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## *Physics*, Chapter 40: Light as a Wave Motion

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# 40

## Light as a Wave Motion

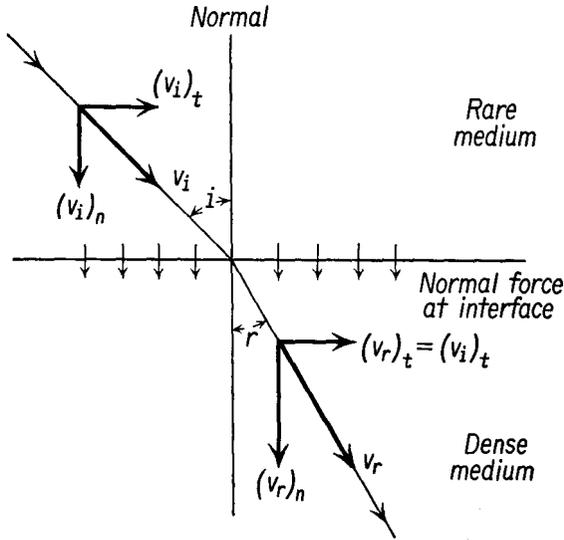
### 40-1 Wave Versus Particle

Let us recount some of the characteristics of the motion of particles and the propagation of waves, with a view toward analyzing the behavior of light. In accordance with Newton's first law, a particle moves in a straight-line path in the absence of external forces. Thus we might infer, as Newton suggested, that light is composed of particles, and that, in a continuous medium, there is no deflecting force on the light particles. At the interface between two media, light may be propagated in a straight line parallel to the interface. Thus even at an interface there is no force on the particles of light unless the light passes through the interface, and in that event the force acting must be perpendicular to the interface. To account for the fact that light is refracted toward the normal on passing from a rare to a dense medium, as from air to water as shown in Figure 40-1, it is necessary to assume that the force is directed from the rare to the dense medium. The normal component of the velocity is increased, while the tangential component remains the same, so that the refracted ray is more nearly directed toward the normal. From this we see that a particle theory of light implies that the velocity of light in the dense medium is greater than the velocity of light in the rare medium.

As we have already seen, a wave theory accounts for refraction by requiring that the velocity in the dense medium is less than the velocity in the rare medium. This particular contest between the wave and particle theories was settled by Focault in 1850 (almost 175 years after the controversy between the points of view of Newton and Huygens was initiated) by measurement of the velocity of light in water and in air. Focault showed that the velocity of light in water was less than its velocity in air, in quantitative agreement with the wave-theory explanation of the index of refraction of water.

A second aspect of the differences between particles and waves lies in their respective principles of *superposition*. When a shotgun is fired at a

target, a certain number of pellets penetrate each square centimeter of target area. When two identical guns are fired at the same target, we infer that an increased number of pellets penetrates each square centimeter of target, and that this number is approximately twice the number of pellets per unit area obtained with one gun; that is, we expect particles to obey an *arithmetic superposition principle*. At a point in a medium where the paths of two waves intersect, the medium is simultaneously displaced by the two waves, so that the resultant displacement is the vector sum of the



**Fig. 40-1** If light were composed of a beam of particles, the velocity  $v_r$  of the refracted ray in an optically dense medium would be greater than the velocity  $v_i$  of the incident ray in the optically rare medium.

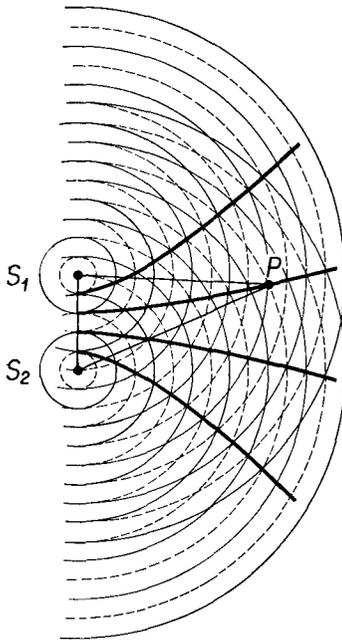
individual displacements. The magnitude of the instantaneous displacement of the medium is obtained from a *vector superposition principle*. We have already seen in Chapters 20 and 21 that the superposition principle was capable of explaining both *beats* and *standing waves*. One vibrating source at the end of a long string generated a wave in the string which caused every particle of the string to vibrate. Two identical vibrating sources at opposite ends of the string did not yield a wave of twice the original displacement at every point of the string, but, instead, nodes were produced at intervals of a half wavelength, at which there was no vibration at all.

The difference between the superposition principles appropriate to particles and to waves results in our expectation that waves must display such phenomena as beats, standing waves, and interference and diffraction

effects, under appropriate conditions, while particles do not exhibit such phenomena. In the succeeding sections we shall examine some of the interference and diffraction effects exhibited by light. These lead us to the conclusion that light is a wave motion.

#### 40-2 Interference of Light from Two Sources

Let us suppose that two vibrating needles are immersed in a ripple tank, and that these vibrate in phase with each other and are driven at the same frequency. Each of the needles is a source of waves which spread out along the surface of the water of the tank as a series of ripples, or Huygens' wavelets. Let us represent the crest of each ripple by a solid line and the



**Fig. 40-2** Ripple system, showing lines of nodes.

trough of each ripple by a dashed line, as in Figure 40-2. Each circular ripple spreading out from a particular source  $S_1$  represents a disturbance which was initiated by the source at some particular time, while a circle of smaller radius represents a disturbance initiated by the source at a later time, when the phase angle of the source vibration was a different value. For example, the phase angle of any dashed circle is  $180^\circ$  greater than the phase angle of the next greater solid circle, concentric with it, and so on. At a point on the water surface where a crest from source  $S_1$  intersects a crest from source  $S_2$ , the amplitude of the resulting disturbance will be the sum of the two separate disturbances. Where the crest from  $S_1$  intersects the trough from  $S_2$ , the resulting disturbance will be of zero amplitude. Another way of saying the same thing is that any point  $P$  on the water surface where the disturbance from  $S_1$  is always in phase with the disturbance from  $S_2$  will vibrate

