $x/a = b$

5. A meter is 39.37 in. Find correct, to the nearest 0.01, the ratio of a meter to a yard.
6. What is the ratio of one side of an equilateral triangle to its perimeter?
7. What is the ratio of the area of a rectangle 8 in. by 6 in. to the area of a rectangle 8 in. by 2 in.?
8. Solder for lead consists of 1 part tin to 1 ½ parts lead; how much of each of these substances is necessary to make 5 lb. of solder?
9. Divide 720 in the ratio of 4 to 5.
10. When 3/8 of an inch on a certain scale represents 3 ft., what does 1 ½ in. on the same scale represent?
11. If the edge of one cube is twice the edge of another, the volume of the larger is how many times that the smaller?
12. If 240 gallons of water will flow through a certain orifice 3 in. in diameter in a certain time, how much water will flow through an orifice 6 in. in diameter in the same time?
13. What is the weight in ounces and the fuel value of the protein in 1 lb. raisins? Of the fat? Of the carbohydrates? What is the fuel value of 1 lb. raisins? (Problem refers to table not reproduced here)
14. When potatoes are 5 lb. for $0.17, how much should a bushel of 60 lb. cost?
15. Which is cheaper, a can of corn containing 1 lb. 2 ½ oz. for$1.21 or 1 lb. 5 oz. for $1.50?
16. James planted 2 rows, 2 ft. apart, in early potatoes. He raised 31 bu. potatoes (60 lb. each), which he retailed at 3 cents a pound. How much did he get for the potatoes?
17. He cut the corn after taking off the ears, and sowed the ground in turnips. He sold 2 bu. (55lb. each) of turnips at 5 cent a pound. How much did he receive for them?
18. George found that 85 % of his seed corn used for planting germinated. His yield was 110 bu. an acre. What would have been his yield if all the seed had germinated?
19. If 31 % of the 474 matured heads were sold for 78 cents a dozen and the remainder for 48 cents a dozen, what amount of money did the cold frame yield?
20. If a man who earns $ 25 per week is saving $ 10 of it, he is saving what ratio of his earnings?
21. Compare the areas of two squares whose sides are respectively a and a' units.
22. State the simplified ratio: 1 kilometer to 1 mi.
23. The side of a square to the diagonal.
24. The circumference of a circle to its radius.

25. If the price of gas is $ .65 per M, what is the charge for 2260 cubic feet of gas?
26. If a mechanically operated hack-saw makes 90 strokes per minute, what is the number per hour?
27. If the yield of alfalfa is one and one-half tons per acre for each cutting, and the crop is cut seven times each year, what is the yield per year on 12 acres?
28. If a train of 30 cars carries 1100 tons, how many cars must be added to the train so that the total load may be 2800 tons?
29. If 104 pounds of seed are used on a 2-acre lot, what amount will be required for 15 acres?
30. What number bears the same ratio to 5.5 that 4.4 bears to 3.3?
31. How much does one lift on the handles of a wheelbarrow if a 100-pound load is placed in the barrow 16 inches from the axle, and the hands 26 inches further away?
32. A cycle car has a 4-inch pulley on the driving shaft belted to a 14-inch pulley fastened to a rear wheel. If the rear wheel is 26 inches in diameter, what is the speed of the car if the engine is making 800 revolutions per minute? Allow 5 per cent for belt slippage.
33. A miller uses 18 bushels of wheat for making 4 barrels of flour. How many barrels of flour can he make from 144 bushels?
34. What per cent is gained in buying oil at 80 cents a gallon and selling it at 12 cents a pint?
35. There were six cloudy days in September, the rest being clear. What was the ratio of cloudy days to clear days?
36. Four boys arrange a three weeks' camping trip. The first boy stays 18 days; the second comes late and stays 12 days; the third is in camp the last two weeks; the fourth stays the three weeks. The total expense is $32.50. What is the share of each?
37. In a certain room of 28 pupils in a school the pupils had invested $ 840 in Liberty Bonds, Baby Bonds, and Thrift Stamps. In order to keep up the same rate of investment, how much should be invested by a room of 20 pupils? Of 42 pupils?
38. Kansas is about 200 mi. by 400 mi. What scale would you use to draw the map upon paper 9 in. by 13 in., which will be a simple scale to work with and still give the largest possible map?
39. Find the unit rate. $6.20 for 5 pounds
40. Write 8.25 as a percent.
41. MULTIPLE CHOICE: The purchase price of a bicycle is $140. The state tax rate is 6 ½% of the purchase price. What is the tax rate expressed as a decimal? A. 6.5 B. 0.65 C. 0.065 D. 0.0065

42. 15 lb = ______ oz
43. Use the percent proportion to find 32.5% of 60.
44. Estimate. 25% of 82
45. A scale drawing has a scale of 1 inch: 10 feet. Use the given actual length to find the length of the object in the scale drawing.   6 feet
46. A fish tank has 5 snails, 6 plants, 10 rocks, and 3 air tubes. Give the ratios in simplest fraction form. Number of plants to number of air tubes
47. Jill Turner earns 5% of sales on all cosmetics she sells. Last week she earned $35.50. What were her sales?
48. 1 out of 6= ___%
49. The Stegosaurus dinosaur weighed about 2000 kilograms. The weight of his brain was only 0.004% of the weight of his body. How much did the Stegosaurus brain weigh?
50. ½ in. =1/16 in. Actual measurement or distance = 1 3/16 in. What is the measurement on the scale drawing?

2.3 Gaining a Perspective: Interpretative Classifications

The sample task set of problems was copied and given to several groups. The first group was a graduate level class that consisted of mathematics educators. The teachers formed groups of two to four. The groups cut the problems apart, leaving one problem per strip of paper. Then, they worked collaboratively to categorize the problems according to their judgment and give each category a descriptive name. If a problem did not seem to fit into any class, the groups were asked to label it as an outlier. The teachers grouped the problems, put them into letter size envelopes and labeled each envelope with their interpretative classification. The envelopes of each group were kept together in a large manila envelope. The following tables list the class descriptors and classes created by each group.

Table 1. Classifications from Textbook Sampling: Group A

Classification	Numbered Problem
Measurement/Conversions	22,25,42
Simple Ratios	1, 2, 3, 4, 5, 8, 9, 14, 20, 28, 29, 30, 33, 35, 37, 46

Table 1 continued

Scale Drawings	10, 38, 45, 50
Geometric	6, 7, 11, 21, 23, 24, 32
Percent	18, 19, 34, 36, 40, 41, 43, 44, 47, 48, 49
Unit Rate	12, 13, 15, 16, 17, 26, 27, 39
Outlier	31

Table 2. Classifications from Textbook Sampling: Group B

Classifications	Numbered Problems
Ratios	1, 2, 5, 6, 7, 9, 20, 21, 22, 23, 24, 30, 35, 45, 46
Simple Proportions	8, 11, 12, 13, 25, 26, 27, 28, 29, 31, 33, 37
Proportional Computation	3, 4, 42
Unit Rate	14, 15, 16, 17, 39
Percent Proportion	18, 19, 32, 34, 40, 41, 43, 44, 47, 48, 49
Scale Drawings	10, 38, 50

Table 3. Classifications from Textbook Sampling: Group C

Classifications	Numbered Problems
Conversions	22, 25, 26, 27, 28, 42
Ratios	1, 2, 5, 6, 20, 23, 24, 35, 46
Proportion	12, 14, 16, 17, 29, 33, 39
Rates	3, 4, 9, 15, 19, 30, 36, 37
Rates of Change	7, 8, 11, 21, 31, 32
Percent	13, 18, 34, 40, 41, 43, 44, 47, 48, 49
Scale	10, 38, 45, 50

Table 4. Classifications from Textbook Sampling: Group D

Classifications	Numbered Problems
Multi-Step Problems	13, 15, 16, 17, 18, 19, 27, 31, 32, 34, 36, 47, 49

Table 4 continued

Ratio/ Proportion	3, 8, 10, 12, 14, 25, 26, 28, 29, 30, 33, 37, 39, 41, 43, 44, 45, 48, 50
Knowledge/Comprehension	4, 9, 40, 42
Simplifying Ratios	1, 2, 5, 7, 20, 22, 35, 46
Geometry-Based	6, 11, 21, 23, 24

The same sample task set of 50 problems was given to several faculty members in the mathematics department at Louisiana State University with the same directions: cut the problems so that there was one problem per slip of paper, categorize the problems and name each group to reflect the criteria and attributes chosen. If there were any problems that did not fit nicely into a category, it could be left out. The mathematicians were given the supplies as well; scissors to cut the problems, letter-sized envelopes for the categories and classifications written on the front of each envelope, and the large manila envelope hold the letter-sized envelopes. The table below illustrates the classifications made by mathematicians.

Table 5. Classifications from Textbook Sampling: Mathematician A

Classifications	Numbered Problems (In Increasing Difficulty)
Conversions	42, 40
Ratios	20, 1, 2, 6, 7, 46, 35, 34, 5, 22, 39
Proportion (Missing-Value)	4, 3, 14, 29, 33, 37, 48, 18, 10, 45, 50, 8, 28, 9
Rates (Multiplying)	44, 43, 47, 25, 41, 49, 36, 16, 17, 19, 27, 26
Area 1/Area2=Length 1/ Length 2	21, 11, 12
Knowledge of Geometry	23, 24
Proportion (Complex)	15, 31, 32, 38

Table 6. Classifications from Textbook Sampling: Mathematician B

Classification	Numbered Problem
Ratios and Rates	1, 2, 7, 20, 22, 35, 38, 39, 40, 41, 46, 48

Table 6 continued

Proportion	3, 4, 9, 10, 12, 14, 15, 16, 17, 18, 21, 25, 26, 29, 30, 31, 33, 34, 37, 43, 44, 45, 47, 49, 50
Multi-Step Problems	8, 19, 27, 28, 32, 36, 42
Constant Ratio	5, 6, 11, 23, 24

Table 7. Classifications from Textbook Sampling: Mathematician C

Classifications	Numbered Problems
Pre-Algebra	1, 2, 5, 10, 18, 25, 26, 27, 29, 33, 34, 35, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50
Algebra	3, 4, 8, 9, 14, 15, 16, 17, 19, 20, 22, 28, 30, 31, 36, 37, 38, 47
Geometry	6, 7, 11, 12, 21, 23, 24, 32

Looking at the existing classifications for ratio and proportion and the classifications created by subject opinion, no standard classifications are evident. The most common shared feature is the use of the words ratio, rate, and proportion.

