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"Reality demonstrates that the more specific rules are, the easier, paradoxically, they make it to navigate around them. Moving ethics out of the realm of values and into the compliance and regulatory space has effectively failed us."

Jean L.P. Brunel – Editor's Letter

NIELS BEKKERS, RONALD Q. DOESWIJK, AND TREVIN W. LAM

Strategic Asset Allocation: *Determining the Optimal Portfolio with Ten Asset Classes*

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Strategic Asset Allocation: *Determining the Optimal Portfolio with Ten Asset Classes*

NIELS BEKKERS, RONALD Q. DOESWIJK, AND TREVIN W. LAM

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Most previous academic studies agree on the importance of strategic asset allocation as a determinant for investment returns. In their frequently cited paper, Brinson, Hood, and Beebower [1986] claim that 93.6% of performance variation can be explained by strategic asset allocation decisions. This result implies that strategic asset allocation is far more important than market timing and security selection.

Most asset allocation studies focus on the implications of adding one or two asset classes to a traditional asset mix of stocks, bonds, and cash to conclude whether and to what extent an asset class should be included to the strategic portfolio; see for example Erb and Harvey [2006] and Lamm [1998]. However, since asset classes were omitted, this partial analysis can lead to sub-optimal portfolios. This is surprising, as pension funds and other institutions have been strategically shifting substantial parts of their investment portfolios towards non-traditional assets such as real estate, commodities, hedge funds, and private equity.

The goal of this article is to explore which asset classes add value to a traditional asset mix and to determine the optimal weights of all asset classes in the optimal portfolio. The article adds to the literature by distinguishing 10 different investment categories simultaneously in a mean-variance analysis as well as a market portfolio approach. It also demonstrates how to combine these two methods. Next to the traditional three asset classes, stocks, government bonds, and cash, we include private equity, real estate, hedge funds, commodities, high yield, credits, and inflation-linked bonds. A study with such a broad coverage of asset classes has not been conducted before, neither in the context of determining capital market expectations and performing a mean-variance analysis, nor in assessing the global market portfolio. The second step in portfolio management—i.e., market timing and security selection—is tactical decision making. These are beyond the scope of this article.

In short, this article suggests that adding real estate, commodities, and high yield to the traditional asset mix delivers the most efficiency, improving value for investors. Next, it shows that the proportion of non-traditional asset classes appearing in the market portfolio is relatively small. The article then reports an empirical and literature analysis to establish long-run capital market expectations for each asset class, which we subsequently use in a mean-variance analysis. Then, we provide an assessment of the global market portfolio. Finally, we show how the mean-variance and market portfolio approaches can be combined to determine optimal portfolios.

METHODOLOGY

Markowitz [1952, 1956] pioneered the development of a quantitative method that takes into account the diversification benefits of portfolio allocation. Modern portfolio theory is the result of his work on portfolio optimization. Ideally, in a mean-variance optimization model, the complete investment opportunity set—i.e., all assets—should be considered simultaneously. However, in practice, most investors distinguish between different asset classes within their portfolio-allocation frameworks. This two-stage model is generally applied by institutional investors, resulting in a top-down allocation strategy.

The first part of our analysis views the process of asset allocation as a four-step exercise, as in Bodie, Kane, and Marcus [2005]. It consists of choosing the asset classes under consideration, then moving forward to establishing capital market expectations, followed by deriving the efficient frontier until finding the optimal asset mix. In the second part of our analysis, we assess the global market portfolio. Finally, we show how the mean-variance and market-neutral portfolio approaches can be combined to determine optimal portfolios.

We take the perspective of an asset-only investor in search of the optimal portfolio. An asset-only investor does not take liabilities into account. The investment horizon is one year and the opportunity set consists of 10 asset classes. The investor pursues wealth maximization, and no other particular investment goals are considered.

We solve the asset-allocation problem using a mean-variance optimization based on excess returns. The goal is to maximize the Sharpe ratio (risk-adjusted return) of the portfolio, bounded by the restriction that the exposure to any risky asset class is greater than or equal to zero and that the sum of the weights adds up to one. The focus is on the relative allocation to risky assets in the optimal portfolio, instead of the allocation to cash. The weight of cash is a function of the investor's level of risk aversion.

For the expected risk premia we use geometric returns with intervals of 0.25%. The interval for the standard deviations is 1%, and for correlations it is 0.1. In our opinion, more precise estimates might have an appearance of exactness, which we want to prevent. We do not take management fees into consideration, except for private equity and hedge funds, as for these asset classes the management fees are rather high relative to the expected risk premia. Other asset classes have significantly lower fees compared to their risk premia. They are therefore of minor importance, especially after taking into account the uncertainty of our estimates. We estimate risk premiums by subtracting geometric returns from one another. Hereby, our estimated geometric returns as well as the risk premiums are both round numbers.

In the mean-variance analysis, we use arithmetic excess returns. Geometric returns are not suitable in a mean-variance framework. The weighted average of geometric returns does not equal the geometric return of a simulated portfolio with the same composition. The observed difference can be explained by the diversification benefits of the portfolio allocation. We derive the arithmetic returns from the geometric returns and the volatility.

DATA

We focus primarily on U.S. data in the empirical analysis. This choice is backed by two arguments. First, the U.S. market offers the longest data series among almost all asset classes. This makes a historical comparison more meaningful. For instance, the high-yield bond market has long been solely a U.S. capital market phenomenon. Secondly, using U.S. data avoids the geographical mismatch in global data. A global index for the relatively new asset class of inflation-linked bonds is biased towards the U.S., French, and U.K. markets, while a global stock index is decently spread over numerous countries. We use total return indices in U.S. dollars.

Asset classes like real estate and private equity are represented in both listed and non-listed indices, while hedge funds are covered only by non-listed indices. Non-listed real estate and private equity indices are appraisal-based, which may cause a smoothing effect in the assumed risk of the asset class. This bias arises because the appraisals do not take place frequently. However, interpolating returns causes an underestimation of risk. Also, changes in prices are not immediately reflected in appraisal values until there is sufficient evidence for an adjustment. Statistical procedures to mitigate these data problems exist, but there is no guarantee that these methods produce accurate measures of true holding-period returns; see Froot [1995]. As these smoothing effects can lead to an underestimation of risk, this article avoids non-listed datasets and instead adopts listed indices for real estate and private equity. The quality of return data of listed indices is assumed to be higher, as they are based on transaction prices. Ibbotson [2006] supports this approach and states, "Although all investors may not yet agree that direct commercial real estate investments and indirect commercial real investments (REITs) provide the same risk-reward exposure to commercial real estate, a growing body of research indicates that investment returns from the two markets are either the same or nearly so." For hedge funds we will use a fund-of-funds index that we unsmooth with Geltner [1991, 1993] techniques. Fung and Hsieh [2000] describe the important role of funds of hedge funds as a proxy for the market portfolio of hedge funds.

Appendices A and B contain our data sources. In Appendix A we discuss our capital market expectations, and in Appendix B we derive the market portfolio from a variety of data sources.

EMPIRICAL RESULTS

Capital Market Expectations

We estimate risk premia for all asset classes based on previous reported studies, our own empirical analyses of data series, and the basic idea that risk should be rewarded. Obviously, estimates like these inevitably are subjective as the academic literature provides only limited studies into the statistical characteristics of asset classes. Moreover, there is generally no consensus among academics and we lack long-term data for most asset classes. Our results should therefore be treated with care, especially since mean-variance analysis is known for its corner solutions, being highly sensitive in terms of its input parameters.