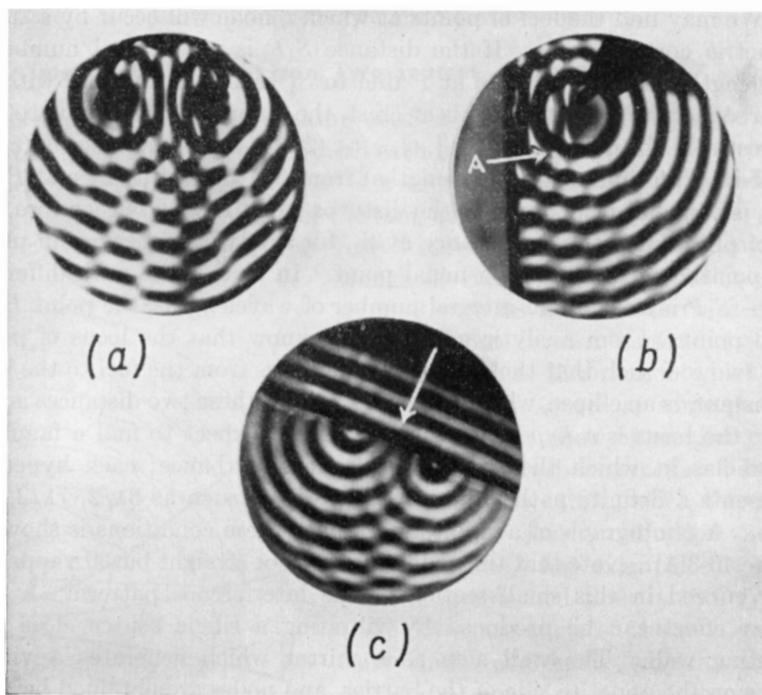
with twice the amplitude due to either source alone, while at any point where the disturbance from  $S_1$  is out of phase with the disturbance from  $S_2$  by  $180^\circ$ , the disturbance will have zero amplitude. This is the two-dimensional analogue of the production of standing waves in a string, and if the two sources are the proper distance apart, that is, an odd number of half wavelengths, a series of nodes and loops will appear along the

line  $S_1S_2$ , just as in the case of the string. Not only will there be nodes where the phase difference is  $180^\circ$ , or  $\pi$  radians, but there must also be nodes appearing wherever the phase difference between the two disturbances is  $\pi, 3\pi, 5\pi, \dots, (2m - 1)\pi$ , where  $m$  is any integer.

We may find the loci of points at which a node will occur by a simple geometric consideration. If the distance  $S_1P$  is an integral number of wavelengths, the disturbance at  $P$  due to  $S_1$  oscillates in phase with the disturbance at  $S_1$ , for when  $S_1$  is at crest, the disturbance at  $P$  due to  $S_1$  is at crest. If the point  $P$  is  $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots, (2m - 1)/2$  wavelengths (called a half-integral number of wavelengths) from  $S_2$ , the disturbance at  $P$  due to  $S_2$  is  $180^\circ$  out of phase with the disturbance at  $S_2$  and is therefore  $180^\circ$  out of phase with the disturbance at  $S_1$ , for the two sources are in phase. The point  $P$  must then be a nodal point. In fact, if the path difference  $S_2P - S_1P$  is an odd half-integral number of wavelengths, the point  $P$  is a nodal point. From analytic geometry we know that the locus of points from two foci such that the sum of the distances from the foci to the locus is constant, is an ellipse, while if the difference of these two distances is constant, the locus is a *hyperbola*. Thus we must expect to find a family of hyperbolas at which there is no resultant disturbance; each hyperbola represents a definite path difference  $S_2P - S_1P$  such as  $3\lambda/2, 7\lambda/2$ , and so on. A photograph of a ripple tank under these conditions is shown in Figure 40-3(a). Note that the nodal lines are not straight but are appropriately curved in this small section of the interference pattern. A very similar effect can be produced by vibrating a single source close to a reflecting wall. The wall acts as a mirror which generates a virtual source on the opposite side of the barrier, and nodes are obtained between the waves from the real source and the virtual source, as shown in Figure 40-3(b).

We cannot examine the behavior of light waves in transit the way we can observe water waves, for the light waves are invisible until they interact with a screen. Since light waves are emitted by independent atoms and molecules which are not in phase with each other, we cannot generate interference effects by setting two sources of light side by side. We may simulate two sources by making use of Huygens' principle, for if a wave is incident upon a slit, each point of the slit acts as a source of wavelets, as was shown in Figure 20-11(a) in connection with our discussion of the diffraction of a wave. In the event that two adjacent slits are placed in a barrier, these slits act as sources, and an interference pattern is produced, as shown in Figure 40-3(c). The light from each atom of a source spreads out as a spherical wave. If a double slit is placed in the vicinity of the source so that the source is equidistant from each slit, the light from each atom arriving at the slits is in phase. When the light from the pair of slits intercepts a screen, we must expect to find adjacent bright and

dark regions on the screen, if light is a wave motion. The dark regions correspond to the positions of nodes, while the bright regions are the positions of antinodes.



**Fig. 40-3** (a) Nodal lines produced by interference of ripples from two sources. (b) Nodal lines produced by interference of ripples from a point source and its virtual image. (c) Nodal lines produced by interference of ripples from two slits. (Photographs courtesy of The Ealing Corporation.)

In Figure 40-4 monochromatic light from a source is limited by a slit  $S$  equidistant from two narrow slits  $S_1$  and  $S_2$ . These two slits act as new sources which emit light of the same frequency and phase. A point  $P$  of the screen will be bright when the light reaching it from the slits  $S_1$  and  $S_2$  is in phase; that is, the light path from  $S_2$  to  $P$  must exceed the light path from  $S_1$  to  $P$  by an integral number of wavelengths, or, if  $m$  is an integer,

$$S_2P - S_1P = m\lambda.$$

This path difference may be found on the figure by swinging an arc of radius  $S_1P$  to intersect the line  $S_2P$  at  $A$ . The path difference between the two rays is  $S_2A$ . In the case of small angles we may approximate the arc by the chord  $S_1A$ , and we may assume that the chord is perpendicular

to the path  $S_2P$  at  $A$ . From the figure we find

$$\sin \theta = \frac{S_2A}{S_1S_2} = \frac{m\lambda}{d},$$

where  $d$  is the distance between the slits, and  $\theta$  is the angle describing the position of  $P$  with respect to the center of the slits. Similarly,

$$\tan \theta = \frac{OP}{CO} = \frac{x}{L},$$

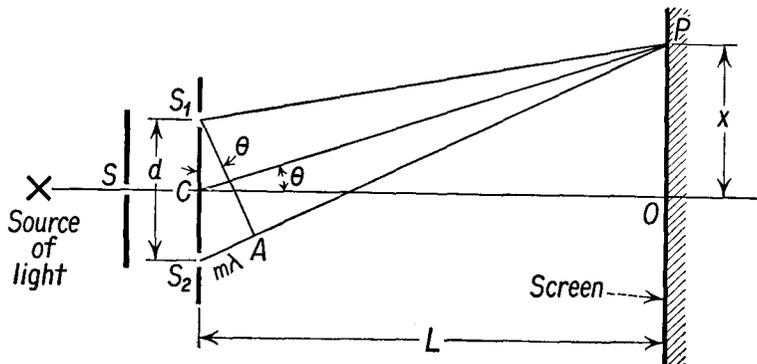


Fig. 40-4

where  $x$  is the distance from the central image to  $P$ , and  $L$  is the distance  $CO$  from the slit to the screen. For small angles  $\sin \theta = \tan \theta$ , so that

$$\frac{m\lambda}{d} = \frac{x}{L},$$

or,

*bright fringes:* 
$$x = m \frac{\lambda L}{d}. \tag{40-1}$$

We may locate the position of the dark fringes on the screen by observing that the path difference for  $P$  to be a node is given by  $S_2A = (2m - 1)(\lambda/2)$ , an odd number of half wavelengths. Thus