The data from the above tables was entered into *Mathematica* and a table was created with one row for each problem and one column for each of the seven subjects. The classification name form problem *n* chosen by subject *A* was placed in cell *nA*. See Appendix B. A program (provided by Dr. Madden) drew a graph with one vertex for each problem. Two vertices were joined by an edge if the corresponding problems were deemed similar by at least 5 subjects. (We say a subject "deemed two problems similar" if that subject put them in the same class.) We call this the 5-similar-graph. A second graph was prepared, where two vertices were joined if the corresponding problems were deemed similar by at least 4 subjects. We call this the 4-similar graph.

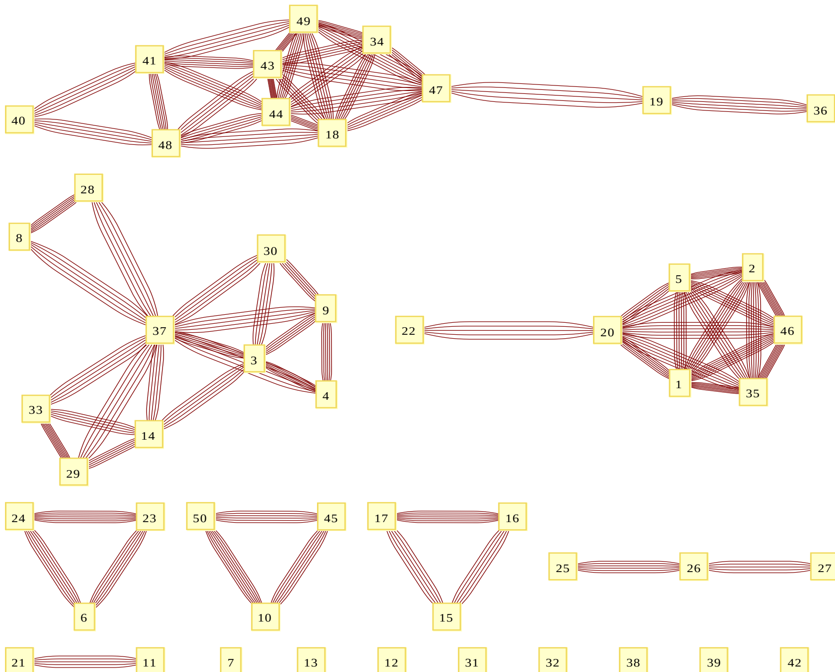


Figure 1. Task Set Classifications (5 Similar)

Figure 1 represents the relationships that were found among the problems listed in the sample task set and the interpretative classifications given by mathematicians and mathematics educators. As explained above, two vertices are linked if more than 5 subjects viewed them similarly. The number of strands in each edge is the actual number of subjects seeing similarity.

While some problems are identified similarly by more than five groups, some are not at all. Looking at the figure, three major groupings are evident, along with five smaller ones. When looking at the classifications given by the surveyed individuals or groups, the three main groupings can be classified as Percents/ Proportions (40, 41, 48, etc.), Rates (8, 28, etc.), and Ratios (22, 20, etc.). The smaller groupings represented can be classified as Geometric Ratios, Scale Drawings, Unit Rates, Conversions, and Rate of Change. The remaining problems in the table were not identified similarly by the people surveyed. (Their independence is not an indication that they are not appropriate tasks for ratio and proportion.) The reader may refer to the list sample task set to draw their own conclusions or devise their own names for the evident groups.

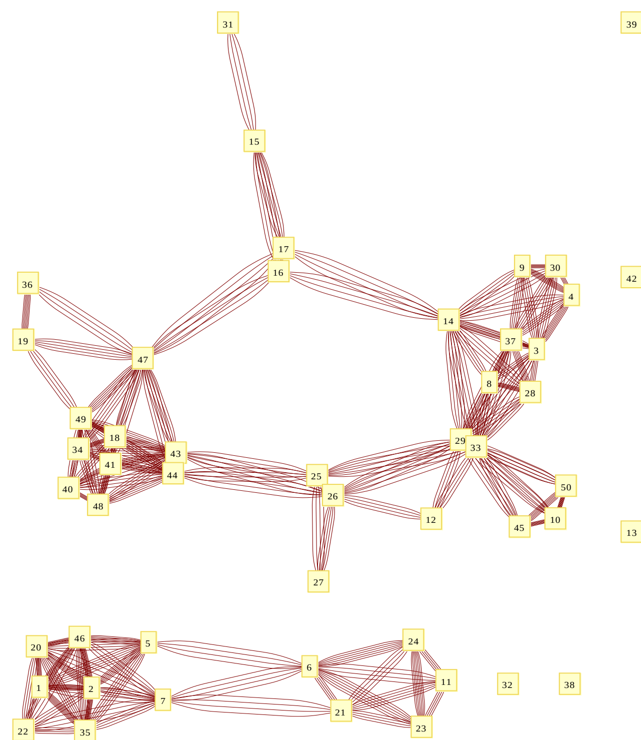


Figure 2. Task Set Classifications 4 Similar)

The 4-similarity graph is depicted in Figure 2. Two major groupings are evident, with some independent problems on the sidelines. Within the first major grouping, four smaller groupings are visible. When comparing the classifications that the groups and individuals surveyed used to identify each problem, the four smaller groups can be labeled Percents/Proportions, Unit Rates, Ratios/Proportions, Scale Drawings and Conversions. The smaller grouping is visibly divided into two sections that can be labeled Ratio Comparisons and Geometric Comparisons. Again, the outlying tasks are not excluded from proportional reasoning just because they were not identified similarly by the groups surveyed. In a general sense, the larger grouping shows all the problems that involve a proportion of some kind while the smaller grouping involves comparisons, which are ratios.

Figure 2 does a better job of showing the connection that proportional reasoning tasks share. In mapping the conceptual field, however, it is important to look within those relationships to draw out the more specific skills required. Figure 1 shows the difference in these skills required to reason proportionally and gives us the ability to have a visible map of the types of tasks that are found in such skill sets. While proportional reasoning is defined by the use of ratios and proportions, by viewing either of these two tables, one can see that it also encompasses geometric comparisons, unit rates, conversions, and scale drawings.

2.4 Louisiana Comprehensive Curriculum

Many of the grade level expectations required in grades sixth through eighth overlap. The curriculum uses the exact same standard in all three grade levels, never giving a level of application or skill and never emphasizing specific goals in proportional reasoning that are any different from the earlier grade levels. The standards are categorized by strands and can become very redundant. While the standards are not precise, the tasks found within the curriculum are more focused and provide the teacher and students with a better look at what is expected in a proportional reasoning task.

The following standards that relate to these skills are copied directly from the Louisiana comprehensive curriculum:

Number and Number Relations

- Use models and pictures to explain concepts or solve problems involving ratio, proportion, and percent with whole numbers (N-8-M)
- Set up and solve simple percent problems using various strategies, including mental math (N-5-M) (N-6-M) (N-8-M)

- Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)
- Determine when an estimate is sufficient and when an exact answer is needed in real-life problems using decimals and percents (N-7-M) (N-5-M)
- Determine and apply rates and ratios (N-8-M)
- Use proportional reasoning to model and solve real-life problems (N-8-M)
- Solve real-life problems involving percentages, including percentages less than 1 or greater than 100 (N-8-M) (N-5-M)

Algebra

- Describe linear, multiplicative, or changing growth relationships (e.g., 1, 3, 6, 10, 15, 21) verbally and algebraically (A-3-M) (A-4-M) (P-1-M)
- Use function machines to determine and describe the rule that generates outputs from given inputs (A-4-M) (P-3-M)
- Describe and compare situations with constant or varying rates of change (A-4-M)
- Explain and formulate generalizations about how a change in one variable results in a change in another variable (A-4-M)

Measurement

- Calculate, interpret, and compare rates such as $/lb., mpg, and mph (M-1-M) (A-5-M)
- Apply rate of change in real-life problems, including density, velocity, and international monetary conversions (M-1-M) (N-8-M) (M-6-M)

Geometry

- Demonstrate conceptual and practical understanding of symmetry, similarity, and congruence and identify similar and congruent figures (G-2-M)
- Predict, draw, and discuss the resulting changes in lengths, orientation, and angle measures that occur in figures under a similarity transformation (dilation) (G-3-M) (G-6-M)
- Solve problems involving lengths of sides of similar triangles (G-5-M) (A-5-M)
- Construct, interpret, and use scale drawings in real-life situations (G-5-M) (M-6-M) (N-8-M)
- Model and explain the relationship between the dimensions of a rectangular prism and its volume (i.e., how scale change in linear dimension(s) affects volume) (G-5-M)

Given the expectations outlined by the Louisiana state comprehensive curriculum, students should gain a full understanding of proportionality both conceptually and procedurally. Not only should students be able to set up a proportion using fractions and convert rates using proportions, they must know when to use these ideas and how to determine the best representation for the problem at hand. Percents and direct proportions, like missing-value problems, can be dealt with by a simple schema (such as the common and over-used "is over of is % over 100") but the student must be able to remember the schema, identify its usefulness in the problem and apply the outcome to real-life situations. Such examples could include finding a markup or markdown, sale prices, tips and taxes, and percent of a number. Applying this skill leads the student to be able to identify unknown products in a problem and inserting a variable. By setting the proportions equal to each other and having a missing piece and applying the procedure to the schema, the student begins the journey into equations. Students can take this information and apply it to a function schema, showing graphical representation of proportion. This skill set illustrates how proportional reasoning is the bridge from elementary mathematics to the advanced understanding of higher mathematics.

Even with the knowledge of the skills necessary to say one can apply proportional reasoning to problem solving, the question still arises in classrooms and teachers' lounges, "What does proportional reasoning look like?" A more in-depth look at Louisiana's curriculum was taken to determine what kinds of tasks are required of students. The grade-level expectations and teaching activities were drawn directly from the comprehensive curriculum (available on the Louisiana Department of Education website; see Appendix A). Proportionality takes on many forms throughout this curriculum. Students are encouraged to use visual and manipulative representations to help them develop conceptual understanding. By participating in these inquiry-based lessons, as exampled by the tasks demonstrated in the curriculum, students begin to use the concepts to express mathematical problems in verbal and symbolic forms that will eventually turn into algebraic expressions and equations. Common representations of proportions are used consistently through all strands. These representations include equivalent fractions, visual and pictorial models, cross multiplication of rates and ratios, percent proportion formula, estimates of size and quantity, changing growth in relationships and sizes, patterns in functions, probability and conversions of units. Resulting changes in similarity transformations and relationships between similar figures and drawing to scale are included in geometry. Algebra uses proportionality to identify linear relationships and functions, geometric growth, dimensional analysis and more.