This article proceeds with the risk premia and standard deviations as shown in Exhibit 1. Appendix A contains the reasoning for these estimates and for the correlation matrix.

Mean-Variance Analysis

Exhibit 2 shows the optimal portfolio based on the mean-variance analysis and its descriptive statistics for a traditional portfolio with stocks and bonds as well as a portfolio with all assets. On top of the traditional asset classes of stocks and bonds, this analysis suggests that it is attractive for an investor to add real estate, commodities, and high yield. The Sharpe ratio increases from 0.346 to 0.396. The allocation to real estate is quite high. To bring this into perspective, we suggest that the proposed portfolio weight is overdone. When one would, for example, be willing to perceive utilities as a separate asset class, it is likely that it also would get a significant allocation as this sector also has a low correlation to the general stock market.

Exhibit 2 also illustrates that mean-variance analysis tends towards corner solutions as it neglects credits, which have characteristics comparable with bonds. However, with these parameters it prefers bonds in the optimal portfolio.

Exhibit 3 shows the benefits of diversification into non-traditional asset classes. In the volatility range of 7% to 20%, the diversification benefits vary between 0.40% and 0.93%. This additional return is economically significant. For example, at a volatility of 10%, the additional return is 0.56%. The efficient frontier of a portfolio with stocks, bonds, and the three asset classes, real estate, commodities, and high yield, comes close to the efficient frontier of an all-asset portfolio. By adding these three asset classes, an investor almost captures the complete diversification benefit.

EXHIBIT 1
Overview of Capital Market Expectations for all Asset Classes

	Geometric Risk Premium	Volatility	Arithmetic Risk Premium*	Geometric Sharpe Ratio	Arithmetic Sharpe Ratio
Stocks	4.75%	20%	6.8%	0.24	0.34
Private Equity	4.75%	30%	9.3%	0.16	0.31
Real Estate	3.75%	16%	5.0%	0.23	0.31
Hedge Funds	1.25%	12%	2.0%	0.10	0.16
Commodities	0.00%	26%	3.4%	0.00	0.13
High Yield	2.50%	11%	3.1%	0.23	0.28
Credits	1.50%	9%	1.9%	0.17	0.21
Bonds	0.75%	7%	1.0%	0.11	0.14
Inflation Linked Bonds	0.50%	7%	0.7%	0.07	0.11

*Note: *We derive arithmetic data by using the equation* $R_A = R_G + 0.5 \times variance$.

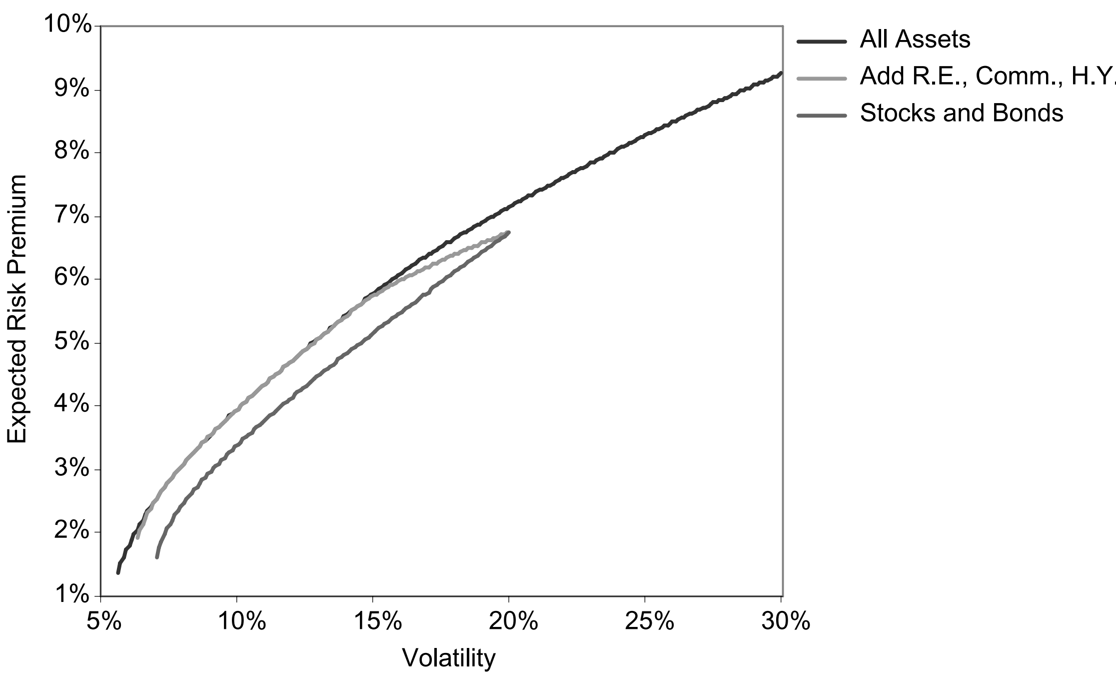
EXHIBIT 2

Optimal Portfolio for a Traditional Portfolio and for an All-Assets Portfolio

Asset Class	Portfolio Weight	Descriptive Statistics	
Traditional			
Stocks	59.2%	Variance	1.6%
Bonds	40.8%	Standard Deviation	12.7%
		Expected Risk Premium	4.4%
		Geometric Return	3.6%
		Sharpe Ratio	0.346
All Asset Classes			
Stocks	26.4%	Variance	1.0%
Private Equity	0.0%	Standard Deviation	10.1%
Real Estate	25.7%	Expected Risk Premium	4.0%
Hedge Funds	0.0%	Geometric Return	3.5%
Commodities	12.7%	Sharpe Ratio	0.396
High Yield	6.6%		
Credits	0.0%		
Bonds	28.6%		
Inflation Linked Bonds	0.0%		

