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## History of cluster structure in nuclei

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**Abstract.** This contribution focuses on the history of clustering in nuclei. Elementary alpha models treat light 4-n nuclei as systems of alpha-particles obeying Bose Einstein statistics. These models neglect the internal structure of the alphas and effects of the Pauli principle between the nucleons in the alpha clusters are taken into account by introducing a short range repulsion between the clusters. The orthogonality condition model and excluded state model treat the alphas as elementary particles, but include effects of the Pauli principle in a more microscopic way. Wheeler's resonating group method is a fully microscopic theory for calculating properties of cluster systems. It makes simplifying assumptions about the internal structure of the clusters but takes the Pauli principle explicitly. Hartree-Fock theory can be used for a microscopic theory of nuclear structure but it is not suitable for light nuclei because there is no well defined mean field. Margenau's microscopic cluster model avoids this problem by using trial wave functions which are antisymmetrized products of parametrized single particle wave functions.

### 1. Introduction

After Gamow's theory of  $\alpha$ -decay [1] it was natural to investigate a model in which nuclei are composed of  $\alpha$ -particles. Gamow developed a rather detailed theory of properties in his book "Constitution of Nuclei" [2] published in 1931 before the discovery of the neutron in 1932. He supposed that 4n-nuclei like  $^8\text{Be}$ ,  $^{12}\text{C}$ ,  $^{16}\text{O}$  .. were composed of  $\alpha$ -particles while other nuclei contained protons and 'electrons' as well as alphas. He recognized that the nuclear 'electrons' had peculiar properties and wrote: " For some unknown reason, although the electrons in the nucleus behave in a peculiar and obscure way, this does not affect very much the laws governing the motion of the nuclear  $\alpha$ -particles and protons; we can treat nuclear processes involving only  $\alpha$ -particles and protons independently of the nuclear electrons."

After the discovery of the neutron proton-neutron models of the nucleus became popular. The paper by Hafstad and Teller [3] which combined alpha- and single particle aspects was quite influential. In the section on alpha-particles other alpha models are discussed. The resonating group method (RGM) which included cluster aspects in a microscopic was invented by Wheeler [4]. The RGM uses antisymmetrized wave functions and takes proper account of the Pauli principle. It also leads to approximate ways of including Pauli principle effects, and allows different cluster structures to co-exist in the same nucleus. Another paper by Margenau [5] which was related to the Hartree-Fock method laid the foundations of microscopic approaches like antisymmetrized molecular dynamics. The Ikeda diagram [6] introduced in 1968 has proved to be very powerful in identifying situations where cluster structure can be observed. The diagram illustrates various cluster structures which could exist

in excited states of light nuclei and makes the hypothesis that particular cluster structure structures will emerge for excitation energies near the corresponding threshold for decay. The review article by von Oertzen, Freer and Kanada-En'yo [7] has nice a discussion of the Ikeda diagram as well as other points of history and recent developments.

## 2. Alpha-particle models

The alpha-particle is a tightly bound structure and it is possible that some light nuclei might be described by as a combination of alphas which obey Bose-Einstein statistics. Models of this kind have been popular since Gamow's original studies.

Hafstad and Teller [3] proposed a cluster model to estimate the binding energies of  $4n$ ,  $4n-1$  and  $4n+1$  nuclei. The alphas in a  $4n$  nucleus are arranged in a close packed structure and interact with nearest neighbours. The binding energy, including zero point energy, is roughly proportional to the number of alphas. There is very good agreement with experimental values except for  ${}^8\text{Be}$  and  ${}^{20}\text{Ne}$  which are somewhat under bound. A  $4n+1$  nucleus consists of  $n$  alphas plus one nucleon; a  $4n-1$  nucleus of  $n$  alphas and a hole. The extra nucleon or hole jumps around from one alpha to the next I a kind of molecular orbital. The estimated binding energies fit experimental values quite well.

In 1940 Dennison [8] proposed a model of  ${}^{16}\text{O}$  with 4 alphas arranged at the corners of a regular tetrahedron. Energy levels were due to the rotation and vibration of this structure. The possible angular momenta were limited by the symmetry. The observed  $0^+$  level observed at 6.06 Mev was predicted to be a breathing mode. By 1954 much more information was available and Dennison was able to fit and predict many excited states of the nucleus. Subsequently the model was abandoned because measurements showed that the 6.06 Mev level had a small monopole matrix element to the ground state and that the breathing mode was much higher in energy.