Missing value or direct proportion is the most prominent form of proportional reasoning in this curriculum. Also known as The Rule of Three, missing-value problems have three given quantities, with two representing a fixed ratio, and the fourth to be determined so that it bears the same ratio to the third. Solving a proportion either through equivalent fractions or cross multiplication is a repeated task for the middle levels. Identifying the relationship between quantities is what gives the students the most trouble. In a story problem, the students have to identify the relationships between the ratios present, when to use a direct proportion formula or other schema. When using a direct proportion they must determine which quantity should be the numerator and which should be the denominator, what unit is on top and what unit is on bottom. The most common examples of this type of proportional representation involve gas mileage, unit rates, and monetary rates. Unit conversions in measurement (and later, conversion of moles to grams in Chemistry) use this type of representation as well.

Using rate of change and constant ratios, students soon discover that they can represent a function through a table or a graph. In a typical activity that illustrates this, students determine the formula for their walking speed through a measured walking demonstration. Groups of four to five students measure out five meters and using a stopwatch, time and record how long it takes to walk the distance. They repeat the demonstration for ten, fifteen, twenty and twenty-five meters. The students are encouraged to represent the data in a way that is suitable, using prior knowledge of tables and graphs to help them determine a way to find their speed. Students discover that a table is suitable but the information is even more easily understood in graphical form. The students evaluate the relationship between their walking times and distances and create a formula to determine their own speed. They graph their speeds with their group members' and compare and contrast.

Balancing weighed items on a scale allows students to manipulate different sides. This is also a visual image, used later to make sense of balancing the sides of an equation. Students learn how to equal out the respective sides and learn that when manipulating one side, the other must change as well or the meaning is lost. Students can test story problems or estimates. This process of balancing proportions can also be simulated on equation mats and manipulating the pieces on both sides.

Students start to show real abstract thought process by developing ways to rationalize the relationships between dimensions of shapes and polyhedra to the respective areas and volumes. Change in size is a theme carried out in similar figures and scale drawings. Students learn to use measurements from each to determine a relationship, usually in a proportional formula of $a/b=c/d$. Relationships between figures and the quantities that describe them grow into proofs for Geometry and basic ideas used in linear and exponential functions in Algebra.

Probability uses ratios to express the likelihood of an event. Students start with an elementary view. For example, they think of the odds of pulling a black marble from a bag using the ratio of the number of black marbles to white marbles included in the bag. They move to the probability of numbers on a number cube, and eventually to complex events whose probability must be calculated using combinations and permutations.

The expectation is that proportional reasoning in all these many guises will lead students to higher order thinking skills, ascending through Bloom's Taxonomy. One hopes that this will create the foundation for algebra and other advanced mathematics and that by threading this common theme through the mathematics curriculum of the middle levels, students can bridge the gap between elementary math skills and abstract thought processes and problem solving. The emphasis at this point in the curriculum changes from procedural knowledge to conceptual understanding and application. Proportionality gives the students a means to look at and compare quantities and the relationships between those quantities. It allows the students to see the building block on which higher mathematics is constructed. The multiple representations of proportions encourage and allow students to grow from visual to symbolic processes in mathematics. The middle grades in the Louisiana comprehensive curriculum teach the students to identify relationships that they carry to higher mathematics and science; thus making proportional reasoning the fundamental bridge. Or such is the plan.

CHAPTER 3 SUMMARY AND CONCLUSION

In this section, we pull together some themes that have dominated the literature and then finally state the main conclusions of this work.

“Understanding the underlying relationships in a proportional situation and working with these relationships has come to be called proportional reasoning,” (Kilpatrick, Swafford & Findell, 2001, pp. 241-254). “One way to define proportional reasoning is to say that it is the ability to recognize, to explain, to think about, to make conjectures about, to graph, to transform, to compare, to make judgments about, to represent or to symbolize relationships of two simple types,” (Lamon, 1999, pg 8). It “is a huge web of knowledge,” in which, “one has to build up competence in a number of practical and mathematical areas,” (Lamon, pg 5). Three main aspects of proportional reasoning that resonates in the literature are: 1) it is about relationships and relationships between relationships (analogies) 2) it is used in real, everyday tasks, and 3) it should be taught with great diligence and practice through experiences.

Cramer et al. highlight in their research that “many aspects of our world operate according to proportional rules” and that “proportional reasoning abilities are extremely useful in the interpretation of real-world phenomena,” (Cramer, Post & Currier, 1993, pp. 159-178). Hoffer agrees with this observation, but suggests, “The acquisition of proportional thinking skills in the population at large has been unsatisfactory.” “…There is evidence that a large segment of our society never acquires them at all,” (Hoffer, 1988, pp. 285-313). Proportional reasoning has to be “…introduced to students in a meaningful manner, with students provided with experiences to enable them to develop their own solution strategies,” (Shield & Dole, 2002).

Cramer et al. point out that, “Proportional reasoning abilities are more involved than textbooks would suggest. Textbooks emphasize the development of procedural skills rather than conceptual understandings,” (Cramer, Post & Currier, 1993, pp. 159-178). “Traditionally, instruction has focused on missing-value problems," (Kilpatrick, Swafford & Findell, pp. 241-254) and "…the traditional cross-multiply and divide algorithm is used…," (Cramer, Post & Currier, pp. 159-178). Cramer et al. believe that this algorithm holds little meaning to the student. Shield and Dole also argue that the traditional method loses the 'connectedness' of the proportional relationships. " The symbolic representation of proportional situations and subsequent manipulation of numbers within proportion equations provides little meaning, either to the real-context of examples presented, or to prior knowledge of other related mathematics topics," (Shield and Dole, 2002, pp. 608-615). Kilpatrick et al. warn that, "moving directly to the cross-multiplication algorithm, without attending to the conceptual aspects of proportional reasoning, can create difficulties for students," (Kilpatrick, Swafford & Findell, 2001, pp. 241-254). This lack of connectedness and push for rote memorization for procedural practices is the cause for poor natural methods for proportional thinking.

Curricula and standards almost universally reflect the belief that proportional thinking skills evolve over time. Students learn ratios and rates in pre-algebra classes then move into functions in algebra and study similarity in geometry. Although the foundation of proportional reasoning is introduced in the lower grades, it remains the bridge that connects the higher mathematics to the fundamental skills allowing students to be able to use mathematics reasoning, namely proportional reasoning, in real life situations.

In development of proportional reasoning, Kilpatrick et al. outline three aspects that students must connect. "First, students' reasoning is facilitated as they learn to make comparisons based on multiplication rather than just addition," (Kilpatrick, Swafford & Findell, 2001, pp. 241-254). This coincides with Vergnaud's idea that identifying the multiplicative structure of proportional situations is the most important step in proportional reasoning, (Vergnaud, 1988, pp. 141-161). "A second aspect is that students' reasoning is facilitated as they distinguish between those features of a proportion situation that can change and those that must stay the same," (Kilpatrick, Swafford & Findell, pp. 241-254). Kilpatrick et al. explains this aspect by using the example "$2 for 3 balloons expresses the same relationship as $4 for 6 balloons," (Kilpatrick, Swafford & Findell, pp. 241-254). The third aspect is the 'connectedness' of the relationship to a real life situation. Just identifying the relationship is not enough. A student must be able to create what Kilpatrick et al. calls a "composite unit". In this example the composite unit is "2-for-3". A student who has reached this aspect would be able to use the composite unit as a comparative base for situation. The student would see that a relationship of 6-for-9 would be the same but a relationship of 12-for-24 would not.

Final conclusions. Based on the information we have gathered, we see a thread running through the conceptual-field of proportional reasoning, a progression that begins with ratios, steps to the more complex relationships involved in rates, incorporates these things in "missing-value" problems, then applies these concepts in geometry and then extends them to functional reasoning.

In the task sample taken from textbooks over the past century, problems have varied but little, except for the contexts in which they are presented. Early textbooks focused on gears, farming, or vocational tools needed to work the land. In more modern scenarios, proportional reasoning is presented in whimsical settings such as balloons and candy, shopping, and entertainment. Even though the real-life situations may differ, the focus on the situations is multiplicative reasoning such as the example of buying 2 items for $3. Also, in the sample tasks that we analyzed, we see that students must make certain assumptions in reasoning, and are not often presented with easily memorized algorithms.

The classifications that prominent researchers have offered for proportional reasoning also suggest this hierarchy of learning. We see that reasoning skills manifest slowly through comparisons, comparisons of two or more relationships, similar items that may have different units, and geometric figures. Kilpatrick's view on how a student should reason is parallel to the classifications of types of tasks that are found in proportional reasoning. Students must first start with the multiplicative comparisons, understand the relationship and then be able to apply that relationship to a larger situation.

Standards and curriculums throughout the country reflect a natural progression similar to Kilpatrick's. Curriculums introduce relationships in the form of ratios and move into direct proportions. Real-life situations are presented throughout, motivating the students to start applying these ideas and skills to other situations. This same progression continues with each activity or each grade level adding onto the previous.

This framework is evident in the reactions of the subjects to the task set of sampled problems. It is also to be seen in existing and interpretative classifications, teaching standards and the Louisiana comprehensive curriculum. (The latter also concur in the idea that students must learn proportional reasoning over a lengthy period of time and through stages.)

That framework for tasks that define proportional reasoning is:

A. Ratios
B. Rates
C. Missing-value Proportions
D. Similarity
E. Constant Ratio/ Functions

Ratio, as a relationship that arises by comparing two like things with respect to size, was given a central place in mathematics by Euclid. After the introduction and use of the real number system, comparisons of unlike things arose- miles and hours, inches and feet, etc. The use of measurement and the comparison of unlike things expands this conceptual field, which ultimately opens the way to higher mathematics.

REFERENCES

Alexander, G. (1921). In John Dewey (Ed.), *The Alexander- Dewey Arithmetic: Advanced Book* (pp. 135-138). New York: Longmans, Green, & Co.

Allen, D. (1997). *Euclid*. Retrieved from http://www.math.tamu.edu/~dallen/history/euclid/euclid.html

Anderson, R. F. (1921). *The Anderson Arithmetic: Book Three* (pp. 283-302). Silver, Burdett, & Company.

Bailey, R., et al. (2005). *Mathematics: Applications and Concepts: Course 2* (pp. 286-330). Columbus, Ohio: Glencoe McGraw-Hill.

Baldridge, S., & Parker, T. (2005). *Elementary Mathematics for Teachers* (pp. 167-168). Okemos, Michigan: Sefton Ash.

Barker, E. H. (1920). *Applied Mathematics for Junior High Schools* (pp. 180-195). Allyn & Bacon.

Beckmann, S. (2005). *Mathematics for Elementary School Teachers* (pp. 299-300). Pearson Education.

Ben-Chaim, D., Fey, J., Fitzgerald, W., Benedetto, C., & J.Miller. (1998). Proportional reasoning among 7th grade students with different curricular experiences. In *Educational Studies in Mathematics* (pp. 36,247-273).