EXHIBIT 3

Efficient Frontier for Different Portfolios



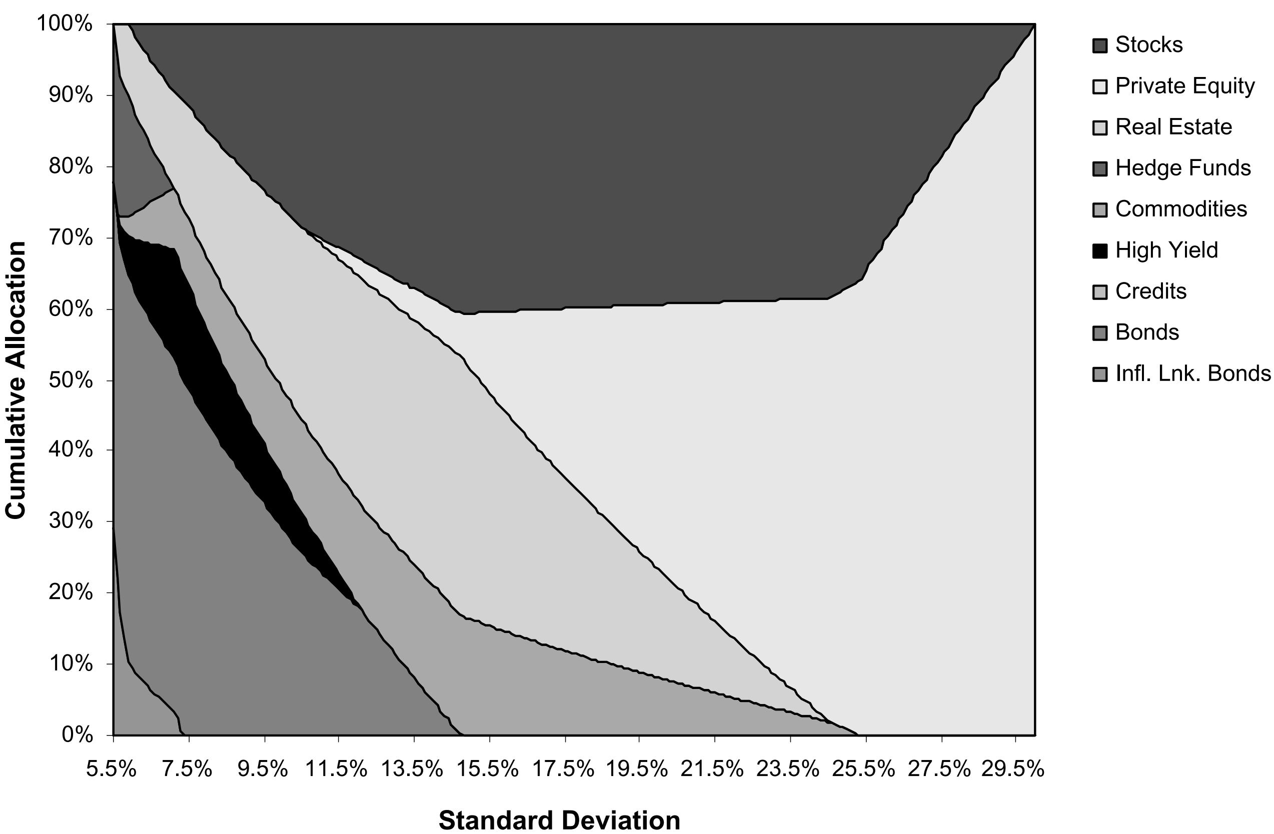
For various reasons, not all investors use cash to (un)leverage their investment portfolio. Therefore, it is interesting to observe the composition of efficient portfolios in a world without the risk-free rate. Exhibit 4 shows the asset allocation on the efficient frontier in an all-asset portfolio starting from a minimum variance allocation towards a risky portfolio. It maximizes the expected excess return constrained by a given volatility.

In the least risky asset allocation, an investor allocates 77.7% of the portfolio towards fixed-income assets. Next to bonds and stocks, real estate and commodities receive a significant allocation in portfolios with a volatility in the range of 7.5% to 12.5%. High yield is also present in most of the portfolios in this range. For riskier portfolios, private equity shows up, and in the end, it ousts bonds, real estate, commodities, and stocks (in that order). For defensive investors, inflation-linked bonds and hedge funds enter the portfolio.

In short, the mean-variance analysis suggests that adding real estate, commodities, and high yield to the traditional asset mix of stocks and bonds creates the most value for investors. Basically, adding these three asset classes comes close to an all-asset portfolio. Private equity is somewhat similar to stocks, but shows up in riskier portfolios, moving along the efficient frontier. This part of the efficient frontier is interesting for investors in search of high returns without leveraging the market portfolio. Hedge funds as a group do not add value. Obviously, when investors attribute alpha to a particular hedge fund, it changes the case for that fund. This also applies to private equity. Credits and bonds are quite similar asset classes, and in a mean-variance context the optimal portfolio tends to tilt to one or another. Inflation-linked bonds do not show up in our mean-variance analysis. The inflation risk premium and the high correlation with bonds prevent an allocation towards this asset class in that setting. However, for defensive investors who primarily seek protection against inflation, this asset class can be very interesting.

EXHIBIT 4

Asset Allocation on the Efficient Frontier in an All-Assets Portfolio



Market Portfolio

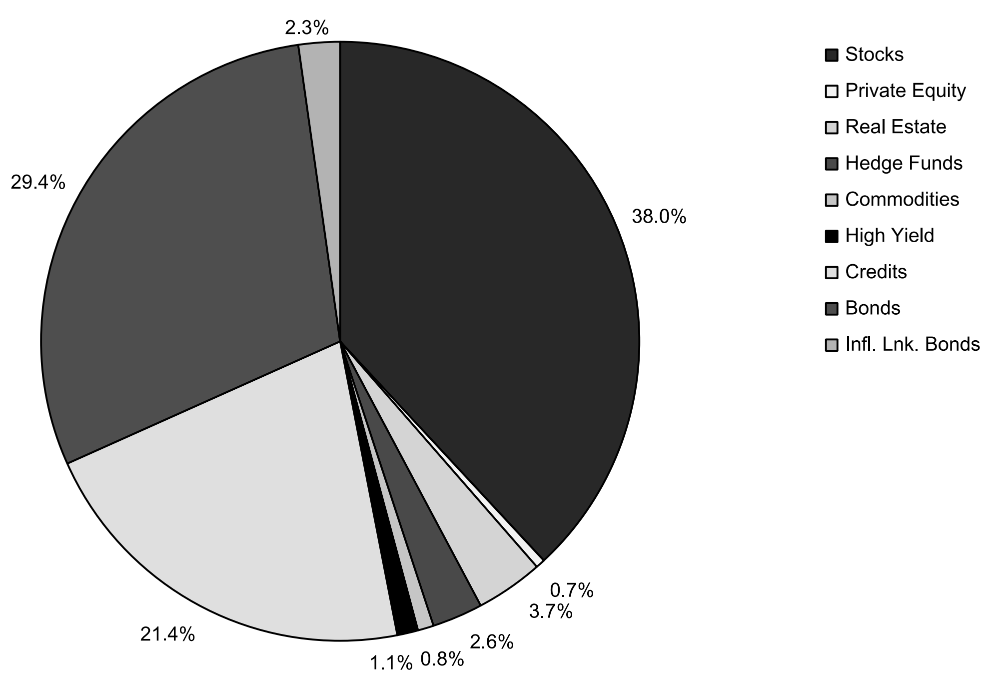
Both academics and practitioners agree that the mean-variance analysis is extremely sensitive to small changes and errors in the assumptions. We therefore take another approach to the asset allocation problem, in which we estimate the weights of the asset classes in the market portfolio. The composition of the observed market portfolio embodies the aggregate return, risk, and correlation expectations of all market participants and is by definition the optimal portfolio. In practice, however, borrowing is restricted for most investors and at the same time borrowing rates usually exceed lending rates. The result is that the market portfolio is possibly no longer the common optimal portfolio for all investors, because some might choose risky portfolios on the efficient frontier beyond the point where no money is allocated to the risk-free rate. In addition, an investor's specific situation could also lead to a different portfolio. Despite this limitation, the relative market capitalization of asset classes provides valuable guidance for the asset allocation problem. In this setting, the market-neutral weight for a particular asset class is its market value relative to the world's total market value of all asset classes.