By the 1960s good experimental data on alpha-alpha scattering was available. Ali and Bodmer [9] made a detailed study of potentials which fitted the scattering phase shifts. They obtained many equally good potentials and looked for ways of distinguishing between them. All their potentials had a repulsive part with a strength  $V_{0\ell}$  which depended on the angular momentum  $\ell$  and an attractive part with a constant strength  $V_\ell$ ,

$$V(r) = V_{0\ell} \exp(-r^2/a^2) - V_\ell \exp(-r^2/a^2).$$

Reference [10] contains an extensive review of the research on  $\alpha$ - $\alpha$  potentials in the 1960s.

Soon after Ali and Bodmer's work on the  $\alpha$ - $\alpha$  potentials several groups postulated that the  ${}^{12}\text{C}$  nucleus was built from three elementary alpha-particles and calculated the structure of its states with Ali and Bodmer potentials. There was particular interest in the excited  $0^+$  state with an energy of 0.38 MeV above the threshold for break up into the 3- $\alpha$  channel which was predicted by Hoyle in 1954 [11]. Visschers and Van Wageningen [12] found that the Ali and Bodmer potentials gave a ground state binding energy which was too small and failed to give a reasonable energy for the Hoyle state. Many authors argued that there were fundamental weaknesses in the elementary alpha model and that the microscopic structure of the  $\alpha$  -particles was important.

Some of the effects of the internal structure of the alpha particles in an alpha model can be taken into account by introducing many body forces. Fedorov and Jensen [13] added a 3-body force to one of the Ali and Bodmer potentials and were able to choose the parameters so that the energies of the ground state and the Hoyle state. Their calculation is interesting because they were able to calculate the decay rate of the Hoyle state and some of it properties.

## 3. The resonating group method

After the discovery of the neutron in 1932 several models were developed in which the nucleus was composed of protons and neutrons. A shell model, which assumed that neutrons and protons moved in individual orbits in an average field produced by the other nucleons, was suggested by Elsassner [14]

in 1933. It predicted magic numbers for  $N=Z=2, 8, 20$  but failed for magic numbers in heavier nuclei because there was no spin-orbit interaction.

The resonating group model (RGM) was an alternative theory developed by Wheeler [4] in 1937. The RGM regards the neutrons and protons as divided into various clusters (such as alpha particles) which were continually being broken up and reformed in various ways. This oscillation from one cluster structure to another is the origin of the term 'resonating group model'.

An example of a resonating group wave function for  $N$  nucleons divided into 2 clusters is as follows. The wave function of the nucleons (1, ...,  $m$ ) in first cluster is  $\varphi_1(\mathbf{r}_1, \dots, \mathbf{r}_m)$  and the wave function of the nucleons ( $m+1$ , ...,  $N$ ) in second cluster is  $\varphi_2(\mathbf{r}_{m+1}, \dots, \mathbf{r}_N)$ . The total wave function is the antisymmetrized product

$$\Psi = A [\varphi_1(\mathbf{r}_1, \dots, \mathbf{r}_m) \varphi_2(\mathbf{r}_{m+1}, \dots, \mathbf{r}_N) \chi(\mathbf{R}_1 - \mathbf{R}_2)]. \quad (2)$$

The vectors  $\mathbf{R}_1$  and  $\mathbf{R}_2$  are the centres of mass of the nucleons in the first and second clusters. The wave function  $\chi(\mathbf{R}_1 - \mathbf{R}_2)$  of relative motion of the clusters can be found by a variational principle and satisfies a Schrödinger equation with a non-local Hamiltonian.

Wildermuth and Kanellopoulos [15, 16] revived the resonating group method in 1958 when the nuclear shell model was well established. They pointed out that, when clusters overlap, resonating group wave functions and shell model wave functions can be very similar after antisymmetrization. On the other hand, when clusters are somewhat separated then resonating group wave functions can include correlations which are not naturally described by shell model wave functions.

Wildermuth and Kanellopoulos [15] argued out that the cluster structure can change from level to level in the same nucleus and gave some examples. One was  $^5\text{He}$  where the ground state was found to have the structure  $\alpha$ -neutron, while the  $3/2^+$  excited state was best described by a deuteron cluster and a triton in relative motion. Another was  $^{19}\text{F}$ . It was concluded that the lowest  $1/2^+$  and  $3/2^+$  levels were best described by an unexcited  $^{16}\text{O}$  core and a triton with orbital angular momenta  $L=0$  and  $L=2$ . On the other hand the excited  $1/2^-$  corresponds to a  $^{15}\text{N}$  core and an alpha cluster in relative motion. Wildermuth and Tang [17] were able to explain the Coulomb energy shifts of levels in the mirror nuclei  $^{19}\text{F}$  and  $^{19}\text{Ne}$  by the differences in cluster structure.