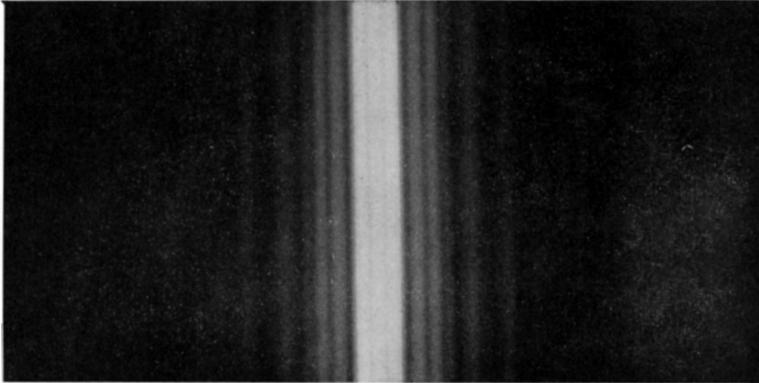
*dark fringes:* 
$$x = \frac{(2m - 1)}{2} \frac{\lambda L}{d}. \tag{40-2}$$

To get an idea of the order of magnitude of the quantities involved, suppose that a sodium lamp emitting monochromatic yellow light illuminates a pair of slits separated by a distance of 0.05 cm, and we find that the first bright line is displaced a distance of 0.24 cm from the central image (for which  $m = 0$ ) when the screen is 200 cm from the slits. We find, from

Equation (40-1), that

$$\lambda = \frac{xd}{L} = \frac{0.05 \text{ cm} \times 0.24 \text{ cm}}{200 \text{ cm}} = 6 \times 10^{-5} \text{ cm}.$$

Such an experiment makes it possible to measure the wavelength of light with a ruler. A two-slit interference pattern produced with light from a mercury arc is shown in Figure 40-5.



**Fig. 40-5** Photograph of the interference pattern produced by the passage of light from a high-pressure mercury arc through two slits which are close together. No lens was used; the light was allowed to fall directly on a photographic plate. (Courtesy of Central Scientific Co.)

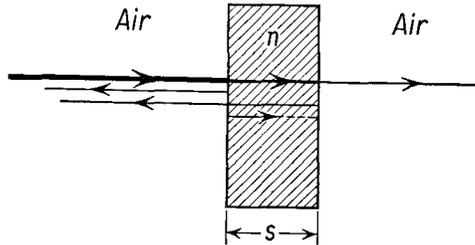
If the slits are illuminated with white light, each color will produce its own set of interference bands, and these will overlap. The central image is produced by light of all wavelengths which are in phase so that the central image is white and bright. Series of colored interference bands appear on either side of the central image. These interference bands may be easily seen by viewing light from a distant source, such as a street lamp, through two pinholes punched in a piece of cardboard. It is interesting to note that if we limit our conception of light to the postulates of geometric optics, we must infer that light has passed through the opaque center of the slits in the two-slit experiment, as in Figure 40-4.

### 40-3 Interference from Thin Films

Let us consider the effect of a thin parallel plate of transparent material upon a light beam normally incident upon the plate, as shown in Figure 40-6. The beam is partially reflected and partially transmitted by each of the two interfaces. The reflected light is made up of two parts, some reflected from the first surface, and some reflected from the second surface

and transmitted back through the first surface. If these interfere *constructively*, that is, if the two parts are in phase or in step, the reflected beam will be bright, while if the two beams are out of phase by  $180^\circ + m(2\pi)$ , or are out of step by an odd number of half wavelengths, the two beams

Fig. 40-6



will interfere *destructively*, and the reflected light will be of zero intensity. The phase difference between the two beams will be due in part to the path difference between the two beams and in part to phase changes which take place on reflection.

Let us suppose that two pieces of string of different masses per unit length are joined together, and that the two are under tension. In accordance with Equation (20-10), the velocity of propagation of a wave in a string is given by  $v = (S/m)^{1/2}$ , where  $S$  is the tension, which is the same in both sections of the string, and  $m$  is the mass per unit length. The velocity of propagation of a wave is different in the two strings, and if a wave is initiated in one of them, it is partially transmitted and partially reflected at the interface, or at the point where the strings are joined. Experimentally we find that the wave is transmitted without a change of phase, no matter in which string the wave starts. When the wave approaches the interface from the denser string, in which the wave velocity is smaller, the wave is reflected without a change of phase. However, when the wave approaches the interface from the lighter string, in which the wave velocity is greater, the reflected wave undergoes a change of  $180^\circ$  of phase with respect to the incident wave, as shown in Figure 40-7. These results are consistent with our previous discussion in Section 20-4, where we saw that a wave reflected from a barrier, which we may conceive of as a very heavy string, changes its phase on reflection by  $180^\circ$ . At normal or nearly normal incidence, these phase changes also occur optically. Thus when light is transmitted from one medium to another, there is no phase change; when light coming from a rare medium is reflected from an interface with a denser medium, it experiences a phase change of  $180^\circ$ ; when light coming from a dense medium is reflected from an interface with a rarer medium, it experiences no phase change on reflection, the phase of the reflected wave being the same as the phase of the incident wave.

When a string or any other medium is set into vibration at a certain frequency  $f$ , the particles of the medium respond at this frequency. Thus when a wave progresses from one medium to another, the frequency remains the same. Since the speed of the wave changes as it goes from one

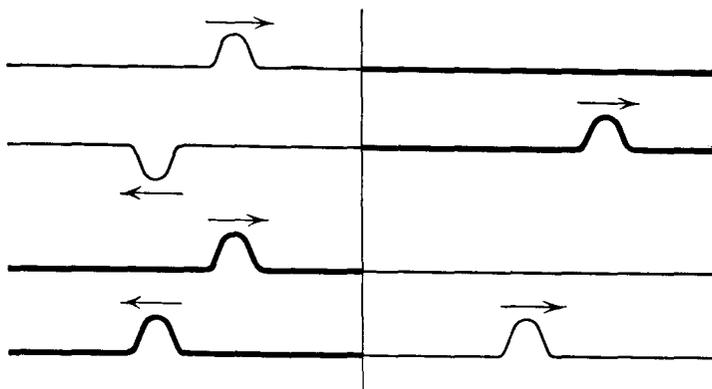


Fig. 40-7

medium to another, the wavelength must change. If  $\lambda_1$  is the wavelength in a medium in which the velocity is  $v_1$ , and  $\lambda_2$  is the wavelength in the medium in which the velocity is  $v_2$ , then

$$\frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2} = \frac{\lambda_1}{\lambda_2}.$$