Cramer, K., Post, T., & Currier, S. (1993). Learning and Teaching Ratio and Proportion: Research Implications. In *Research Ideas for the Classroom* (pp. 159-178). New York: Macmillan Publishing Company.

Connecticut State Department of Education. (2009). Retrieved from http://www.sde.ct.gov/sde/cwp/view.asp?a=2618&q=320872

Euclid. *Euclid's Elements Book V: Theory of Abstract Proportions*.

Ford, W., & Ammerman, C. (1920). *Second Course in Algebra* (pp. 166-177). New York: The Macmillan Comapny.

Georgia State Department of Education. (2009). Retrieved from http://public.doe.k12.ga.us

Goozner, C. (1973, 1988). *Arithmetic Skills Worktext* (2nd ed., pp. 285-303). New York: Amsco School Publications.

Grattan-Guiness, Ivor. (2000). *The Rainbow of Mathematics: A History of the Mathematical Sciences* (pp. 56-64). W. W. Norton & Company.

Hamilton, S. (1919,1920). *Hamilton's Essentials of Arithmetic: Higher grades* (pp. 140-146). American Book Company.

Hoffer, A. (1988). Ratios and Proportional Thinking. In T. Post (Ed.), *Teaching mathematics in Grades K-8: Research Based Methods* (pp. 285-313). Boston: Allyn & Bacon.

Hoyt, F. S., & Peet, H. E. (1920). *Everyday Arithmetic* (pp. 245-251). Cambridge, Massachusetts: Houghton Mifflin Company.

Jensen, G. R. (2003). *Arithmetic for Teachers: With Applications and Topics from Geometry* (pp. 241-242). Providence, Rhode Island: American Mathematical Society.

Karplus, R., Pulos, S., & Stage, E. K. (1983). Proportional Reasoning of Early Adolescents. In Robert Lesh and Marsha Landau (Ed.), *Acquisition of Mathematics Concepts and Processes.* (pp. 45-90). New York: Academic Press Inc.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). Developing Fluency with Other Numbers: Proportional Reasoning. In *Adding It Up: Helping Children Learn Mathematics* (pp. 241-254). Washington D.C.: National Academy Press.

Lamon, S. J. (1999). *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers* (pp. 1-19). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Larson, R., Boswell, L., Kanold, T., & Stiff, L. (2005). *Pre-algebra* (pp. 266-324). McDougal Littell.

Lesh, R., Post, T., & Behr, M. (1988). Proportional Reasoning. In M. Behr and J. Hiebert (Ed.), *Number Concepts and Operations in the Middle Grades*: *Volume 2* (pp. 93-118). Reston, Virginia: The National Council of Teachers of Mathematics.

Lindquist, T. (1920). *Junior High School Mathematics: Book II* (pp. 188-196). Charles Schribners Sons.

Louisiana State Department of Education. (2009). Retrieved from http://www.doe.state.la.us/

Malloy, C., Price, J., Willard, T., & Sloan, L. (2005). *Pre-algebra* (pp. 262-322). Columbus, Ohio: Glencoe McGraw- Hill.

Marcy, S., & Marcy, J. (1978). *Pre-algebra with Pizzazz!: Book BB* (pp. 29-54). Palo Alto, CA: Creative Publications.

Mathematics Framework for California Public Schools. (2009). Retrieved from http://csmp.ucop.edu/downloads/cmp/math.pdf

Mississippi Content Standards for Mathematics. (2009). Retrieved from http://www.mde.k12.ms.us/ACAD/ID/Curriculum/Math/index.htm

O'Daffer, P. (2005). *Mathematics for Elementary School Teachers* (p. 350).

Paddock, C. E., & Holton, E. E. (1922). *Vocational Arthimetic* (pp. 94-101). New York: D. Appleton & Company.

Porter, Theodore M. (1995). *Trust in Numbers: The Pursuit of Objectivity in Science and Public Life* (p. 57) Princeton, New Jersey: Princeton University Press.

Price, J., Rath, J. N., & Leschensky, W. (1982). *Merrill Pre-algebra: A Problem Solving Approach* (pp. 270-298). Columbus, Ohio: Charles E. Merrill Publishing Comapny.

Proportionality. (2009). Retrieved from http://en.wikipedia.org/wiki/Proportionality_(mathematics)

Shield, M. and Dole, S. (2002). Investigating Textbook Presentations of Ratio and Proportion. In: Barton, B., Irwin, K.C., Pfannkuch, M. and Thomas, M. O. J., *Mathematics Education in the South Pacific. Proceedings of the Twenty-fifth Annual Conference of the Mathematics Education Research Group of Australasia*. *MERGA 25*, Auckland, N.Z., (608-616). 7-10 July 2002.

Stone, J. C. (1922). *Junior High School Mathematics: Book II* (pp. 31-37). Benj. H. Sanborn & Co.

Texas Essential Knowledge and Skills for Mathematics. (2009). Retrieved from http://ritter.tea.state.tx.us/rules/tac/chapter111/ch111b.html

Vergnaud, G. (1983). Multiplicative Structures. In Robert Lesh and Marsha Landau (Ed.), *Acquisition of Mathematics Concepts and Processes.* (pp. 128-174). New York: Academic Press Inc.

Vergnaud, G. (1988). Multiplicative Structures. In M. Behr and J. Hiebert (Ed.), *Number Concepts and Operations in the Middle Grades*: *Volume 2* (pp. 141-161). Reston, Virginia: The National Council of Teachers of Mathematics.

Wentworth, G., & David Eugene Smith. (1919, 1920). *School Arithmetics: Book Three* (pp. 103-112). Ginn & Company.

APPENDIX A: LOUISIANA COMPREHENSIVE CURRICULUM ACTIVITIES

Sixth Grade- LA Comprehensive Curriculum

13. Use models and pictures to explain concepts or solve problems involving ratio, proportion, and percent with whole numbers (N-8-M)

20. Calculate, interpret, and compare rates such as $/lb., mpg, and mph (M-1-M) (A-5-M)

Activity 8: Grocery Math (GLEs: <u>10</u>, 13, <u>20</u>)

Materials List: grocery ads, Grocery Ad BLM, pencil, paper, calculators

Provide students with grocery ads from a current newspaper or the Grocery Ad BLM. Have students work in pairs with a specified task: Given $20 to go to the grocery store, buy a variety of fruits and vegetables. Direct students to select a combination of at least four different fruits and/or vegetables, estimate the cost, estimate the tax at 10% and record the estimate total. Then have the students calculate the total cost and present their findings to the class. The findings should include the estimated cost and the final cost. Sketch a rectangle to represent the $20. Divide the rectangle into approximate parts to show prices (i.e. if $5.00 is spent on apples, $\frac{1}{4}$ of the rectangle should be marked and labeled “apples”). Use the rectangle as a visual representation of the information. Considering the purchases, have students respond to the following: Do you have enough money to buy 3 lbs. of each? 4 lbs. of each? 5 lbs. of each? Use a calculator to determine the cost of each fruit/vegetable at 3, 4, and 5 lbs. If one item can be bought in three-pound bags for $2.99, select one item and decide if it is cheaper to buy by the bag or by the pound? Instruct students to record answers as a rate: 1lb. for $.997.

(Teacher Note: When computing 10% tax, mental math should be encouraged. Students should be able to explain how they arrived at the answer. Accept answers that show a clear understanding that “moving the decimal 1 place to the left” is because they are multiplying by .1 (one tenth or 10- hundredths).

Activity 9: Tangram Ratio (GLE: 13)

Materials List: Tangrams BLM, one paper square for each student, scissors, pencil

A tangram puzzle is made up of seven pieces: 2 large triangles, 1 medium triangle, 2 small triangles, 1 parallelogram, and 1 square. A large square can be formed using all 7 tangram pieces. Have each student make his/her own set of tangrams so that he/she can have a direct understanding of the relationships between the parts and whole and among the pieces to one another. Simple directions for creating the tangrams are given below. Directions with visual representations are on the Tangrams BLM.

Fold and cut a square sheet of paper by following these instructions:

1. Fold the square in half diagonally, unfold, and cut along the crease into two congruent triangles.
2. Take one of these triangles. Fold in half, unfold, and cut along the crease. Set both of these triangles aside.
3. Take the other large triangle. Lightly crease to find the midpoint of the longest side. Fold so that the vertex of the right angle touches that midpoint, unfold and cut along the crease. You will have formed a middle-sized triangle and a trapezoid. Set the middle-sized triangle aside with the two large-size triangles.
4. Fold the trapezoid in half, unfold, and cut. To create a square and a small-sized triangle from the other trapezoid half, fold the acute base angle to the adjacent right base angle and cut on the crease. Place these two shapes aside.
5. To create a parallelogram and a small-sized triangle, take one of the trapezoid halves. Fold the right base angle to the opposite obtuse angle, crease, unfold, and cut.
6. You should have the 7 tangram pieces: 2 large congruent triangles
   1 middle-sized triangle

2 small congruent triangles

1 parallelogram

1 square

7. The pieces may now be arranged in many shapes. Try recreating the original square. After a quick review of the terms *area and ratio*, have students determine the ratio of the area of each piece to that of the other pieces by comparing the sizes of the pieces. For example, students should determine the ratio of a small triangle's area compared to the medium triangle. Next, ask students to write the ratios of each of the tangram pieces to the whole (the completed puzzle). As an example, students should find that the ratio of a large triangle to the large square (the completed puzzle) to be 2 to 4, which reduces to a ratio of 1 to 2. This ratio compares the area of a large triangle to the area of the square (the completed puzzle). Have the students use the ratios to write proportions. When students complete the activity, have them rewrite the ratios of area of single pieces to the whole as fractions and then as percents.

Activity 11: Vacation Math (GLE: 20)

Materials List: Internet access, maps, or atlases, paper, pencil

We're going on vacation! Allow students, in groups of 2, to make use of the Internet, maps, or atlases to locate the distance from home to a destination of their choice. Have the students predict how long it will take to drive at the posted speed limit. This distance with a variety of speeds will be used to determine trip length. Class discussion should focus on the distance formula with students discovering the formula instead of having it given to them. Questions student should explore include the following: If we are going to drive to visit our location, how long will it take to get there if we drive 60 mph? If the car we're using gets 30 miles to the gallon, how much gas will we use to get there and back? If the price of gas is $2.50 per gallon, how much will it cost to go on our trip? Have each group make a presentation to the class sharing information.