Exhibit 5 shows the global market portfolio based on a variety of data sources. Appendix B provides details about the market portfolio and its dynamics for the period 2006–2008. The asset classes stocks and investment grade bonds (government bonds and credits) represent more than 85% of the market for these years. At the end of 2008 we estimate this number at 88.8%. This means that the size of the average remaining asset class is less than 12.0%. Based on this analysis, we conclude that the proportion of non-traditional asset classes appearing in the market portfolio is relatively small.

Combination of Market Portfolio and Mean-Variance Analysis

The mean-variance analysis can be combined with the market portfolio. Here, we choose to take the market portfolio as a starting point, which we subsequently optimize with turnover and tracking error constraints. We choose to take the market portfolio as a starting point, as it embodies the aggregate return, risk, and correlation expectations of all market participants without the disadvantage

EXHIBIT 5
Pie Chart of the Market Portfolio at the End of 2008



of delivering the corner solutions of the mean-variance analysis.

Exhibit 6 shows the optimal portfolios with different tracking error constraints and a maximum turnover of 25% (single count) relative to the market portfolio. In other words, in this example we limit ourselves to finding optimized portfolios with portfolio weights that do not differ more than 25% from the market portfolio, calculated as the sum of the absolute difference between the market portfolio and the optimized portfolio for each asset class. Focusing on the 0.25% tracking error constraint, it appears that the analysis recommends especially adding real estate, commodities, and high yield, and removing hedge funds and inflation-linked bonds. This is logical, as the results from the mean-variance analysis are applied in this market-portfolio-adjustment process. There is a 12.5% shift in portfolio weights. Obviously, fewer constraints result in a higher risk premium and a higher Sharpe ratio, until we end up with the theoretically optimal portfolio from the mean-variance analysis. Within this methodology, investors must determine their own individual constraints, while the market portfolio and the portfolio optimized by mean-variance are considered as the boundaries for the asset classes.

SUMMARY AND CONCLUSIONS

Our mean-variance analysis suggests that real estate, commodities, and high yield add the most value to the traditional asset mix of stocks, bonds, and cash. Basically, adding these three asset classes comes close to an all-asset portfolio. The portfolio with all assets shows a diversification benefit along the efficient frontier that varies between 0.40% and 0.93% in the volatility range of 7% to 20%. That is an economically significant extra return for free.

Another approach to the asset allocation problem is assessing the weights of the asset classes in the market portfolio. Based on this analysis we conclude that the proportion of non-traditional asset classes appearing in the market portfolio is relatively small.

One can combine the mean-variance analysis with the market portfolio. Within this methodology, investors must determine their own individual constraints, while the market portfolio and the portfolio optimized by mean-variance are considered as the boundaries for the asset classes.

EXHIBIT 6

Optimal Portfolio for Different Tracking Error Constraints and a Maximum Turnover Constraint of 25% (Single Count), and the No Contraints Optimal Portfolio That Represents the Results of the Mean-Variance Analysis

	Market Portfolio	0.5% TE	1.0% TE	1.5% TE	No constraints
Stocks	38.0%	37.4%	36.3%	35.9%	26.4%
Private Equity	0.7%	0.6%	0.5%	0.7%	0.0%
Real Estate	3.7%	6.0%	8.7%	11.2%	25.7%
Hedge Funds	2.6%	0.0%	0.0%	0.0%	0.0%
Commodities	0.8%	2.8%	4.0%	5.8%	12.7%
High Yield	1.1%	3.2%	3.8%	0.8%	6.6%
Credits	21.4%	21.0%	16.8%	16.1%	0.0%
Bonds	29.4%	29.0%	29.9%	29.4%	28.6%
Inflation Linked Bonds	2.3%	0.0%	0.0%	0.0%	0.0%
Expected Risk Premium	3.64%	3.76%	3.81%	3.87%	3.99%
Standard Deviation	10.3%	10.3%	10.2%	10.3%	10.1%
Sharpe Ratio	0.354	0.364	0.372	0.378	0.396
Tracking Error (TE)	0.0%	0.5%	1.0%	1.5%	4.4%
Turnover (Single Counted)	0.0%	12.5%	22.4%	25.0%	78.7%

APPENDIX A

Capital Market Expectations

Risk premia for stocks and bonds are well documented, and long-term data series extending over 100 years are available. We will therefore start with the risk premia for stocks and bonds. Then, we derive the risk premia of other asset classes by comparing historical performance data and consulting the literature. In order to estimate volatilities and correlations, we rely more on our own historical data, due to a lack of broad coverage in the literature. Below, we discuss returns and standard deviations for each asset class. Afterwards, we estimate correlations among all asset classes.

Stocks. Extensive research on the equity-risk premium has been conducted in recent years. Fama and French [2002] use a dividend discount model to estimate an arithmetic risk premium of 3.54% over the period 1872–2000 for U.S. stocks, while the realized risk premium for this period is 5.57%. In the period 1951–2000, the observed difference is even larger. They conclude that the high 1951–2000 returns are the result of low expected future performance. However, the United States was one of the most successful stock markets in the 20th century, so a global perspective is important to correct this bias. Dimson, Marsh, and Staunton [2009] use historical equity risk premia for 17 countries over the period 1900–2008. They conclude that their equity risk premia are lower than frequently cited in the literature, due to a longer timeframe and a global perspective. Exhibit A1 provides an overview of historical risk premia and volatilities.

Both Fama and French [2002] and Dimson, Marsh, and Staunton [2003, 2009] find that the historical equity premium was significantly higher in the second half of the 20th century than it was in the first half. Dimson, Marsh, and Staunton [2009] expect a lower equity premium in the range of 3.0% to 3.5% going forward. In this article we use an equity risk premium of 4.75%. This is slightly above the average of countries in a long timeframe and corresponds well with consensus estimates among finance professors as documented by Welch [2008] and among CFOs as reported by Graham & Harvey [2008].

The other estimate we need is stock market volatility. Dimson, Marsh, and Staunton [2009] find a standard deviation of 17.3% for global equity during the 109-year period 1900–2008. Over the period 1970–2008 the global MSCI index[1] had a volatility of 18.8% and 22.0% expressed in dollars and euros, respectively. We average these last two figures and estimate the volatility of stocks at 20%.

Government bonds. Dimson, Marsh, and Staunton [2009] also evaluate the risk premium of bonds over cash. Their data point to a lower risk premium than the Barclays Government Indices, which have been available since 1973; see Exhibit A2. The last decades have been extremely good for government bonds. We use a geometric risk premium of 0.75% for government bonds over cash, in line with the long-term historical average from Dimson, Marsh, and Staunton [2009].

The volatility of bonds has been significantly lower in recent decades compared to longer timeframes, as Exhibit A2 shows. Over the last 25 years and the last 10 years, it has come down to 6.3% and 5.5%, respectively. We think a volatility of

EXHIBIT A1

Overview of Historical Risk and Return Characteristics for Stocks

Source	Country	Risk Premium on Cash	St. Dev.	Ann. St. Dev. of Monthly Returns	Data
MSCI U.S.	U.S.	3.1%	18.4%	15.4%	1970–2008
MSCI World	World	3.0%	18.8%	14.8%	1970–2008
Fama and French [2002]*	U.S.	3.9%	18.5%	N.A.	1872–2000
	U.S.	2.5%	19.6%	N.A.	1872–1950
	U.S.	6.0%	16.7%	N.A.	1951–2000
Dimson, Marsh and Staunton [2009]	U.S.	5.2%	20.2%	N.A.	1900–2008
	U.K.	4.2%	21.8%	N.A.	1900–2008
	World	4.4%	17.3%	N.A.	1900–2008
MSCI World in Euros			22.0%	16.0%	1970–2008

*Note: *Standard deviation of the risk premium instead of the standard deviation of the nominal return. We derive geometric data by using the equation* $R_G = R_A - 0.5 \times variance$.