From Snell's law the index of refraction is

$$n = \frac{v_1}{v_2}; \quad (37-4)$$

therefore

$$n = \frac{\lambda_1}{\lambda_2}. \quad (40-3)$$

In studying the interference effects in thin films, the effect of the distance traversed in the medium upon the difference in phase between the ray reflected at the first surface and the ray reflected at the second surface is most easily accounted for by expressing that distance in wavelengths. But here we must be careful to use the wavelength of the light in the medium in which the light is moving rather than the wavelength in vacuum. If  $s$  is the thickness of the film, the light traverses a distance  $2s$  in its passage into and away from the second reflecting interface. The difference in phase between light reflected from the second interface and light newly incident upon the first interface, because of the path

difference, is  $2s \frac{2\pi}{\lambda'}$ , where  $\lambda'$  is the wavelength in the film. In addition, if the film is immersed in a medium of different refractive index, either the wave reflected at the first interface or the wave reflected at the second interface is shifted in phase by reflection, by  $180^\circ$  or  $\pi$ . Thus the total phase shift  $\Delta\phi$  between the two beams is

$$\Delta\phi = 2s \frac{2\pi}{\lambda'} + \pi.$$

If the film is an air wedge between two glass plates, the wavelength in the air film is the same as that measured by a spectrometer, and we may drop the prime from  $\lambda'$ , yielding

$$\text{air film:} \quad \Delta\phi = 2s \frac{2\pi}{\lambda} + \pi. \quad (40-4)$$

If the film is a wedge of material of refractive index  $n$ , the wavelength  $\lambda'$  may be obtained from Equation (40-3) as

$$\lambda' = \frac{\lambda}{n},$$

and we have

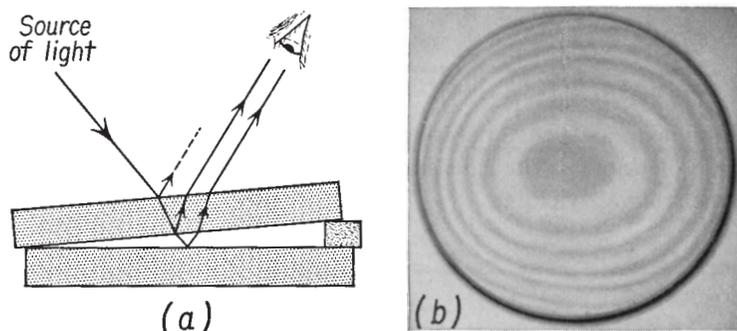
$$\text{dielectric film:} \quad \Delta\phi = 2ns \frac{2\pi}{\lambda} + \pi. \quad (40-5)$$

When the phase difference between the two reflected beams is an integer times  $2\pi$ , the waves reinforce each other, but when the phase difference is an odd integer times  $\pi$ , the waves interfere destructively, and no light is reflected from the film.

The interference produced by thin films has been used to reduce the loss of light by reflection which takes place at the surfaces of lenses in optical instruments. A thin film of lithium fluoride or calcium fluoride of thickness about one quarter the wavelength of sodium light is deposited on the lens. The refractive index of this material is intermediate between that of the air and the glass of the lens. On the way into the lens, the light is reflected from a denser medium, so that a phase change of  $\pi$  is produced at the air-fluoride interface, and the same phase change is produced at the fluoride-glass interface. Thus the total phase difference between the two reflected rays is entirely due to the path difference. This is made equal to half a wavelength to make the rays interfere destructively. White light is made of many wavelengths, so that reflection is not eliminated at all wavelengths. The deposited film is generally appropriate for yellow light, so that coated lenses look purple (white minus yellow) by reflected light.

Interference bands can easily be produced by making a wedge-shaped

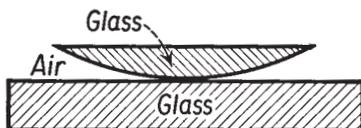
air film between two plane pieces of glass, as shown in Figure 40-8(a). When this wedge is illuminated with monochromatic light, a series of parallel bright and dark lines will be observed if the two glass surfaces *A* and *B* are perfectly plane. If one glass surface is known to be plane, as in an *optical*



**Fig. 40-8** (a) Thin air film between glass plates. (b) "Contours" or interference pattern produced in testing the surface of a telescope objective; the oval rings indicate that the surface is not quite spherical. (Photograph courtesy of Bausch & Lomb Optical Co.)

*flat*, the flatness of the second surface may be determined by observing the interference pattern produced when a wedge-shaped film of air is set up between them. The interference pattern then takes on the appearance of a contour map, as shown in Figure 40-8(b) with the dark lines representing the loci of equal thickness of the air film.

Newton observed and studied the interference produced by a thin film of air between the convex surface of a lens and a plane piece of glass,

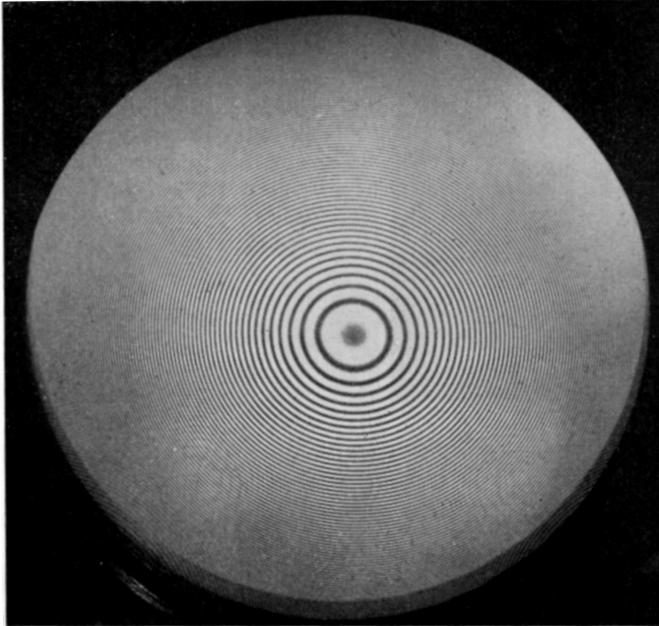


**Fig. 40-9** Apparatus for producing Newton's rings.

as shown in Figure 40-9. Because of the circular symmetry of the arrangement, a series of bright and dark rings may be seen when the system is illuminated with monochromatic light. These are called *Newton's rings*. Thomas Young first explained these rings on the basis of the wave theory of light and explained why the central spot should be dark on the basis of the change in phase on reflection. He reproduced Newton's experiment by using an oil film between a crown-glass lens and a flint-glass plate. The oil had an index of refraction intermediate between the crown glass and the flint glass. In this case the central spot was white and the other bright and dark regions were shifted in a corresponding manner.