Seventh Grade- LA Comprehensive Curriculum

10. Determine and apply rates and ratios (N-8-M)

11. Use proportions involving whole numbers to solve real-life problems (N-8-M)

Activity 11: Rates (GLE: 10)

Materials List: sale papers/grocery items that can be used to figure unit cost and/or a copy of Grocery Shopping BLM, pencil, paper

Provide the students with a list of items they can purchase along with the prices. These items can be 6-packs of soft drinks, ounces of potato chips, pounds of peanuts, and so on. Be sure to use items that can be used to figure unit cost. Have the students calculate the unit price of each item. Also, extend this to include rates such as $45.00 for 8 hours of work, driving 297 miles in 5 hours, reading 36 pages in 2 hours, and so on. Have the students figure unit rates for these types of problems also. Put students in small groups, and give the students 5-10 minutes to review the information from the activity and to respond to one or both of the following situations. They should also write at least 3-5 questions they anticipate being asked by their peers and 2-5 questions to ask other experts. When time is up, the teacher will randomly select groups to assume the role of p*rofessor know-it-all* and provide their answers and reasoning for the situation. They will also have to provide "expert" answers to questions from their peers about their reasoning.

Situation 1: Lucy and CJ are in charge of buying chips for a class party. They plan to purchase 1.5 to 2 oz of chips for each of the 24 students. Use the information below to help them make the best purchase.

Big Al's Grocery

1 – 1.75oz can for $0.75

12 – 1.75oz cans for $8.95

1 – 6oz can for $2.52

Solution:

Size	*Total Ounces*	*Total Cost*	*Cost per Ounce*	
1 – 1.75 oz can	*1.75*	*$0.75*	*0.428*	
12 – 1.75 oz cans	*21*	*$8.95*	*0.426*	
1 – 6 oz can	*6*	*$2.55*	*0.425*	*X*

Situation 2: Kenneth and Jena are in charge of buying sodas for a class party. They plan to purchase 6 oz of soda for each of the 26 students. Use the information in the table to help them make the best purchase.

PJ's Grocery

Container Size	Capacity in ounces	Cost
1 Liter	33.8 oz	$1.09
2 Liter	67.6 oz	$1.29
3 Liter	101.4 oz	$1.99

Solution: This table shows the different combinations of containers the students may use to get their target of 156 ounces. Be sure students are able to defend their choices; they may not choose the overall lowest unit cost which requires them to purchase additional soda. They will need to purchase a minimum of 156 ounces of soda.

Quantity	*Size*	*Total Ounces*	*Total Cost*	*Cost per Ounce*	
5	*1-L*	*169 oz*	*$5.45*	*$0.03*	
2	*2-L*				
1	*1-L*	*169 oz*	*$3.67*	*$0.021*	
3	*2-L*	*202.8 oz*	*$3.87*	*$0.0190*	*X*
1	*3-L*				
1	*2-L*	*169 oz*	*$3.28*	*$0.0194*	
2	*3-L*	*202.8 oz*	*$3.98*	*$0.0196*	
1	*3-L*				
2	*1-L*	*169 oz*	*$4.17*	*$0.024*	

Activity 12: Ratio Patterns (GLEs: 10, 11)

Materials List: pattern blocks or pieces of paper in 5 colors with squares, rectangles and triangles, scissors, pictures of quilts, patterns, repeating patterns

Show the class pictures of quilts and patterns that have a repeating pattern such as an AB, ABA, or ABC pattern, and pass out five different colors of paper (pattern blocks, if available) marked with varying shapes including squares, rectangles, and triangles about 2 inches in size. Divide students into groups, and have them cut out the shapes. Show students two shapes - an equal number of red squares and blue triangles. Discuss the ratio of red pieces to blue pieces or squares to triangles. Group different color pieces and shapes to create designs. Discuss how repeated patterns are pleasing to the eye. Ask volunteers to come forward to create a pattern with pieces. Have the volunteers give the ratio of the colors or shapes. Divide students into groups to create their own patterns using different color ratios. Next, give each group a different ratio of reds to greens and blues to yellow, etc. (e.g., the ratio of 3 blue to every 4 green or 2 red for every 5 yellow) and have students create a pattern and demonstrate how their ratio was used to create the pattern. Introduce the concept of *proportion* for the patterns the students have created (e.g. 4 green for every 2 red is the same as 8 green for every 4 red). Demonstrate how to set up a proportion: $\frac{4}{2}=\frac{8}{4}$. Help students realize the two fractions are equivalent; the second has only increased by a common factor of 2. Cross multiply to create an equation that shows the cross products are equal: $4\times4=2\times8$. Instruct students how to solve the equation; in this example it is just a matter of simplifying each side. Now tell the students the ratio of yellow to red is 3 to 4. If I have 20 red I want to use, how many yellow will I need? Again, demonstrate how to set up and solve a proportion to find the solution. Vary the difficulty of the task by specifying the pattern (e.g., the same colors cannot touch on more than two sides).

Challenge the students to create their own quilt pattern and to determine the ratio/proportion of the colors they used. Have the students change the ratio of the colors used to a percent of colors used.

Activity 13: What's the Recipe? (GLEs: 7, <u>11</u>)

Materials List: a different recipe for each pair of students, What's the Recipe BLM

Discuss situations where a recipe may need to be reduced or increased. Discuss the fact that all ingredients must be increased or decreased *proportionally* in order for the recipe to turn out correctly. (For example, use a recipe for making chocolate chip cookies that makes 24 cookies, but the recipe needs to be increased so that everyone in a class of 36 gets a cookie.) If the given recipe produces 2 dozen cookies, what would the recipe be for producing 1 dozen cookies? 4 dozen? 6 dozen? 7 dozen? $\frac{1}{2}$ dozen? Give each pair of students a different recipe or the What's the Recipe BLM, and have them reduce or increase the recipe proportionally by two given amounts. Also, have students create new recipes based on a given percent of the original recipe. A hot chocolate recipe is a good choice; after calculating how much of each ingredient, the students could make hot chocolate for the class. Water could be heated in a coffee pot.

Activity 9: Problem Solving Triangle Puzzle (GLEs: 5, 7, 10, 11)

Materials List: Triangle Puzzle BLM for each student, scissors, tape, pencil, paper

Provide students with Triangle Puzzle BLM formed of equilateral triangles. Have students cut the triangles apart, and match each problem to the solution. The triangles will form a symmetrical shape when each problem is answered correctly. Students can then tape the pieces together.

Directions for teacher to make additional puzzles: Cut out several equilateral triangles and place together to form a symmetrical shape. Write a problem along one side of a triangle. Find the triangle that shares this side and write the solution along the side of this triangle. Continue this process until all triangles have either an answer or a problem written on each side. Include ratio, proportions, order of operations, percents, decimals, fractions and mixed number situations on the puzzle. Make sure that the same answer is not used more than once as this makes the puzzle very difficult to solve.

Activity 13: Cooperative Problem Solving (GLEs: 5, 7, 10, 11)

Materials List: Cooperative Problem Solving BLM, pencil, paper

To prepare for this activity, copy the Cooperative Problem Solving BLM, and cut the pieces apart. Problems may be separated by placing them in sandwich bags to be distributed to the student groups. Have students work in groups of two or three to solve real-life situations. Each group should be given one sandwich bag which contains the pieces of one word problem. Each student takes at least one card and keeps it in his/her possession. Have students in each group take turns sharing the information on their cards, then work together to find a solution to the situation. This is a good tool to get all students involved in the problem solving process. Even the weakest students have a part, because they must contribute the information on their cards and read them to the group in order for the problem to be solved.

Example of a set of cards that one group would solve:

The seventh graders are planning to sell cups of hot chocolate at the basketball games this	If 6 spoonfuls of mix make a cup of hot chocolate,	How many spoonfuls of mix will be needed to make 42 cups of hot chocolate?

Allow time for student groups to share their problem and solution. Lead a class discussion about the different methods used to solve the problems. Students should be able to identify the quantities being compared; point out that when two quantities are compared and written as a fraction, that fraction is called a ratio. Have students find equivalent ratios in the problem they worked. Model how to set up and solve proportions using the problem set in this activity. Remind students that each proportion is two equivalent ratios.

Activity 14: Common Ratios (GLEs: 5, 10, 11)

Materials List: a measuring tape for each group, Common Ratios BLM for each student, pencils, calculators

Remind students how they used proportions in the previous activity to solve problems when equivalent ratios could be written. Include examples to reinforce this concept.

Students will compare their heights with other measurements of their body to the nearest millimeter to determine if there is a common ratio. Be careful with the division of groups during this activity. Give each student a copy of the Common Ratios BLM for recording measurements. Have students work in groups of three or four. Ask them to take turns with one student measuring, one student recording on the given chart, and the third being the person measured. Have students complete the chart by measuring the distances described in the chart, and then finding each ratio. Have the students compare their findings to the findings of the other students in the group. Then have the students complete the questions as a group.

Have students research the work of Leonardo da Vinci to see if their proportions relate to his ratios. Have students use da Vinci's ratios to predict their measurements. What is The Golden Ratio? How do artists use these ratios today? Websites which provide information on the Golden Ration include these:

http://www.geocities.com/jyce3/, http://members.tripod.com/mropfer/the_golden_webquest.htm, and http://en.wikipedia.org/wiki/Golden_ratio.

Activity 15: In Another World! (GLEs: 5, 11)

Materials List: In Another World! BLM for each student, rulers and/or measuring tapes, pencils, calculators

You are a 65-inch tall Earthling who has landed on the world of Gianormas. Immediately upon arrival, you meet Leonardo who is 50 ft tall! As you look around, you notice that everything in this new world is Leonardo's size. You assume that everything is to the same scale as it is on Earth. Students will measure items that may be found in the classroom to the nearest quarter-inch. Then they will use proportions to find the measurement of the same items in a world of giants, then in a world of miniatures. Each student will need a ruler or measuring tape, a pencil, and a calculator to complete the In Another World! BLM. Remember to convert inches into feet or vise versa when necessary.

Eighth Grade- LA Comprehensive Curriculum

7. Use proportional reasoning to model and solve real-life problems (N-8-M)
9. Find unit/cost rates and apply them in real-life problems (N-8-M) (N-5-M) (A-5-M)
29. Solve problems involving lengths of sides of similar triangles (G-5-M) (A-5-M)

30. Construct, interpret, and use scale drawings in real-life situations (G-5-M) (M-6-M) (N-8-M)

Activity 5: The Better Buy? (GLE: 9)

Materials List: The Better Buy BLM, Choose the Better Buy? BLM, pencils, paper, math learning log, grocery ads (optional)

Begin this activity by putting a transparency of The Better Buy? BLM on the overhead. Cover the bottom portion that gives group directions. Using a modified *SQPL*, (view literacy strategy descriptions) have students independently write questions that this statement *(One potato chip costs $0.15.)* might suggest to them. After about one minute, have the students get into pairs, compare questions and write at least two of their questions to post on the class list.