EXHIBIT A2

Overview of Historical Risk and Return Characteristics for Government Bonds

Source	Country	Risk Premium on Cash	St. Dev.	Ann. St. Dev. of Monthly Returns	Data
Barclays Treasuries	U.S.	2.2%	6.5%	5.4%	1973–2008
	U.S.	3.6%	6.3%	5.0%	1984–2008
	U.S.	3.0%	5.5%	4.8%	1999–2008
Dimson, Marsh, and Staunton [2009]	U.S.	1.2%	8.3%	N.A.	1900–2008
	U.K.	0.4%	11.9%	N.A.	1900–2008
	World	0.8%	8.6%	N.A.	1900–2008

7% is the best proxy for government bonds. This accounts for the inflation-targeting monetary policy introduced by major central banks in the early 1980s, while it is in line with the observed volatility in the last decades.

Private equity. For private equity one could expect a risk premium relative to stocks due to low liquidity. Willshire [2008] estimates the risk premium for private equity over stocks at 3% using a combination of each U.S. retirement system's actual asset allocation and its own assumptions; see Exhibit A3.

Kaplan and Schoar [2005] find average returns equal to that of the S&P 500, but they did not correct for sample biases. Using 1,328 mature private equity funds, Phalippou and Gottschalg [2007] conclude that performance estimates found in previous research overstate actual returns. They find an underperformance of 3% compared to the S&P 500 (net of fees). In a literature overview, Phalippou [2007] finds support to Swensen's [2000] claims that private equity generates poor returns and that the low risk observed is the result of a statistical artefact.

We use LPX indices that represent listed private equity. Survivorship bias is assumed to be negligible, since the index takes into account that companies are bought or merged, change their business model, or are delisted. Our data also show an underperformance, but this concerns a short sample period. Since we do not have enough support from existing literature that private equity returns (net of fees) exceed public equity returns, we assume the risk premia of stocks and private equity to be equal to 4.75%.

Historical returns show private equity had more risk than stocks, and other research find a beta for private equity greater than one; see Phalippou [2007]. Based on annual standard deviations we should adopt standard deviations for private equity that are almost twice the standard deviations of stocks. However, because our data history is short, we focus on annualized standard deviations of monthly returns. Then, averaged over the United States and Europe, the standard deviation is 50% higher for private equity than for stocks. Therefore, we estimate the volatility of private equity at 30%.

Real estate. Of all alternative asset classes, real estate probably received most attention from academics in the past. A literature review by Norman, Sirmans, and Benjamin [1995] tries to summarize all findings. Overall, they find no consensus for risk and return characteristics for real estate. However, more

EXHIBIT A3

Overview of Historical Risk and Return Characteristics for Private Equity

Source	Risk Premium on Cash	Risk Premium on Stocks	St. Dev.	Ann. St. Dev. of Monthly Returns	Data
Stocks: MSCI U.S.	–2.5%		21.9%	16.0%	1998–2008
Private Equity LPX America	–3.8%	–1.3%	45.6%	28.9%	1998–2008
Stocks: MSCI Europe	1.5%		24.7%	16.6%	1994–2008
Private Equity LPX Europe	1.0%	–0.5%	35.0%	19.7%	1994–2008
Wilshire [2008]		3.0%	26.0%	N.A.	Prospective
Phalippou and Gottschalg [2007]		–3.0%	N.A.	N.A.	1980–2003
Kaplan and Schoar [2005]		0.0%	N.A.	N.A.	1980–1997

than half of the consulted literature in their paper reported a lower return for real estate compared to stock.

Exhibit A4 reports an overview of real estate risk and return characteristics. Wilshire [2008] and Fugazza, Guidolin, and Nicodano [2006] also show lower risk premia for real estate than for stocks. We proceed with a risk premium of 3.75% relative to cash, which is 1% lower than our estimate for stocks. Compared to the long-run U.S. history, our estimate seems rather low; but compared to Wilshire [2008], it seems high. It is in line with Fugazza et al. [2006].

Norman, Sirmans, and Benjamin [1995] conclude that most studies found risk-adjusted returns for real estate that are comparable to stocks. We take this into account and estimate the volatility of real estate at 16%, whereby ex-ante Sharpe ratios are roughly the same for stocks and real estate, while the ratio of the standard deviations is in line with Fugazza et al. [2006].

Hedge funds. The academic literature reports extensive information on biases in hedge fund indices, as shown in Exhibit A5. However, estimates for the market portfolio of hedge funds are scarce. Funds of hedge funds are often considered to be a good proxy for the market portfolio, since they have fewer biases than typical hedge funds. However, their returns are affected by the double counting of management fees. Fung and Hsieh [2000] estimate the portfolio management costs for a typical hedge fund of funds portfolio to be between 1.3% and 2.9%.

Exhibit A6 reports historical risk premia for hedge funds of funds. We use the HFRI fund of funds composite index, which is equally weighted and includes over 800 funds. Furthermore, it is broadly diversified across different hedge fund styles. As McKinsey [2007] suggests, we find a weakening performance of hedge funds over cash. When we average the aggressive and conservative estimate of the risk premium over cash, we find a risk premium of roughly 1.25%. This is the estimate that we use in this article.

Over the period 1990–2008, the volatility of hedge funds was slightly higher than half the volatility of stocks. Taking into account our estimate of the volatility for stocks of 20%, we estimate the volatility of hedge funds at 12%—i.e., 11.9%/20.5% multiplied by 20%.

Commodities. An unleveraged investment in commodities is a fully collateralized position that has three drivers of returns: the risk-free rate, the spot return, and the roll return. Erb and Harvey [2006] point out that the roll return has been a very important driver of commodity returns, but it is unclear what the sign of roll returns will be in the future.[2] In their extensive study they find that the average individual compound excess return of commodity futures was zero. They argue that individual commodities are not homogeneous and that their high volatility and low mutual correlations result in high diversification benefits. The diversification benefit comes from periodically rebalancing the portfolio and is reflected in the high historical performance of the GSCI index compared to the return from individual commodities, as can be seen in Exhibit A7. We use the GSCI index, since it represents the majority of open interest in the future market (Masters [2008])

EXHIBIT A4

Overview of Historical Risk and Return Characteristics for Real Estate

Source	Risk Premium on Cash	Risk Premium on Stocks	St. Dev.	Ann. St. Dev. of Monthly Returns	Data
Stocks: MSCI U.S.	3.0%		18.9%	16.8%	1972–2008
Real Estate: Nareit U.S.	2.5%	–0.5%	21.9%	15.4%	1972–2008
Fugazza et al. [2006] European Stocks	5.7%		16.9%		1986–2005
Fugazza et al. [2006] European Real Estate	4.7%	–1.0%	13.2%		1986–2005
Wilshire [2008]		–2.5%	15.0%		Prospective