#### 4. Pauli excluded states

Wildermuth and Kanellopoulos made an important observation on the effects of antisymmetrization on the RGM wave functions. If the two cluster wave functions  $\varphi_1$  and  $\varphi_2$  in  $^8\text{Be}$  are constructed from harmonic oscillator wave functions then antisymmetrization causes the RGM wave function  $\Psi$  to vanish if the wave function  $\chi(\mathbf{R}_1 - \mathbf{R}_2)$  is an oscillator state with principle quantum number  $N < 4$ . These are called Pauli-excluded states. The first non-vanishing states  $\chi$  are degenerate in the oscillator approximation and have  $L = 0, 2, 4$ . Pauli excluded states occur in many other two cluster systems. For example in  $^{20}\text{Ne}$  with an  $(\alpha + ^{16}\text{O})$  cluster structure the states with  $N < 16$  are excluded if the clusters and relative motion wave functions are built from oscillator wave functions with the same oscillator parameters. The lowest allowed states have  $N = 16$  and  $L=0, 2, 4, 6, 8$ .

One simplification of the RGM is Saito's orthogonality condition model [18]. The Schrödinger for the relative motion wave function  $\chi(\mathbf{R}_1 - \mathbf{R}_2)$  in a two cluster problem is solved in a deep local potential  $V(r)$  of folding type with the condition that the solutions are orthogonal to the Pauli forbidden states.

Another, more drastic, simplification to a two cluster problem is to solve the wave equation for  $\chi(\mathbf{R}_1 - \mathbf{R}_2)$  in  $V(r)$  and to reject the states in  $V(r)$  which correspond to the Pauli excluded states. The theoretical background for this simplification was given by Buck, Friedrich and Wheatley [19] who gave an application to  $(\alpha, \alpha)$  scattering with  $V(r) = V_0 \exp(-a r^2)$  plus a Coulomb potential ( $V_C = 4e^2 \text{erf}(\beta r)/r$ ) with  $\beta = 0.75 \text{ fm}^{-1}$ . The excluded states were 0S and 1S, 0D and the potential parameters were chosen to give the 2S-resonance state at 92.12 keV and the correct behaviour of the  $L = 2$  phase

shift near 3 MeV. The calculated s-, d-, and g-phase shifts were in excellent agreement with measured values for centre of mass energies up to 30 MeV.

Buck, Dover and Vary [20] were the first to apply this Pauli excluded state method. They gave an excellent account of the low collective states of  $^{20}\text{Ne}$  in an  $(\alpha, ^{16}\text{O})$  cluster model and the low excited band in  $^{16}\text{O}$  as an  $(\alpha, ^{12}\text{C})$  cluster model.

### 5. Margenau's method

The resonating group method is a general method for constructing wave functions for cluster nuclei. It includes antisymmetrization and allows for a general wave function of relative motion of the clusters. The wave function proposed by Margenau [5] is simpler but less general than the RGM wave function but easier to use. He made an application to  $^8\text{Be}$  in an  $(\alpha, \alpha)$  cluster model by constructing a Slater determinant for 8 nucleons with two Gaussian wave packets centred at points  $\mathbf{R}_1$  and  $\mathbf{R}_2$  and calculated the expectation value of the energy  $E(\mathbf{R})$  where  $\mathbf{R} = |\mathbf{R}_1 - \mathbf{R}_2|$  is the separation of the two centres. Then he used the energy  $E(\mathbf{R})$  as an interaction potential to calculate  $(\alpha, \alpha)$  scattering. This application was not successful, partially because the measured scattering phase shifts available at the time were inaccurate.

Margenau's wave function could be improved by using the generator coordinate method proposed by Hill and Wheeler [21] in 1953. If Margenau's wave function is denoted by  $\Psi(\mathbf{r}_i, \mathbf{R})$  then the generator coordinate wave function is a superposition of Margenau functions

$$\Psi(\mathbf{r}_i, f) = \int d\mathbf{R} \Psi(\mathbf{r}_i, \mathbf{R}) f(\mathbf{R}).$$

With a suitable choice of the weight function  $f(\mathbf{R})$  the GCM is equivalent to a RGM wave function. The Margenau wave function is not an eigenstate of angular momentum. Angular momentum projection is a special case of the GCM (Griffin and Wheeler [22]).