When a thin, wedge-shaped film or a Newton's ring apparatus is illuminated with white light, colored fringes are formed as a result of the destructive interference of first one wavelength, then another. The pattern of Newton's rings formed with monochromatic light is shown in Figure 40-10.

The colors observed in soap bubbles, oil slicks, butterfly wings, and even some minerals are associated with the interference effects from thin films rather than with pigments.

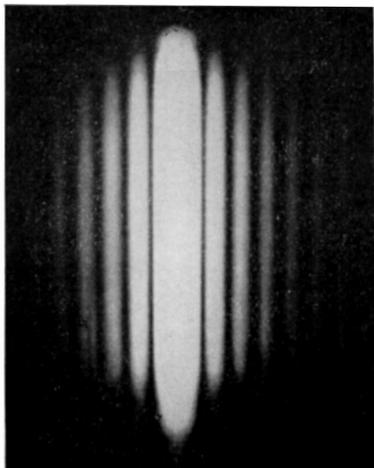


**Fig. 40-10** Newton's rings; pattern produced with sodium light. (Courtesy of Bausch & Lomb Optical Company.)

#### 40-4 Diffraction of Light

The terms "interference" and "diffraction" are used rather loosely. In general, "interference" is used to describe wave effects involving a limited number of rays, as in the two-slit interference pattern, while "diffraction" is used when a broad wave front is limited by a barrier or an aperture and an infinite number of elements of the wave front must be considered. We have already noted that, in the case of sound, the waves bend around corners and also spread out in passing through a narrow aperture; that is, sound waves are *diffracted*. Since interference experiments show definitely that light is propagated as a wave motion, we must expect to find diffraction

effects associated with light. As in the case of the two-slit interference pattern, we shall find that the angular aperture in which diffraction effects are observed is associated with the parameter  $\lambda/d$ , where  $d$  is the linear dimension of the obstacle. In our daily experience we observe diffraction effects with sound, for the dimensions involved are of the order of the wavelength of sound. Diffraction effects are not commonly observed with



**Fig. 40-11** Photograph of a diffraction pattern produced by the passage of light through a narrow slit. (Reproduced by permission from *College Physics*, 2nd ed., by Sears and Zemansky, 1952; Addison-Wesley Publishing Company, Inc., Reading, Mass.)

light because the wavelength of light is much smaller than the dimensions of common objects.

When a beam of light from a distant source passes through a narrow slit and falls upon a screen at some distance from it, the pattern on the screen consists of a bright image of the slit and a series of bright and dark fringes on either side of the central bright fringe, as shown in Figure 40-11. Only a small portion of the incident wave front passes through the narrow slit to produce this diffraction pattern. The appearance of the bright and dark regions on the screen can be explained by assuming that each point of this section of the wave front acts as a source of light. The slit is divided into imaginary elements. The amplitude and phase of the disturbance generated at the screen by each of these elements is computed, and the resultant disturbance is determined by the superposition principle. Since both amplitude and phase must be taken into account, the resultant is computed by vector methods, similar to those used in finding the voltage in an a-c circuit.

Imagine  $AB$  of Figure 40-12 to be the edges of the slit, greatly magnified, and the wave front approaching it to be a plane monochromatic wave, so that each point of the wave front incident upon the slit is vibrating in the same phase. Point  $C$  is the center of the slit, and  $CO$  is a perpendicular

line from the slit to the screen. We may locate the dark fringes by dividing the slit into an even number of elements such that the light from one element reaching the screen just cancels the light from an adjacent element.

Let us divide the slit into two equal parts,  $AC$  and  $CB$ , and consider the conditions under which a point  $D$  on the screen is dark. If the point  $D$  is so located that the light from a small element  $i$  near  $A$  is  $180^\circ$  out of phase with the light from a small element  $i'$  near  $C$ , these two elements

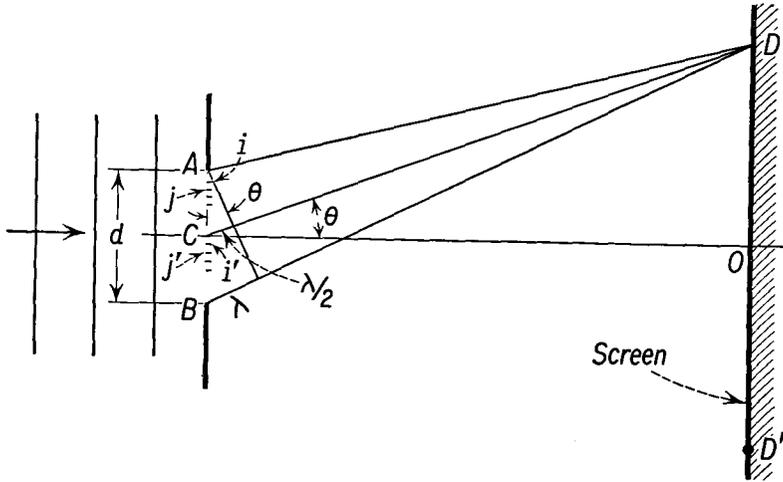


Fig. 40-12

together will contribute no illumination to the screen. In the same way the pair of elements  $j$  and  $j'$  will interfere destructively at  $D$ , and so on. This will be the case when the path differences  $CD - AD = \lambda/2, \dots, BD - CD = \lambda/2$ , for each pair of elements across the slit. Thus the path difference between the light reaching  $D$  from the bottom of the slit and the light reaching  $D$  from the top of the slit is equal to a wavelength. We have