EXHIBIT A5

Biases in Hedge Fund Indices

Source	Bias	Magnitude	Data
Fung and Hsieh [2000]	Backfill	0.70%	1994–1998
	Survivorship	1.40%	1994–1998
Posthuma and van der Sluis [2003]	Backfill	2.27%	1996–2002
Amin and Kat [2005]	Survivorship	0.63%	1994–2001

Exhibit A6

Overview of Historical Risk and Return Characteristics for Hedge Funds

Source	Risk Premium on Cash	Risk Premium on Stocks	St. Dev.	Ann. St. Dev. of Monthly Returns	Data
Stocks: MSCI U.S.	3.8%		20.5%	14.6%	1990–2008
Hedge Funds: HFRI FOF Composite	3.8%	0.0%	11.9%	8.9%	1990–2008
Agressive Estimate	2.5%	–1.3%			1990–2008
Conservative Estimate	0.2%	–3.6%			1990–2008
Hedge Funds: HFRI FOF Composite	7.8%	–7.1%	12.0%	9.0%	1990–1999
Hedge Funds: HFRI FOF Composite	–0.3%	–6.7%	9.9%	8.6%	2000–2008

Notes: For the conservative estimate we subtract a 2.27% backfill bias and a 1.40% survivorship bias from the arithmetic return, while the aggressive estimate uses 0.70% and 0.63%, respectively. Now the geometric risk premiums on cash are 2.5% and 0.2%, respectively.

Exhibit A7

Overview of Historical Risk and Return Characteristics for Commodities

Source	Risk Premium on Cash	Risk Premium on Stocks	St. Dev.	Ann. St. Dev. of Monthly Returns	Data
Stocks: MSCI U.S.	3.1%		18.4%	15.4%	1970–2008
Commodities: GSCI	3.9%	0.8%	25.6%	19.9%	1970–2008
Gorton and Rouwenhorst [2006]	5.0%	0.0%		12.1%	1959–2004

and offers the longest historical return series since 1969. Gorton and Rouwenhorst [2006] create an equally weighted monthly rebalanced portfolio of commodity futures that had returns like stocks over the period 1959–2004.

The historical geometric risk premium for the GSCI commodity index was 3.9% over the period 1970–2008, which exceeds the MSCI US by 0.8%. Erb and Harvey [2006] raise questions to the representativeness of both the equally weighted portfolio and the GSCI index. On the one hand, they show that an equally weighted stock index would by far outperform a market-cap weighted index. On the other hand, the GSCI index composition has changed dramatically over time and allocates heavy weights to energy commodities. They suggest that a simple extrapolation of historical commodity index returns might not be a good estimate for future returns. Lummer and Siegel [1993] and Kaplan and Lummer [1998] claim that the long-run expected return of commodities equals the return on Treasury bills. Many theories for commodity risk premia exist, but most of those are not measurable.[3] Since we do not find enough support for a forward-looking positive risk premium, we proceed with a commodity return equal to the risk-free rate, in line with Kaplan and Lummer [1998].

Erb and Harvey [2006] show that the average annual standard deviation of commodities was 30%. A portfolio of commodities, however, diversifies away part of the risk. We take the volatility of the S&P GSCI index during 1970–2008 as our measure of risk. Therefore, our estimate for the volatility of commodity returns is 26%.

High yield and credits. Exhibit A8 shows historical risk premia for high yield and credits. According to Elton et al. (2001), the credit spread comprises the following three components: default risk compensation, tax premium, and systematic risk premium. Additionally, de Jong and Driessen [2005] find a liquidity premium in credit spreads. High-yield bonds require a higher default risk premium than corporate bonds due to lower creditworthiness of the issuers or subordinate debt.

We estimate the risk premium of credits over government bonds at 0.75%, as we think the findings of Altman [1998] are far closer to the true premium than the historical excess return findings in Barclays indices. Altman [1998] also examines the return from U.S. high-yield bonds compared to U.S. treasuries over the period 1978–1997. The excess return of high yield (over Treasuries) during the 20-year period 1978–1997 is 2.47%. We believe that this figure significantly overstates the

EXHIBIT A8

Overview of Historical Risk and Return Characteristics for High Yield and Credits

Source	Risk Premium on Cash	Risk Premium on Bonds	St. Dev.	Ann. St. Dev. of Monthly Returns	Data
Barclays Government Bonds U.S.	2.2%		6.5%	5.4%	1973–2008
Barclays Credit U.S.	1.9%	–0.3%	9.2%	7.4%	1973–2008
Altman [1998] High Grade Corporate U.S.*		0.8%		5.4%	1985–1997
Barclays Government Bonds U.S.	3.6%		6.3%	5.0%	1984–2008
Barclays Credit U.S.	3.6%	0.0%	7.3%	5.7%	1984–2008
Barclays High Yield U.S.*	2.8%	–0.8%	14.0%	8.4%	1984–2008
Altman [1998] High Yield U.S.*		2.5%		5.20%	1978–1997

*Note: *Arithmetic risk premia.*

risk premium of high yield. At the start of the sample period the high-yield market was still immature, which leaves room for liquidity problems and biases. Our sample period from 1984 to 2008 even has a negative risk premium for high yield. Obviously, ex-ante this cannot be the case. We proceed with a 1.75% premium over government bonds.

Barclays indices show that the volatilities of corporate bonds and high yield have been higher than that of government bonds. This study moves forward with a 9% volatility for credits, as seen in the period 1973–2008. This is 2% higher than for bonds. High yield has shown a substantially higher standard deviation than bonds and credits. The difference between the standard deviation of annual returns and monthly returns is large. We therefore also attach weight to the annualized monthly data. We take 11% volatility as our proxy.

Inflation-linked bonds. The interest rate on inflation-linked bonds comprises the real interest rate and the realized inflation. This differs from the return on bonds, which consists of a real interest rate, expected inflation, and an inflation risk premium. The cost of insurance for inflation shocks should be reflected by a discount on the risk premium for inflation-linked bonds relative to nominal bonds. Theoretically, the inflation risk premium should be positive.

Over the last 11 years the inflation risk premium has been absent; see Exhibit A9. Grishchenko and Huang [2008] point to liquidity problems in the TIPS market as the reason for the negative inflation risk premium they document. After adjusting for liquidity in TIPS they find an inflation risk premium of 0.14% over the period 2004–2006. Hammond [1999] estimates the risk premium at 0.5%. On the basis of these findings we estimate the inflation risk premium at 0.25%.

Over the sample period, government bonds were slightly less volatile than inflation-linked bonds, but forward-looking volatilities should not differ much. Therefore, we estimate the volatility of inflation linked bonds at 7%, equal to government bonds.

Exhibit 1 provides an overview of all expected returns and standard deviations for the asset classes discussed above. As we use the parameters in a one period mean-variance analysis, we also show the Sharpe ratio based on the arithmetic return.