Once the class questions are posted, give the students ten minutes and have the pairs of students determine method(s) of answering at least three of the class questions. Circulate as students are answering their questions, and be sure that any misconceptions are addressed before they begin independent work. Have the students answer the initial question after they have completed work with their partners. Next, provide students with Choose the Better Buy? BLM. Have students work individually to find the unit rates to determine the better buy in each situation. Students should verify results with a partner. Give opportunities for questions if students have a problem that they do not agree upon. Extend the activity by giving grocery ads from different stores carrying the same items to each group of four students. Give students a list of items to purchase and have student groups of four make projections about savings on groceries by shopping at store A versus store B over a year. Have students present their findings to another group or the class. Have students record in their math *learning log* (view literacy strategy descriptions) what they understand about unit prices.

Activity 6: Refreshing Dance (GLE: 9)

Materials List: Refreshing Dance BLM, pencils, paper

Have students work in groups of four to prepare a cost-per-student estimate for refreshments at an 8th grade party. Distribute Refreshing Dance BLM. Have students complete the chart and determine the total cost of refreshments for each student and the total cost of the dance if they plan for 200 students. Students will present their proposals and the answers to the questions to the class using a modified *professor know-it-all* (view literacy strategy descriptions) strategy. Students will answer questions about their proposal from the class. Using *professor know-it-all,* the teacher will call on groups of students randomly to come to the front of the room and provide "expert" answers to questions from their peers about their proposal. The teacher should remind the students to listen to the questions and to think carefully about the answers received so that they can challenge or correct the *professor know-it-alls* if the answers the "experts" give are not correct or need elaboration and amending. Students should be able to justify not only the cost of the refreshments but also the amount that needs to be ordered.

Activity 7: My Future Salary (GLE: 8, 9, 39)

Materials List: grid paper for students, My Future Salary BLM, paper, pencil, Internet access

Introduce *SQPL* (view literacy strategy descriptions) by posting the statement "An electrical engineer earns more money in one year than a person making minimum wage earns working for 5 years." Have students work in pairs to generate questions that they would like to have answered about this statement. Have students share questions with the class and make a class list of questions. Students must make sure that a question relating to a comparison of job salaries is asked. Give students time to research the information needed to answer the question. A site that has recent top salaries can be found at

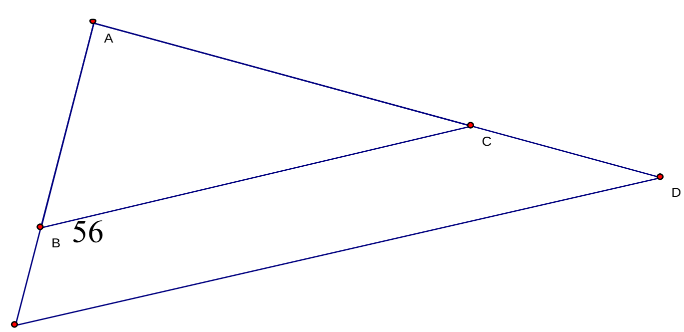
http://money.cnn.com/2005/04/15/pf/college/starting_salaries/

The students can share their information with the class by using *professor know-it-all* (view literacy strategy descriptions). The research group will go to the head of the class and report their findings to the class and answer questions from the group about their findings. Give other groups time to share their findings, also. Ask the students why the minimum hourly wage is considered a unit rate (*amount of money paid per hour of work*). Distribute the My Future Salary BLM and have students make observations about what has happened to the minimum wage in the years since 1960. Lead a discussion with students about how the minimum wage has changed through the years. Have students create a graph of the minimum wage from the information in the chart and predict the minimum wage for the year 2010. Have the students calculate what a person working a minimum wage job working 40 hours per week made in 2003 and what that person would make using their prediction for the year 2010. Discuss how the graph helps with making predictions. The information on the My Future Salary BLM is also found on the following website: http://www.workinglife.org/wiki/Wages+and+Benefits%3A+Value+of+the+Minimum+Wage+%281960-Current%29. The web site http://www.bls.gov/bls/blswage.htm gives current wages of jobs listed with the labor division of the U. S. government. The Louisiana Board of Regents has an e-portal designed specifically for Louisiana students: https://www.laeportal.com/main.aspx. This portal was designed to be used by eighth grade students as they make a five year academic plan. There is a teacher section which provides links to careers, salaries and other information that would be applicable to this activity.

Activity 8: Similar Triangles (GLEs: 7, 29)

Materials List: 6 drinking straws for each pair of students, scissors, pencils, paper, math learning log, ruler

Have students work in pairs to create an equilateral triangle using drinking



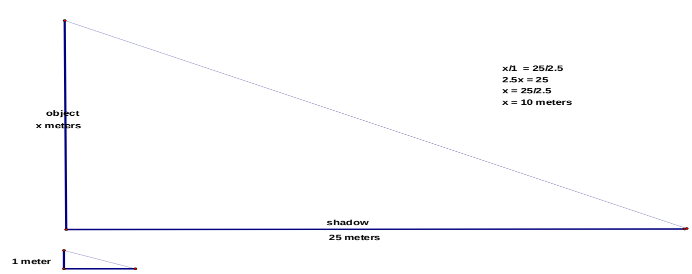
straws for sides. Ask students to explain how they know they have created an equilateral triangle. (*They have three straws the same length).* Have them measure and record the side length. Instruct students to make a second equilateral triangle with sides of different length than those of triangle one. Have students measure with rulers the sides of their new triangle. Ask them to determine a way to prove that the two triangles are similar using what they have learned about proportions. Students should understand that the triangles are similar because the sides are of proportionate lengths. Triangle one has sides twice as long as triangle two and the angles measure the same because they are equilateral triangles. Equilateral triangles are also equiangular. Lead students to write a conjecture about the relationship of proportionate sides and equal angles in two equilateral triangles. Ask them if it seems possible that this relationship will hold true with other triangle types.

Next, have students construct or draw a triangle with all three sides of different lengths (scalene). Have students label the triangle with the measure of each of the side lengths and each angle measure. Instruct students to select one vertex of their new triangle and label the vertex A. Have students extend the sides of the triangle from vertex A so that the side is $\frac{3}{2}$ the length of the original side. Repeat this with the other side from vertex A. Instruct students to connect the two endpoints of the new sides for their triangle. Have students make some observations about the two triangles that they have formed. Challenge students to use proportions to prove that the two triangles are proportional. Discuss how the angles of these two triangles are congruent but the side lengths are proportionate. Tell the students that the symbol to show similarity is '~'. We call the two triangles 'similar triangles' because the angles are congruent and the side lengths are proportionate. Next, have them construct or draw a triangle using a ratio provided to them, perhaps a ratio of $\frac{3}{4}$ and determine if the same conjecture holds true for triangles with sides of different lengths. For example, if they create a triangle with side lengths of 3 inches, 4 inches, and 5 inches, a triangle with sides of 2.25 inches, 3 inches, and 3.75 inches would meet the requirement. Once they have constructed the triangle, the students should set up a proportion to verify proportionality. Be sure to look for and clear up any misconceptions about using the correct angles when the figures are not oriented the same way. Have students record in their math *learning log* (view literacy strategy descriptions) what they know about similar triangles.

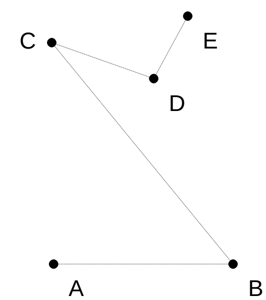
Activity 9: Proportional Reasoning (GLEs: 7, 29)

Materials List: Proportional Reasoning BLM, meter sticks, objects to measure outside, pencils, paper, calculator

This activity allows students to apply the concept of similar triangles. Distribute the Proportional Reasoning BLM. Students will calculate the height of various objects by measuring the object's shadow and the shadow of a meter stick placed vertically on the ground. Following the directions on the BLM, lead students to understand that they can solve the problems by creating a proportion between the corresponding parts of the right triangle formed by the object and its shadow with the right triangle formed by the meter stick and its shadow. Have students sketch and label dimensions of the corresponding parts of the similar triangles formed with these objects.



Once the students have returned to the classroom, have different groups put their proportions on the board and make observations. Students should be able to see that the ratios found by the groups should be close to the same.



Activity 10: Scaling the Trail (GLE: 7)

Materials List: Scaling the Trail BLM, pencils, paper, ruler

Provide each student with a Scaling the Trail BLM. Have the students find the length of the trail using the information given on the BLM. Discuss segment notation ($\overline{AB}$) so that students record information accurately. Challenge the students to add another $1\frac{1}{4}$ miles to the trail by extending the trail in any direction from point A so that the trail leads closest to point C. This will require students to determine the length of the segment that needs to be added to the diagram in inches.

Activity 5: The Theorem (GLE: 30, 31)

Materials list: The Theorem BLM, pencils, paper, calculators, graph paper

Provide students with the side lengths of several right triangles missing the length of one of the sides. Discuss the use of the formula as it applies to the missing lengths in the triangles. Extend this activity to include real-life situations that require students to find the length of one of the sides of a right triangle with situations by distributing The Theorem BLM. Have students verify their solutions to the BLM by comparing answers with another student and discussing any results that differ.

Activity 6: How Big is This Room Anyway? (GLE: 30)

Materials list: meter sticks or tape measures, newsprint or other large paper for blueprint, rulers, scissors, pencil, paper

Assign different groups of students the task of measuring the classroom dimensions. Have the class determine a scale that would fit on a piece of newsprint or poster board, and then have someone draw the room dimensions to scale on the poster. Tell students that the class will make a classroom blueprint. Divide students into groups of three to five. Assign each group a different object in the classroom to measure (file cabinets, book shelves, trash can, etc. - remember only length and width of the top of the object is needed for the blueprint). Have students convert actual measurements using the scale measurements determined earlier. Instruct students to measure draw and cut out models from an index card. Have each student measure his/her own desktop and make a scale model for the classroom blueprint. Remind students to write their names on the desktop model. Ask, "What is the actual area of your desktop? What is the scale area of your desktop? What comparisons do you see as you make observations of the areas of your room and desktop? List your observations." Have groups submit their scale models of the classroom objects (not desks at this time) for the blueprint. Discuss methods used to determine the measurements of the models, and then glue the models in the correct position on the classroom blueprint. Have students, one group at a time, place their desktop models on the classroom blueprint, working so that those who sit in the center of the room can add their models first. Post blueprints/scale models on the wall for all classes to compare. Using the class scale model of the classroom, have students make predictions about distance from various points in the room *(i.e., If the distance from the teacher's desk to the board is 5 inches on the scale model and the scale is 1 inch represents 4 feet, then the that the actual distance is 20 feet.)* Have student measure the actual distance(s) to check for accuracy of the scale model of the classroom.