$$BD - AD = \lambda,$$

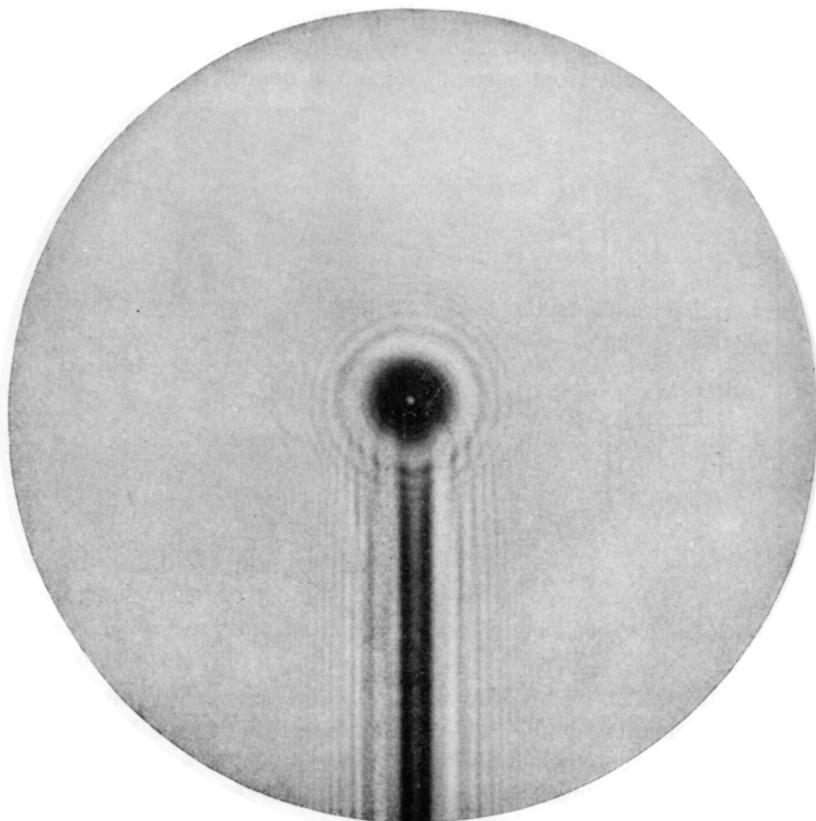
and, using the same small angle approximations we used in our analysis of the two-slit interference effect, we find

$$\sin \theta = \frac{\lambda}{d} \tag{40-6}$$

for the location of a dark band, or fringe. Clearly, a dark fringe also appears at the symmetrical point  $D'$ .

To find the location of the second dark band, we divide the slit into four equal parts and repeat the above argument for the top two parts and

for the bottom two parts separately. Thus we observe that the second dark fringe will occur when the path difference between the top and bottom of the slit is  $2\lambda$ . In general, the  $m$ th dark fringe is found at the screen when



**Fig. 40-13** Photograph of the shadow of a ball bearing supported on a rod. Note the diffraction pattern around the rod and ball bearing and the bright spot in the center of the shadow of the ball bearing. (Reproduced by permission from *College Physics*, 2nd ed., by Sears and Zemansky, 1952; Addison-Wesley Publishing Company, Inc., Reading, Mass.)

the path difference between light reaching the screen from the top and bottom of the slit is  $m\lambda$ , or, from Equation (40-6), we have

$$m\text{th dark fringe:} \quad \sin \theta = m \frac{\lambda}{d}. \quad (40-7)$$

Thus the diffraction pattern of a single slit consists of a central maximum with alternating dark and bright fringes on either side of the central region.

Thus we note that the parameter  $\lambda/d$  is characteristic of the width of the diffraction pattern.

The diffraction pattern produced by plane waves incident upon a circular aperture consists of a bright central disk surrounded by fainter circular rings. If  $d$  is the diameter of the aperture, it can be shown that the angle  $\theta$  subtended by the radius of the first dark ring is given by

$$\theta = 1.22 \frac{\lambda}{d}. \quad (40-8)$$

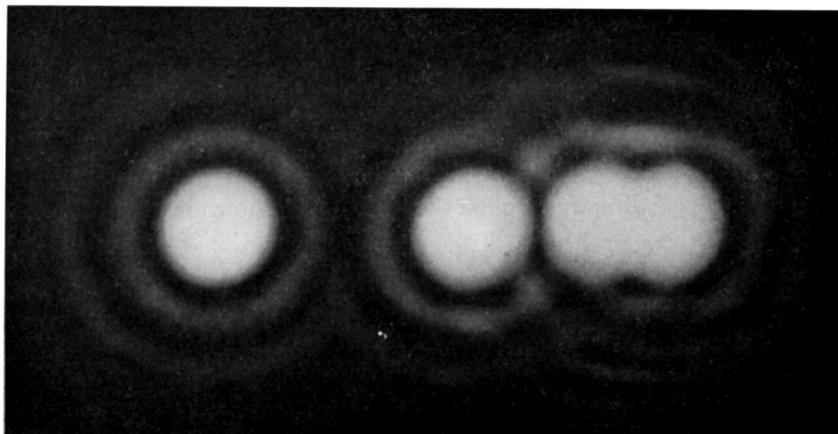
When plane waves of light are limited by an obstacle, such as a disk or ball bearing, rather than by an aperture, waves diffracted from the edge of the obstacle reach a point in the center of the shadow in phase with each other, for the center of a circular shadow is equidistant from all points on the rim. A bright spot will be found at the center of a circular shadow, as though light passed directly through the obstacle, as shown in Figure 40-13.

#### 40-5 Diffraction and Resolving Power

Diffraction phenomena can also be observed when light passes through large apertures, such as the lenses of microscopes and telescopes. The effect of such phenomena is to limit the *resolving power* of the instrument; that is, the ability of the instrument to show increasingly greater detail at higher magnifications. If light from a point source is focused by a converging lens, the image will not be a sharp point even if the lens has been perfectly corrected for all aberrations. The very best image which can be formed will be determined by the diameter of the lens opening, according to Equation (40-8), as shown in Figure 40-14.

The size of the diffraction pattern will be determined by the wavelength of the light used, the diameter of the lens, and the focal length of the lens used. Thus the best astronomical telescope forms the smallest image of a star. If we consider two points sending light through an optical system, the image of each point will be a diffraction pattern. If the points are close together, these patterns may overlap, so that it will not be possible to distinguish them as two separate points. The images are said to be resolved if the dark ring of one pattern passes through the center of the other pattern, or if the two central disks are separated a distance equal to the radius of one of them. If two points cannot be resolved by an instrument, merely increasing the magnification serves no useful purpose, for one simply obtains a larger, fuzzier image. These considerations provide us with a fundamental limit on the magnification of a microscope or a telescope. In practice, the smallest separation of two point sources which can be resolved by a microscope with visible light is about  $1.8 \times 10^{-5}$  cm, and the highest magnifications used are generally less than 2,000  $\times$ . To obtain higher

resolving power, it is necessary to use shorter wavelengths. The 200-in. reflector at Mt. Palomar can distinguish between two stars which are separated by  $1.3 \times 10^{-7}$  radian or  $2.6 \times 10^{-2}$  sec of arc. This may be compared to the eye whose pupil has a diameter of about 3 mm, so that the theoretical resolving power is such that the eye might resolve objects separated by 47 sec of arc. Actually, the average person can distinguish objects separated by about 1 min of arc.