EXHIBIT A9

Overview of Historical Risk and Return Characteristics for Inflation-Linked Bonds

Source	Risk Premium on Cash	Inflation Risk Premium	St. Dev.	Ann. St. Dev. of Monthly Returns	Data
Barclays Government Bonds U.S.	3.2%		5.3%	4.8%	1998–2008
Barclays INFL.Linked Bonds U.S.	3.2%	0.0%	5.8%	6.1%	1998–2008
Hammond [1999]		0.5%			–
Grischenko and Huang [2008]		0.1%			2004–2006

Correlations. In a study with n asset classes, the correlation matrix consists of $(n(n-1))/2$ different entries, or 36 in this study. We derive all correlation estimates from historical correlations, but in order to fill the complete correlation matrix we have to make assumptions at some point. This is mainly the case for asset classes with short sample periods like private equity and inflation-linked bonds, and to a lesser extent for hedge funds. We focus mainly on correlations of annual returns. However, we also attach weight to the correlations of monthly returns. Our ex-ante correlation matrix is positive definite and thereby meets the condition of Ong and Ranasinghe [2000].

Correlations are time-varying; see for example Li [2002], who documents time-varying correlations in a study that covers G7 countries from 1958–2001. Over this period, the average correlation between stocks and government bonds was approximately 0.2, similar to the 0.17 that we observe for the period 1973–2008 as shown in Exhibit A10. We take 0.2 as the estimate for the correlation between stocks and government bonds; see Exhibit A11.

Private equity experienced a high correlation with stocks, a fact that is also supported by Ibbotson [2007]. Looking forward, we estimate the correlation between private equity and stocks at 0.8. Because of the strong correlation between stocks and private equity, we assume similar individual correlations with other asset classes. This is supported by correlations of monthly returns.

We estimate the correlation of stocks and real estate at 0.6, in line with the reported historical average of 0.56. For stocks and hedge funds we also estimate the correlation at 0.6, again in line with the 0.58 that we observed over the period 1990–2008. This correlation is the result of the exposure of hedge funds to the stock market. The mutual correlation of real estate and hedge funds was lower at 0.41. Therefore, we estimate the correlation at 0.4.

High yield and credits both showed somewhat higher correlations with stocks than government bonds did, and this may be attributable to the credit risk embedded in these bonds. Going forward, we expect correlations of 0.6 and 0.4, respectively. Over the period 1998–2008, inflation-linked bonds showed a slightly lower correlation with stocks than government bonds did, both on annual and monthly returns. In a forward-looking context, we therefore consider a correlation of 0.0 between stocks and inflation-linked bonds to be justified.

Commodities had close to zero correlation with all asset classes, other than hedge funds and inflation-linked bonds. An explanation for the positive relationship with hedge funds could be their investment positions in commodities. The positive relationship with inflation-linked bonds can be explained by their common driver—unexpected inflation. As Exhibit A11 shows,

EXHIBIT A10

Overview of Historical Correlations of Annual Returns (Lower Left Part of the Matrix) and Monthly Returns (Upper Right Part)

	Stocks (1970)	Private Equity (1998)	Real Estate (1972)	Hedge Funds (1990)	Commodities (1970)	High Yield (1984)	Credits (1973)	Bonds (1973)	Inflation Linked Bonds (1998)
Stocks (1970)	1.00	0.76	0.55	0.54	0.02	0.57	0.35	0.14	–0.01
Private Equity (1998)	0.76	1.00	0.62	0.62	0.21	0.68	0.16	–0.26	0.11
Real Estate (1972)	0.56	0.36	1.00	0.26	0.04	0.60	0.38	0.16	0.28
Hedge Funds (1990)	0.58	0.90	0.41	1.00	0.29	0.41	0.30	0.01	0.09
Commodities (1970)	–0.09	0.32	–0.17	0.39	1.00	0.08	–0.05	–0.07	0.27
High Yield (1984)	0.70	0.65	0.71	0.56	0.00	1.00	0.50	0.15	0.28
Credits (1973)	0.48	–0.17	0.45	0.32	–0.19	0.60	1.00	0.87	0.79
Bonds (1973)	0.17	–0.91	0.11	–0.15	–0.21	0.18	0.82	1.00	0.71
Inflation Linked Bonds (1998)	–0.18	–0.21	0.29	0.07	0.58	0.19	0.75	0.35	1.00

Note: The effective period for each correlation starts with the date of the shortest dataset.

EXHIBIT A11
Overview of Estimated Correlations

	Stocks	Private Equity	Real Estate	Hedge Funds	Commodities	High Yield	Credits	Bonds	Inflation Linked Bonds
Stocks	1.0								
Private Equity	0.8	1.0							
Real Estate	0.6	0.6	1.0						
Hedge Funds	0.6	0.7	0.4	1.0					
Commodities	0.0	0.1	0.0	0.4	1.0				
High Yield	0.6	0.7	0.7	0.6	0.0	1.0			
Credits	0.4	0.4	0.4	0.3	0.0	0.5	1.0		
Bonds	0.2	0.2	0.1	–0.1	0.0	0.2	0.8	1.0	
Inflation Linked Bonds	0.0	0.1	0.3	0.1	0.3	0.3	0.8	1.6	1.0

we take this into account in our estimated correlations. The correlation between commodities and private equity was driven solely by 2008. As mentioned before, we assume similar correlation for stocks and private equity with other asset classes.

For high yield and credits, we mostly round the annual correlations to get our estimates. For bonds we round the correlation with real estate, and for the correlation with hedge funds and inflation-linked bonds we also take the monthly correlations into account. For the last two remaining cells of the correlation matrix, the correlation of inflation-linked bonds with real estate and hedge funds, we round the annual correlations, as they are in line with the monthly correlations.

APPENDIX B

Market Portfolio

We derive the market portfolio from a variety of sources that we consider best in providing an assessment of the market size of an asset class. As most markets were rather depressed in 2008, we estimate the market size over the period 2006–2008 to illustrate the dynamic character of the market portfolio.

For stocks we use the market capitalization of the MSCI All Countries Index, summing the standard index and the small-cap index. We then subtract the weight of REITs as they are part of the real estate asset category in this study. At the end of 2008 we estimate the market capitalization of stocks at USD 20.51 billion[4] as shown in Exhibit B1. In contrast, McKinsey Global Institutes [2008][5] estimates far higher figures for stocks in the previous years than we derive from MSCI. For example, they come up with a capitalization of USD 54 trillion at the end of 2006, while we estimate the market size at that moment to be USD 34 trillion. The difference arises when McKinsey does not adjust the figures for free float, and this is what makes MSCI values more representative for the investable universe in this study. Ibbotson [2006] estimates stocks at USD 29.1 trillion in its market value approach, which is close to MSCI market capitalizations.

EXHIBIT B1
Estimate of the Market Portfolio from 2006 to 2008 (USD Bln)

	2006	2007	2008
Stocks	33752	36071	20510
Private Equity	1105	1044	355
Real Estate	3960	3649	2025
Hedge Funds	1500	1900	1400
Commodities	237	330	452
High Yield	1020	996	612
Credits	9582	11017	11555
Bonds	12755	13728	15913
Inflation Linked Bonds	995	1229	1222
Total Market Capitalization	64906	69964	54043

The fixed-income estimates result from market capitalizations of Barclays indices (previously Lehman indices). The Barclays Multiverse Index comprises all fixed-income asset classes.