Activity 12: Scale Drawings (GLE: 30)

Materials list: Scale Drawings BLM, pencil, paper

Provide the students with the problems to practice scale drawing problems by distributing Scale Drawing BLM. Give students time to work through these situations and then divide students into groups of four to discuss these situations. Have students in groups come to consensus on the solutions to these problems and then have them prepare for a discussion using *professor know-it-all* (view literacy strategy descriptions). With this strategy, the teacher selects a group to become the "experts" on scale drawing required in the situation that is selected. The group should be able to justify its thinking as it explains its proportions or solution strategies to the class. All groups must prepare to be the "experts" because they are not told prior to the beginning of the strategy which group(s) will be the "experts" and ask questions about scale drawings.

APPENDIX B: INTERPRETATIVE CLASSIFICATIONS OF THE TASK SET

1	Simple Ratios	Ratios	Ratios	Simplifying Ratios	Ratios	Ratios and Rates	Pre-Algebra
2	Simple Ratios	Ratios	Ratios	Simplifying Ratios	Ratios	Ratios and Rates	Pre-Algebra
3	Simple Ratios	Proportional Computation	Rates	Ratio/Proportion	Proportion (Missing-Value)	Proportion	Algebra
4	Simple Ratios	Proportional Computation	Rates	Knowledge/Comprehension	Proportion (Missing-Value)	Proportion	Algebra
5	Simple Ratios	Ratios	Ratios	Simplifying Ratios	Ratios	Constant Ratio	Pre-Algebra
6	Geometric	Ratios	Ratios	Geometry-Based	Ratios	Constant Ratio	Geometry

11	10	9	8	7
Geometric	Scale Drawings	Simple Ratios	Simple Ratios	Geometric
Simple Proportions	Scale Drawings	Ratios	Simple Proportions	Ratios
Rates of Change	Scale	Rates	Rates of Change	Rates of Change
Geometry-Based	Ratio/Proportion	Knowledge/Comprehension	Ratio/Proportion	Simplifying Ratios
Area 1/Area2=Length 1/Length 2	Proportion (Missing-Value)	Proportion (Missing-Value)	Proportion (Missing-Value)	Ratios
Constant Ratio	Proportion	Proportion	Multi-Step Problems	Ratios and Rates
Geometry	Pre-Algebra	Algebra	Algebra	Geometry

16	15	14	13	12
Unit Rate	Unit Rate	Simple Ratios	Unit Rate	Unit Rate
Unit Rate	Unit Rate	Unit Rate	Simple Proportions	Simple Proportions
Proportion	Rates	Proportion	Percent	Proportion
Multi-Step Problems	Multi-Step Problems	Ratio/Proportion	Multi-Step Problems	Ratio/Proportion
Rates (Multiplying)	Proportion (Complex)	Proportion (Missing-Value)	-----	Area 1/Area2=Length 1/Length 2
Proportion	Proportion	Proportion	-----	Proportion
Algebra	Algebra	Algebra	-----	Geometry

21	20	19	18	17
Geometric	Simple Ratios	Percent	Percent	Unit Rate
Ratios	Ratios	Percent Proportion	Percent Proportion	Unit Rate
Rates of Change	Ratios	Rates	Percent	Proportion
Geometry-Based	Simplifying Ratios	Multi-Step Problems	Multi-Step Problems	Multi-Step Problems
Area 1/Area2=Length 1/Length 2	Ratios	Rates (Multiplying)	Proportion (Missing-Value)	Rates (Multiplying)
Proportion	Ratios and Rates	Multi-Step Problems	Proportion	Proportion
Geometry	Algebra	Algebra	Pre-Algebra	Algebra

26	25	24	23	22
Unit Rate	Measurement/Conversions	Geometric	Geometric	Measurement/Conversions
Simple Proportions	Simple Proportions	Ratios	Ratios	Ratios
Conversions	Conversions	Ratios	Ratios	Conversions
Ratio/Proportion	Ratio/Proportion	Geometry-Based	Geometry-Based	Simplifying Ratios
Rates (Multiplying)	Rates (Multiplying)	Knowledge of Geometry	Knowledge of Geometry	Ratios
Proportion	Proportion	Constant Ratio	Constant Ratio	Ratios and Rates
Pre-Algebra	Pre-Algebra	Geometry	Geometry	Algebra

32	31	30	29	28	27
Geometric	Outlier	Simple Ratios	Simple Ratios	Simple Ratios	Unit Rate
Percent Proportion	Simple Proportions	Ratios	Simple Proportions	Simple Proportions	Simple Proportions
Rates of Change	Rates of Change	Rates	Proportion	Conversions	Conversions
Multi-Step Problems	Multi-Step Problems	Ratio/Proportion	Ratio/Proportion	Ratio/Proportion	Multi-Step Problems
Proportion (Complex)	Proportion (Complex)	-----	Proportion (Missing-Value)	Proportion (Missing-Value)	Rates (Multiplying)
Multi-Step Problems	Proportion	Proportion	Proportion	Multi-Step Problems	Multi-Step Problems
Geometry	Algebra	Algebra	Pre-Algebra	Algebra	Pre-Algebra

38	37	36	35	34	33
Scale Drawings	Simple Ratios	Percent	Simple Ratios	Percent	Simple Ratios
Scale Drawings	Simple Proportions	-----	Ratios	Percent Proportion	Simple Proportions
Scale	Rates	Rates	Ratios	Percent	Proportion
-----	Ratio/Proportion	Multi-Step Problems	Simplifying Ratios	Multi-Step Problems	Ratio/Proportion
Proportion (Complex)	Proportion (Missing-Value)	Rates (Multiplying)	Ratios	Ratios	Proportion (Missing-Value)
Ratios and Rates	Proportion	Multi-Step Problems	Ratios and Rates	Proportion	Proportion
Algebra	Algebra	Algebra	Pre-Algebra	Pre-Algebra	Pre-Algebra

44	43	42	41	40	39
Percent	Percent	Measurement/Conversions	Percent	Percent	Unit Rate
Percent Proportion	Percent Proportion	Proportional Computation	Percent Proportion	Percent Proportion	Unit Rate
Percent	Percent	Conversions	Percent	Percent	Proportion
Ratio/Proportion	Ratio/Proportion	Knowledge/Comprehension	Ratio/Proportion	Knowledge/Comprehension	Ratio/Proportion
Rates (Multiplying)	Rates (Multiplying)	Conversions	Rates (Multiplying)	Conversions	Ratios
Proportion	Proportion	Multi-Step Problems	Ratios and Rates	Ratios and Rates	Ratios and Rates
Pre-Algebra	Pre-Algebra	Pre-Algebra	Pre-Algebra	Pre-Algebra	Pre-Algebra

50	49	48	47	46	45
Scale Drawings	Percent	Percent	Percent	Simple Ratios	Scale Drawings
Scale Drawings	Percent Proportion	Percent Proportion	Percent Proportion	Ratios	Ratios
Scale	Percent	Percent	Percent	Ratios	Scale
Ratio/Proportion	Multi-Step Problems	Ratio/Proportion	Multi-Step Problems	Simplifying Ratios	Ratio/Proportion
Proportion (Missing-Value)	Rates (Multiplying)	Proportion (Missing-Value)	Rates (Multiplying)	Ratios	Proportion (Missing-Value)
Proportion	Proportion	Ratios and Rates	Proportion	Ratios and Rates	Proportion
Pre-Algebra	Pre-Algebra	Pre-Algebra	Algebra	Pre-Algebra	Pre-Algebra

APPENDIX C: A SAMPLING OF A HUNDRED YEARS OF TEXTBOOKS

The Anderson Arithmetic: Book Three By Robert Anderson Pages 283- 302

5. State at sight the ratio of:

4 yd to 2 yd	6 pk to 2 pk	8 lb to 2 lb	8 lb to 4 lb
2 ft to 4 ft	0.02 to 0.06	0.02 to 0.08	$6 to $8
10 in to 2 ¼ in	12 ½ rd to 2 ½ rd	1 yd to 4 ft	1 ½ ft to 8 in

10. Solve for x:

3/x = ½ x/4 = 3/4 x/2 = ½ 5/x = 1/3 x/a = b

15. A meter is 39.37 in. Find correct to the nearest 0.01, the ratio of a meter to a yard.

20. What is the ratio of one side of an equilateral triangle to its perimeter?

25. What is the ratio of the area of a rectangle 8 in x 6 in to the area of a rectangle 8 in x 2 in?

30. Find the ratio of the area of a triangle whose base is 6 in and altitude 8 in to the area of a triangle whose base are 6 in and altitude 4 in.

35. In problem 34, what is the ratio of the area of the first triangle to the area of the second when b = 8, a = 6, L= 9, and w = 12?

5. Solder for lead consists of 1 part tin to 1 ½ parts lead; how much of each of these substances is necessary to make 5 lb of solder?

10. Divide 720 in the ratio of 4 to 5.

5. Solve for x: 6/8 = x/24 10. Solve for x: x/8 = ¾ 15. Solve for x: x/6 = 2/10

20. Solve for x: 9 : x = 36 : 24 25. Solve for x: 0.5 : x = 0.25 : 1

5. When 3/8 of an in on a certain scale represents 3 ft, what does 1 ½ in on the same scale represent?

10. If a train running 40 mi an hour runs a certain distance in 5 hr what time will it run in the same distance at the rate of 50 mi per hour?

5. If the edge of one cube is twice the edge of another, the volume of the larger is how many times that of the smaller?

10. If 240 gallons of water will flow through a certain orifice 3 in. in diameter in a certain time, how much water will flow through an orifice in the same time?

5. What is the weight in ounces and the fuel value of the protein in 1 lb raisins? Of the fat? Of the carbohydrates? What is the fuel value of 1 lb of raisins?

Hamilton's Essentials of Arithmetic: Higher Grades By Samuel Hamilton Pages 140-146

5. Find the missing value: 40 : ? = 72 : 18   10. Find the missing value: 7.5 : 1.5 = 2.5 : ?

15. When potatoes are 5 lb. for 17 cents, how much should a bushel of 60 lb. cost?

20. Which is cheaper, a can of corn containing 1 Ib. 2£ oz. (181 oz.) for 121 cents, or 1 lb. 5 oz. (21 oz.) for 150?

5. James planted 2 rows, 2 ft. apart, in early potatoes. He raised 31 bu. potatoes (60 lb. each), which he retailed at 3 cents a pound. How much did he get for the potatoes?

10. He cut the corn after taking off the ears, and sowed the ground in turnips. He sold 2 bu. (55 lb. each) of turnips at 5 cents a pound. How much did he receive for them?