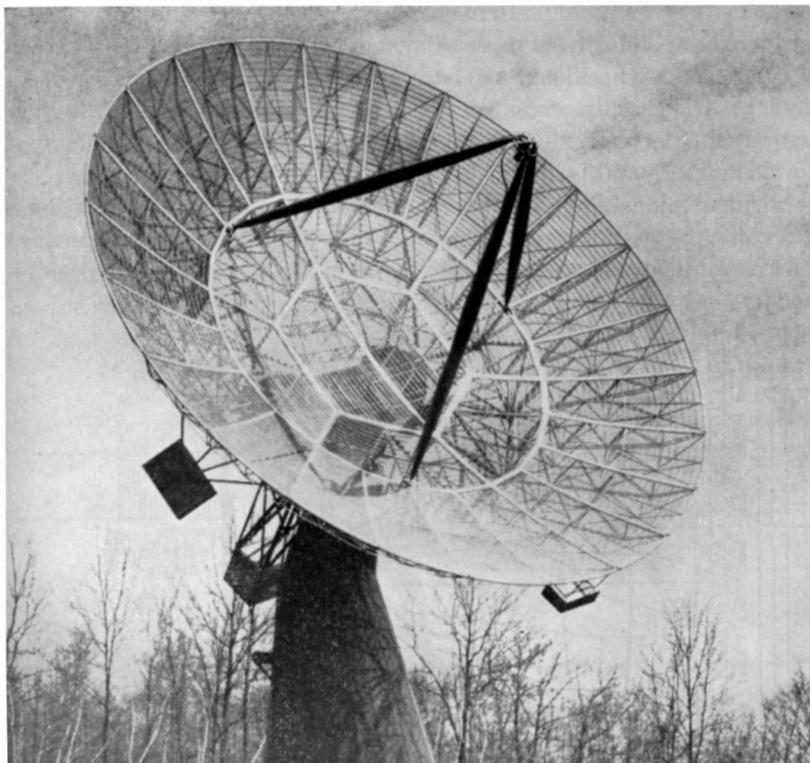


**Fig. 40-14** Photograph of the diffraction patterns of light produced by a lens. Four point sources of light were used. The two patterns on the right can just be resolved as due to two sources. (Reproduced by permission from *College Physics*, 2nd ed., by Sears and Zemansky, 1952; Addison-Wesley Publishing Company, Inc., Reading, Mass.)

Identical considerations apply to the design of radar reflectors and the reflectors used in radiotelescopes which are now being used to study sources of radio noise and line emission of microwaves from the sun and other cosmic sources. An antenna having a reflector of 60 ft diameter must be used with 21-cm radiation to resolve the sources of radiation to an angle of about  $1^\circ$ . This particular wavelength is emitted by neutral hydrogen atoms in the hydrogen clouds of the galaxy. The largest radiotelescope now in operation in the United States is the 60-ft paraboloid of Harvard University, shown in Figure 40-15, while reflectors three and four times this diameter are under construction.

#### 40-6 The Diffraction Grating

The *diffraction grating* is widely used for the measurement of the wavelength of light and for spectrum analysis. Diffraction gratings are used as reflection gratings or as transmission gratings. A reflection grating consists of a

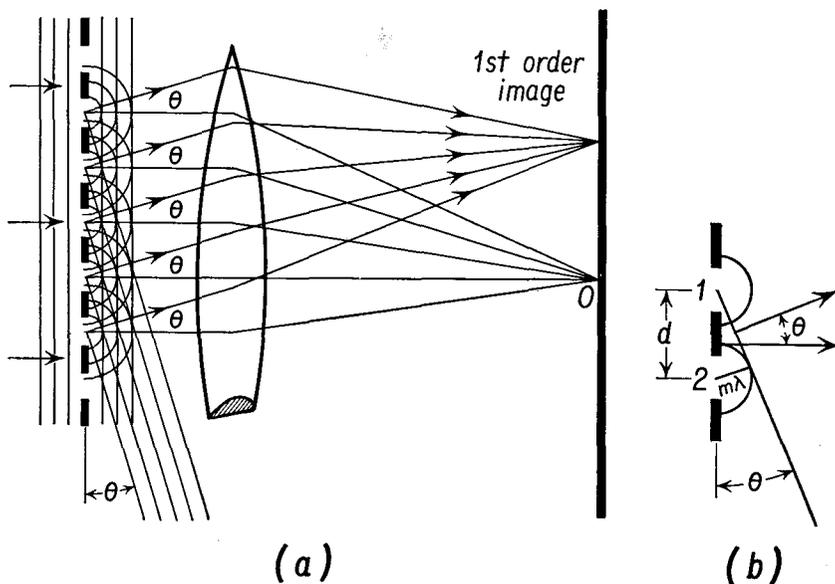


**Fig. 40-15** The George R. Agassiz 60-foot Radio Telescope of Harvard College Observatory at the G. R. Agassiz Station, Harvard, Mass. (Photo by Robert D. Cox, Sky Publishing Corp; reproduced by permission of Harvard College Observatory.)

series of parallel rulings or scratches made on a polished reflecting surface. The number of rulings varies from about 400 per centimeter in some gratings to about 6,000 per centimeter in other gratings. A transmission grating has a series of parallel rulings made on a flat glass surface. The light is transmitted through the spacings between the scratches. Good gratings are difficult to prepare, and, for ordinary purposes, replicas are used. These replicas can be made by pouring a solution of collodion in ether over a ruled grating. After the ether has evaporated, the collodion layer is stripped off and is cemented to one side of an optical flat. The collodion retains the impression and acts as a fairly good diffraction grating.

To understand the action of a grating, let us consider a set of plane parallel waves incident on a transmission grating, as shown in Figure 40-16. The parallel light is generally produced by placing the light source in front of the slit of a collimating telescope. The spaces between the rulings can be considered as a series of equally spaced narrow slits, a few of which are

shown in the figure. The light which passes through the grating can be considered as coming from these slits, and, according to Huygens' principle, the slits can be considered as sources of waves. These waves will be of circular section, as drawn in a plane perpendicular to the rulings. For the sake of simplicity, let us assume that the incident light is monochromatic of wavelength  $\lambda$  and is directed normal to the plane of the grating. Since the incident plane wave front is parallel to the plane of the grating, the light emerging from the slits at any one time is all in phase and spreads out in wave fronts of circular section from each slit as center.



**Fig. 40-16** (a) Action of a diffraction grating on a parallel beam of monochromatic light. The relative dimensions of the grating and the lens are drawn out of correct proportion in order to illustrate the effect of the lens on the diffracted beam. (b) The relationship between the grating space  $d$ , the wavelength  $\lambda$ , and the angle of diffraction  $\theta$ .