The Bloch-Brink wave function [23] is a generalization of the Margenau wave function from 2- to  $n$ - $\alpha$  clusters. It is an antisymmetrized of  $n$  Gaussian wave packets on  $n$  centres. There is a simple formula for calculating the expectation value of a nuclear Hamiltonian with two-body force. Expressions for the matrix elements were given by Brink in 1955. They were discovered earlier by Claude Bloch but not published. The Bloch-Brink wave functions go over to harmonic oscillator shell model wave functions when the alpha cluster wave functions have a strong overlap.

### 6. Antisymmetrized Molecular Dynamics

Antisymmetrized molecular dynamics (AMD) [24] and Fermion molecular dynamics (FMD) [25] were invented to describe heavy ion break up reactions. They combined a quantum wave packet treatment of the structure of the interacting with a classical description of the propagation of the wave packets. The initial state of the ions was a bound cluster wave function of the kind introduced by Margenau. During the collision the cluster wave functions developed in time and could break up or remain together depending on the interactions and the intensity of the collision.

The first studies used simple structures for the initial states of the ions, but then it was realized that the approach could yield very good structure wave functions. In the applications to study the structure of light nuclei. At the simplest level the wave function of a nucleus was approximated by an antisymmetrized product of Gaussian wave functions, one for each nucleon. The best wave function was found by minimizing the energy. A better wave function is obtained by projecting angular momentum and minimizing after projection. A still better approximation is to take a linear combination of these wave functions. By taking different Gaussians for each nucleon it is possible to include the effects of spin-orbit and tensor forces.

### 7. Experimental observation of cluster structure

The Hoyle state in  $^{12}\text{C}$  with spin-parity  $0^+$  and excitation energy 0.38 MeV above the threshold for decay into the  $3\alpha$  channel is a striking example. The calculations of Fedorov and Jensen [13] predict

that the  $\alpha$  clusters are well separated. The calculated decay width of the state predicted in ref.[13] is 20 eV very sensitive to the spatial extension of the state and agrees well with the experimental value of 8.3 eV.

Recent experiments by Kawabata et al [26] indicate that the weakly bound second excited  $3/2^-$  state in the nucleus  $^{11}\text{B}$  with excitation energy  $E_x = 8.56$  MeV has a  $2\alpha + t$  cluster which is very similar to the  $3\alpha$  structure of the Hoyle state in  $^{12}\text{C}$ . The measured values of the isoscalar quadrupole transition strength to this state is smaller than typical shell model values while the isoscalar monopole strength  $B(E0:IS) = 96 \text{ fm}^4$  is large and comparable with the corresponding value  $B(E0:IS) = 121 \text{ fm}^4$  in  $^{12}\text{C}$ . Also the Coulomb energy shift between the mirror states in  $^{11}\text{B}$  and  $^{11}\text{C}$  is large which indicates a dilute structure of the 8.56 MeV state in  $^{11}\text{B}$  which is consistent with a  $2\alpha + t$  cluster structure.

Wuosmaa et al [27] found evidence for exotic clustering in the scattering of  $^{12}\text{C}$  by  $^{12}\text{C}$  for centre of mass energies  $E_{\text{cm}} \approx 32$  MeV. They found that the  $0_2^+$  Hoyle state with excitation energy 7.65 MeV is excited quite strongly and that the double excitation  $^{12}\text{C} + ^{12}\text{C} \rightarrow ^{12}\text{C}(0_2^+) + ^{12}\text{C}(0_2^+)$  cross section has a resonance with peak energy  $E_{\text{cm}} \approx 32.5$  MeV and width  $\Gamma_{\text{cm}} \approx 4.7$  MeV. Their data can be explained if the excited compound nucleus  $^{24}\text{Mg}^*$  has a cluster structure  $^{12}\text{C}(0_2^+) + ^{12}\text{C}(0_2^+)$  with relative angular momentum  $L = 16$ . The excitation energy is far above the threshold energy 15.7 MeV for the reaction but the Ikeda threshold condition is satisfied if the angular momentum and Coulomb barriers are taken into account. Some other examples of experiments are given in ref.[7].

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