15. George found that 85% of his seed corn used for planting germinated. His yield was 110 bu. an acre. What would have been his yield if all the seed had germinated?

20. If 31% of the 474 matured heads were sold for 78 cents a dozen and the remainder for 48 cents a dozen, what amount of money did the cold frame yield?

Everyday Arithmetic: Advanced Book By Hoyt and Peet Pages 245-251

5. State the ratio: 4:3   10. State the ratio: .25 : .5

15. State the ratio: 4 in. : 2 yd.

20. It requires 9 lb. of cucumbers to furnish as much nourishment as 1 lb. of beef. What is the ratio of the amount of nourishment in cucumbers to that in beef?

5. Solve for x: 4.5 : 20.25 = 18 : x 10. Solve for x: 319.7 : 742 = 3.24 : x

15. Land sold at the rate of $1900 for 12 acres would cost how much for 75 acres?

20. The record of a herd of 24 cows showed a yield of 3250 Ib. of milk per week. How much milk would be produced per week if 10 cows of the same average capacity should be added to the herd?

5. If AC = *2ft.*, CF = 3 ft., .EC = 5 ft., BC = 20 ft., EF = 15 ft., what does AB equal?

10. One boy in the picture is holding a 5-ft. pole. The other boy finds its shadow to be 2 ft. long. They find that the telegraph pole beside them casts a shadow 20 ft. long. Two similar triangles are thus formed whose sides are proportional.

Let x = the height of the pole.
$x: 5 = 20:2$
$x = ?$

Junior High School Mathematics By John C. Stone Pages 31-37

5. State the ratio: 27 gal to 18 gal 10. State the ratio: 700 ft to 175 ft

15. A certain rectangular garden is 60 ft wide and 80 ft long. What is the ratio of its width to its length?

20. If a man who earns $25 per week is saving $10 of it, he is saving what ratio of his earnings?

5. Find the relation of two rectangles having equal bases. Let a and b be the dimensions of one, and c and 5 of the other, for the base b is the same in both.

10. Compare a triangle whose dimensions are a and b, with one whose dimensions are a' and b'.

15. Compare the areas of two squares whose sides are respectively a and a' units.

20. If one square has a side three times that of another, how do their areas compare?

25. If the linoleum for a square floor costs $40, how much will the same grade of linoleum cost for a square floor but half as long, not considering waste?

Applied Mathematics for Junior High Schools By Eugene Henry Barker Pages 181-195

5. State the ratio: 5 ft. to 1 yd. 10. State the ratio: $7.50 to $3.75

15. State the ratio: 1 oz. to 1 lb. 20. State the ratio: 1 kilometer to 1 mi.

25. The volume of a 1-ft. cube to the volume of a 2-ft. cube.

30. The side of a square to the diagonal.

35. The circumference of a circle to its radius.

5. If the price of gas is $ .65 per M, what is the charge for 2260 cubic feet of gas?

10. If there are 3 quarter notes or their equivalent, per measure, how many quarter notes or equivalent are there in a piece of music of 32 bars ?

15. If a mechanically operated hack-saw makes 90 strokes per minute, what is the number per hour?

20. If the yield of alfalfa is one and one-half tons per acre for each cutting, and the crop is cut seven times each year, what is the yield per year on 12 acres?

5. If a train of 30 cars carries 1100 tons, how many cars must be added to the train so that the total load may be 2800 tons?

10. If 104 pounds of seed are used on a 2-acre lot, what amount will be required for 15 cents acres?

15. What number bears the same ratio to 5.5 that 4.4 bears to 3.3?

5. How much does one lift on the handles of a wheel barrow if a 100-pound load is placed in the barrow 16 inches from the axle, and the hands 26 inches further away?

5. A cycle car has a 4-inch pulley on the driving shaft belted to a 14-inch pulley fastened to a rear wheel. If the rear wheel is 26 inches in diameter, what is the speed of the car if the engine is making 800 revolutions per minute? Allow 5 per cent for belt slippage.

5. In a tractor demonstration test, the following were the results. Express in each case as a per cent, the ratio of the drawbar pull to the weight of the tractor.

TRACTOR	TYPE	WEIGHT	PLOWING SPEED	PLOWS	DEPTH PLOWED	DRAWBAR PULL
A	2-wheel	3380	3.5	2-14"	9.22	2705
B	4-wheel	5100	3	3-14"	8.08	1510
C	4-wheel	3250	3	3-14"	8.58	2300
D	Crawler	3500	2.5	3-14"	8.91	2455
E	Crawler	6900	2.5	3-14"	9.47	3450
F	4-wheel	5200	2.5	4-14"	8.12	3125
G	4-wheel	7800	2	4-14"	7.19	2800
H	Crawler	7400	2.25	3-14"	8.42	2600
I	3-wheel	4000	2.5	3-14"	8.31	2000

The Alexander- Dewey Arithmetic: Advanced Book By George Alexander and Edited By John Dewey Pages 135-138

5. Mr. Ward bought a pile of wood containing 12 cords for $45. He purchased another pile containing 60 cords at the same rate. How much did he pay for it?

10. How much must be paid for 18 tulip bulbs selling at *72* cents a dozen?

5. If 3 packages of paper cost $5.25, what will 12 packages cost?

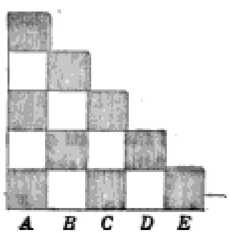
10. Make a bill for material which a teacher of sewing ordered at the beginning of this term. There were 18 girls in the class and she ordered the following for each: 21 yards sateen at 22 cents; 17 spools of thread at 5 cents a spool; 14 yards of elastic at 4 cents; 1 doz. buttons at 15 cents *a* dozen.

15. A miller uses 18 bushels of wheat for making 4 barrels of flour. How many barrels of flour can he make from 144 bushels?

20. If Charles Denny can make 12,600 peach baskets in 6 weeks, how many baskets can he make in 45 weeks?

25. What per cent is gained in buying oil at 80 cents a gallon and selling it at 12 cents a pint?

Wentworth-Smith Mathematical Series, School Arithmetics: Book Three By George Wentworth and David Eugene Smith Pages 103-112



5. State the ratio: $2E$ to D 10. State the ratio: $5E$ to A

15. State the ratio: 5 to $.4 + E$ 20. State the ratio: 5 to $A + D + E$

5. Find x. x/10 =1 10. Solve for x. 2/x = 1/3

15. The ratio of the height of a certain door to the width is 3 to 4. If the door is 7 ft. high, how wide is it?

5. There were six cloudy days in September, the rest being clear. What was the ratio of cloudy days to clear days?

10. Four boys arrange a three weeks' camping trip. The first boy stays 18 days; the second comes late and stays 12 days; the third is in camp the last two weeks; the fourth stays the three weeks. The total expense is $32.50. What is the share of each?

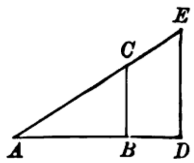
5. If 9 men can complete some work in 20 hr., how many men could complete it in 16 hr. more time?

5. Solve for x. 27: x = 243: 36 10. Solve for x. 2 : 21.7= 2.4: 3.3

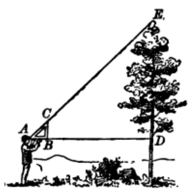
15. If you are thinking of a number which has to 17 the ratio 35:119, what is the number?

20. When a sum of money is divided equally among seven persons, each receives \$16.80. How much would each receive if the same sum were divided equally among eight persons?

5. In the same figure, if AD = 13/16 in, AC = 10/16 in, or 5/8 in, and AB = 9/16 in, what is the length of AE?



5. Using the triangle of Ex. 3, find the height of the tree if AD is 70 ft. and the point D is 5 ft. 2 in. above the ground.



5. A boy weighing 100 lb. just balances a seesaw by sitting 4 ft from the stick on which the plank rests. If the plank is 9 ft long and we neglect the weight of the plank, how much does the boy on the other end weigh?

5. How high is a tree if it casts a shadow 30 ft. long when a post 6 ft. 3 in. high casts a shadow 3 ft. 9 in. long?

Second Course in Algebra By Ford and Ammerman Pages 166-177

5. The dimensions of a certain grain bin are 3 feet by 6 feet by 7 feet. What is the ratio of its cubical contents to that of a bin whose dimensions are 3 feet 6 inches by 5 feet by 1/2 yards?

5. State such proportions as you can make out of the following: 1 pint, 1 quart, 1 gallon, 2 gallons.

5. Is it proportional? 17:19 = 21:23 10. Solve the proportion. x:4 =3:2

15. What number bears the same ratio to 4 as 16 does to 6? [HINT. Let x represent the unknown number and form a proportion.]

20. Prove that no four consecutive numbers, as n, $n+1$, $n+2$, $n+3$, can form a proportion in the order given

Vocational Arithmetic By Paddock and Holton Pages 94-101

5. State the ratio: 23 1/5 to 17 3/5   10. State the ratio: 3 pks. 7 qts. 1 pt. to 2 bu. 1 pk. 3 qts.

5. Find x. 23:25 = 25 :x   10. Find x. x : 13 1/2 gals. = 3 qts. : 4 qts. 1 pt.

5. It requires 875 feet of shafting for a shop 100 feet long. How many feet of shafting will be required at the same rate for a shop 57 feet long?

5. Twelve men in 5 days can erect 11 electric motors. How many men can erect 33 electric motors in 18 days?

Junior High School Mathematics: Book II By Theodore Lindquist Pages 188- 196

5. Ruth paid $ 16.68 for 4 Baby Bonds; what would she have to pay for 9 at the same rate? For 12 ? For 17?

10. In a certain room of 28 pupils in a school the pupils had invested $ 840 in Liberty Bonds, Baby Bonds, and Thrift Stamps. In order to keep up the same rate of investment, how much should be invested by a room of 20 pupils? Of 42 pupils?

5. The distance a person travels is proportional to the product of the rate and the time travelled. $D = rt$.

10. The volume of a cone varies as the square of the radius of the base, if the altitude remains constant.

15. The volume of a cube is proportional to what?

5. Measure your schoolroom and make a plan of it to scale.

10. Kansas is about 200 mi. by 400 mi. What scale would you use to draw the map upon paper 9 in. by 13 in., which will be a simple scale to work with and still give the largest possible map?

VITA

Christina Vincent is a resident of Baton Rouge, Louisiana. Christina works as a high school mathematics teacher. She has taught elementary and middle school mathematics, as well as art, reading, study skills and tutoring in all subjects. Christina is a graduate of Louisiana State University. She began work in the Master of Natural Sciences program in the summer of 2007. She is interested in mathematical pedagogy and successful teaching strategies.