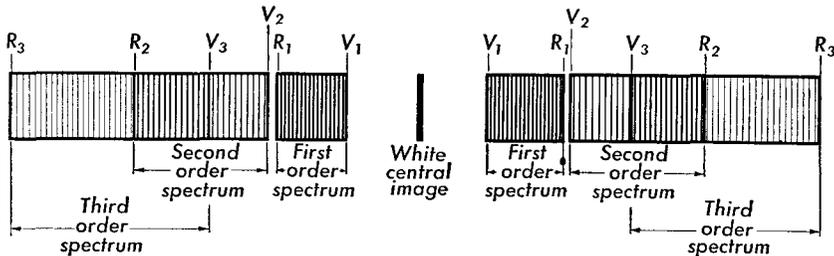
When parallel light composed of plane wave fronts is incident upon a lens, it is brought to a focus at a point in the focal plane of the lens. If the incident parallel light is everywhere in phase across a plane perpendicular to the direction of propagation of the light, it will be in phase at the focus, and a bright spot will be produced. Let us consider the phase relationships between light from adjacent slits incident upon the lens in various directions making an angle  $\theta$  with the normal to the grating.

Light leaving the grating in the direction of the normal is in phase in a plane perpendicular to its direction of motion. This light will be brought to a focus by a converging lens and will produce what is known as the

central image at  $O$ , in the focal plane of the lens and on the principal axis of the lens. Light leaving the grating at an angle  $\theta$  with the normal to the grating will interfere constructively, provided that the path traversed by light from adjacent slits differs in length by an integral multiple of  $\lambda$ . From Figure 40-16(b) it can be seen that constructive interference will occur when

$$\sin \theta = \frac{m\lambda}{d}, \tag{40-9}$$

where  $m$  is an integer, and  $d$  is the distance between adjacent slits. The image for which  $m = 1$  is called the first-order image, and so on. Thus if violet light and red light are incident on the grating, the first-order image



**Fig. 40-17** Relative positions of the first three orders of spectra produced by a diffraction grating on either side of the central white image. Notice that the second and third orders overlap considerably.

of violet light will be deviated through a smaller angle than that of red light, since the wavelength of violet is shorter than that of red light. If white light is incident on a diffraction grating, a series of continuous spectra will be obtained on each side of the central image. The central image itself will be white, since all of the wavelengths from the source are focused in it. In any one order, the spectrum produced by a diffraction grating has the colors in the reverse order from that produced by a prism. In a diffraction-grating spectrum the violet is deviated least, while in a prism spectrum the red is deviated least.

The diffraction grating is often used as the dispersing element in a spectrometer, in place of a prism, producing images of several orders, as shown in Figure 40-17. Because the wavelength of visible light ranges from about 3,800 Å to about 7,500 Å, a factor of slightly less than 2, there is a break between the first-order spectrum and the second-order spectrum, but the third-order spectrum overlaps the second-order spectrum.

The prism spectroscope has one important advantage over the diffraction-grating spectroscope in that all the energy which passes through the prism is concentrated in a single spectrum. In a diffraction-grating spectroscope the energy from the source of light is spread over several

orders, and a large fraction of this energy is concentrated in the zero order, or central image. On the other hand, the diffraction grating provides a direct means of measurement of wavelength from the measurement of an angle and the spacing between the rulings of the grating. The dispersion produced by the grating can be calculated from Equation (40-9), while the dispersion of a prism does not follow a simple law. For most optical glass the dispersion is much greater in the violet region than in the red region of the spectrum.

### Problems

40-1. Yellow sodium light whose wavelength is 5,893 Å comes from a single source and passes through two slits 1 mm apart. The interference pattern is observed on a screen 175 cm away. (a) How far apart are two adjacent bright bands? (b) Taking the distance between the first minima on either side of the central maximum as the "shadow" of the solid region between the two slits, how wide is this shadow?

40-2. Light from a mercury arc is passed through a green filter and then falls upon two narrow slits 0.06 cm apart. The interference pattern is formed on a screen 250 cm away. The distance between two adjacent green lines is found to be 2.27 mm. Determine the wavelength of the light.

40-3. Five per cent of the incident light striking a glass-air surface is reflected back. What percentage of the incident light is transmitted after passage through an optical system containing eight surfaces?

40-4. Calculate the thickness, in centimeters, of a nonreflecting film of refractive index 1.40 to be used for coating a glass plate. Assume that sodium light is incident upon it.

40-5. Prove that if an object of thickness  $T$  is placed at one edge of a glass plate, and a second plate is placed atop the first so as to make an air wedge, then the number  $N$  of dark lines produced in light of wavelength  $\lambda$  is  $N = 2T/\lambda$ .

40-6. Calculate the number of dark lines that will be produced when green light of wavelength 5,461 Å is incident normally upon a wedge-shaped air film produced by inserting a piece of steel 0.02 cm thick between two glass plates, at one end.

40-7. Two glass plates 10 cm long are in contact at one end and are separated by a thin sheet of paper at the other end, forming a wedge-shaped air film. Red light of wavelength 6,600 Å is incident normally upon the glass. Experiment shows that there are 17 dark lines per centimeter. (a) Calculate the thickness of the paper. (b) How many lines per centimeter would be produced by green light of wavelength 5,400 Å?

40-8. In a Newton's ring apparatus a planoconvex lens is placed upon a flat glass plate, convex side down. The diameter of the first dark ring is observed to be 0.1 cm. What is the radius of curvature of the lens? Monochromatic light for which  $\lambda = 6,000$  Å is used in normal illumination. [HINT: If  $r$  is the radius of the Newton ring,  $s$  the thickness of the air at this position, and  $R$  the

radius of curvature of the lens, then  $r/s = (2R - s)/r$ , and, to a first approximation,  $r^2 = 2Rs$ .]

40-9. Light from a distant source is incident normally upon a single slit. The wavelength of the incident light is 5,893 Å. It is found that the width of the central maximum on a screen located 1 m from the slit is 0.1 cm. What is the width of the slit?

40-10. What must be the diameter of the reflector of a radar antenna in the form of a parabolic mirror if the antenna is to separate two airplanes  $1^\circ$  apart? The wavelength to be used is 3 cm.

40-11. A diffraction grating has 6,000 lines per centimeter. White light is incident on the slit of a diffraction-grating spectrometer so that the collimated beam falls normally on this grating. At what angle will the blue light of 4,400 Å wavelength be found (a) in the first order and (b) in the second order?

40-12. In the grating of Problem 40-11, at what angle will the red light of 7,200 Å wavelength be found (a) in the first order and (b) in the second order? (c) What is the highest order spectrum in which this red light will be found?

40-13. Blue light of 4,500 Å is used to determine the number of lines on a grating. When this grating is used with a spectrometer, the second-order image is found at an angle of  $30^\circ$  from the central image. Determine the number of lines per centimeter on the grating.

40-14. The yellow line in the spectrum of sodium, sometimes called the *D* line, consists of two lines very close together when viewed with a spectroscope of moderate resolving power. The wavelengths of these lines are  $D_1 = 5,896$  Å and  $D_2 = 5,890$  Å. Determine the angular separation of these lines when viewed with a diffraction grating having 6,000 lines/cm and viewed (a) in the first order and (b) in the second order.