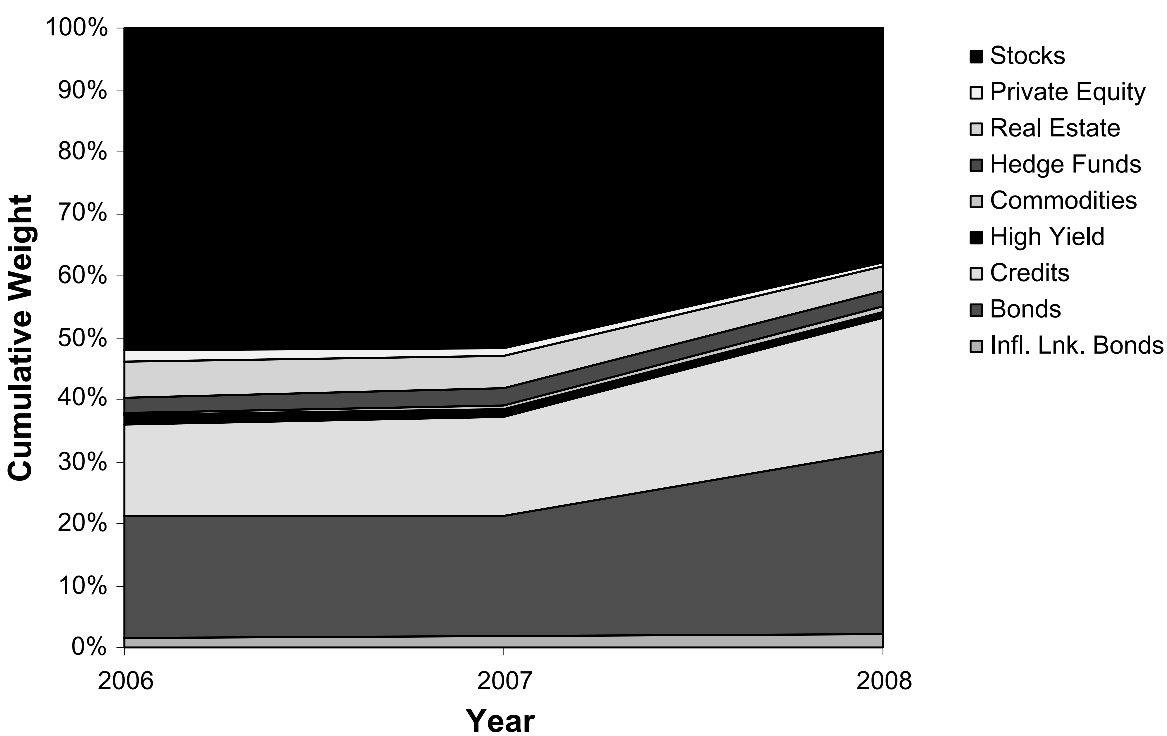
Within this universe we use the market capitalization of the Barclays Multiverse Government Index minus inflation-linked bonds as a proxy for government bonds. This amounts to USD 15.913 billion at the end of 2008. The market capitalization of the Barclays Global Inflation Linked Index was USD 1.222 billion at the end of 2008, a figure that we use for our estimate of the size of the inflation-linked bonds market. For high yield this is USD 612 billion, derived from the Barclays Global High Yield Index. The remaining market value within the Barclays Multiverse Index consists primarily of corporate debt and mortgage-backed securities (MBS), which we assign to the asset class credits and have a worth of USD 11.555 billion. Ibbotson [2006] applies a geographical composition of the bond market and valuates the total market at USD 21.4 trillion, which is somewhat smaller than our estimate of USD 24.4 trillion for the total fixed income market at the end of 2006.

For private equity we use the 2006 year-end estimate by McKinsey [2007]. As we lack other data sources, we adjust this figure for 2007 and 2008, multiplying it by the cumulative performance of the LPX50, a global index that measures the performance of 50 listed private-equity companies. We estimate the size of the private-equity market at USD 355 billion at the close of 2008. The observed presence of private equity in financial markets is greater because of their high leverage. This also applies to hedge funds, for which we use data from Hedge Fund Research. They estimate the unleveraged assets under management at USD 1.4 trillion at the end of 2008.

The real estate market needs further discussion. Within the real estate market, a first distinction can be made when it comes to commercial and residential real estate. The residential market would be much bigger than the commercial market, were it not for the fact that a large portion of this market is the property of the occupiers or residents. Hordijk and Ahlqvist [2004], as an extreme example, estimate that only 5% of all residential real estate in the U.K. is available to investors. Added to investability constraints, most individual investors already have an exposure to residential real estate that exceeds the money they have available for investments, simply because they own a home. Therefore, this study focuses on commercial real estate only. Going forward there are three measures for the size of the market. The broadest measure comprises all real estate, both investment grade and non-investment grade. The investable universe then consists of all investment grade real estate, and includes real estate that is owner-occupied. The invested universe, the smallest one, differs from investable, because it measures the market that is actually in the hands of investors. Liang and Gordon [2003] estimate the investable market at USD 12.5 trillion.

EXHIBIT B2

Estimate of the Market Portfolio from 2006 to 2008



The commercial real estate market is valued by using data from RREEF Real Estate Research; see Hobbs [2007]. Different from other sources, RREEF divides the market estimate of real estate into the four quadrants of public equity, private equity, public debt, and private debt. At the end of 2006, they estimate the investable and invested markets at USD 16.0 and 9.8 trillion, respectively. The 9.8 trillion estimate is the total market and includes both equity and debt. The equity component of invested real estate, which is the universe suitable for comparison in this framework, is USD 4.0 trillion. The estimate is close to the figure given by Ibbotson [2006], who estimates this measure of the real estate market at USD 4.6 trillion. Real estate debt, such as MBS, can be considered as part of the fixed-income asset class and is in fact largely captured by the estimate for credits. We obtain the figures for 2007 and 2008 by adjusting the 2006 figure for the change in the global market capitalization of REITs, as measured by MSCI.

The growth of commodity markets in recent years is evident and observable, but unfortunately hard to qualify. According to Doyle, Hill, and Jack [2007] from the FSA Markets Infrastructure Department, even the most important market participants were unable to accurately measure the commodity market. Masters [2008] uses open interest in commodity futures as a proxy for the market value. We use these data and estimate the commodity market at USD 452 million in 2008.[6]

Exhibit B2 shows the market portfolio from 2006 to 2008.

ENDNOTES

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[1]We use the MSCI World Index until 1988, afterwards the MSCI All Countries Index.

[2]The upward or downward sloping term structure of futures prices creates the possibility of a roll return. It arises when an almost expiring future is rolled over to a future with a longer maturity.

[3]See Erb and Harvey [2006] for a literature overview on commodity market theories.

[4]This number contains some double counting, as private equity and hedge funds also have positions in stocks. In the case of private equity, the double counting is likely negligible. Hedge funds are private pools of capital, which makes correction impossible.

[5]McKinsey [2007] also estimates other market values for bonds, but due to the lack of transparency of these figures, this study uses other sources.

[6]Although this study does not treat derivatives as an asset class, the commodity market is gauged with the futures market since that is the only investment proposition for this asset class. The estimate is an average of the daily value of open interest during 2006 and 2007, adjusted for the stake of physical hedgers in Masters [2008]. Note that it concerns the average daily dollar value of open interest instead of year-end estimates that we have for the other asset classes. We derive the 2008 estimate from the first quarter of 2008 figure from Masters [2008] by multiplying it with the cumulative performance of the GSCI, the same methodology as for private equity.

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