Wallpaper symmetries

1. You should be able to identify symmetry of a given wallpaper pattern. You don't have to memorize all the groups – an identification key similar to one on Wikipedia will be available to you.
    a. Examples of wallpaper patterns that you should be able to analyze:

http://euler.slu.edu/escher/index.php/Wallpaper_Patterns

Note: You will be allowed to use the reference table for finding wallpaper symmetries.

Regular Tessellations

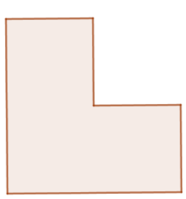
You can use a tessellation explorer to answer some of the questions:
https://www.geogebra.org/m/rMMQxGDP

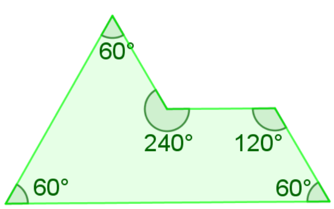
1. What is a regular tessellation.
2. Which regular polygons tessellate? Explain why they tessellate
3. Why a regular pentagon does not tessellate?
4. Why no regular polygon with more than 6 sides cannot tessellate?
5. Describe a semi-regular (Archimedean) tessellation.
6. Draw examples of four different semiregular tessellations.
    a. Explain why your combinations of regular polygons in 5 work.
7. Calculate the interior angle of a regular dodecagon and use it to explain if such octagon and equilateral triangle can form a semi-regular tessellation.

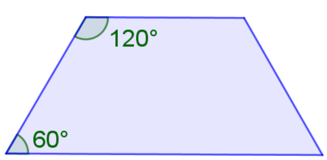
Note: You will be allowed to use actual polygons or applet for any of the questions above.

Rep-tiles

8. Describe what a rep-(n) tile in geometry is.
9. Show how the following shapes can be thought of rep-(n) tiles.







In the following problems you will be asked to show that certain triangles are rep-tiles. Besides drawing a picture showing how it is a rep-tile, don't forget to provide some arguments why a bigger copy is similar to the smaller one. (The corresponding angles must be congruent and sided proportional).

10. Any triangle can also be a rep-4 tile. Show how.
11. A 30-60-90 triangle can also be a rep-3 tile. Show how.
12. One type of an isosceles triangle is a rep-2 triangle. Show how and calculate the angles in such a triangle.
13. Calculate the dimensions of a rectangle that is also a rep-2 tile.

Note: You will be allowed to use either actual pieces or transparency for the questions in this section.

Symmetries and algebraic structures

1. Review the sets of numbers (Natural, whole, integers, rational, irrational, real).
2. How do we define a rational number? How can you tell if a decimal represents a rational number?
3. Is the repeating decimal 3.12121212… a rational number? Explain.
4. What was strange about the decimal 0.9999999… ? What's your take on it?
5. List and describe important properties of structures and operations.
6. Explain what a Cayley table is and how it reflects properties of operations/structures.
7. Symmetry systems. In each symmetry system, you should be able to:
    a. List all symmetries
    b. Generate a Cayley table. For larger systems (such as square symmetries) you will not be asked to generate the whole table, just its parts.
    c. Discuss properties of the system and justify your answers. Decide if a structure is a group or Abelian group and justify your answer.
    d. Generate and solve equations pertinent to that system. For example in the system of square symmetries, solve the equation: $R_{90^\circ} \star X \star D_1 = R_{180^\circ}$. For larger systems, Cayley table will be shown on the visualizer.
8. You should be able to do #3 for various symmetry systems:
    a. For symmetries of a square (you don't have to generate the whole table here).
    b. For symmetries of a letter H.
    c. For symmetries of an equilateral triangle.

9. Discuss the properties of operations and decide if a structure is a group or Abelian group if the structure is given just by the operation table (it can be any system, not just a symmetry one):

	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	5	4	7	6	1	8	3
3	3	8	5	2	7	4	1	6
4	4	3	6	5	8	7	2	1
5	5	6	7	8	1	2	3	4
6	6	1	8	3	2	5	4	7
7	7	4	1	6	3	8	5	2
8	8	7	2	1	4	3	6	5

	0	1	a	b
0	0	0	0	0
1	0	1	a	b
a	0	a	b	1
b	0	b	1	a

	0	1	a	b
0	0	1	a	b
1	1	0	b	a
a	a	b	0	1
b	b	a	1	0

In the following problems, think of „gaining a property“ this way. If we have natural numbers and an operation (addition or multiplication), then the identity and inverse properties for neither operation hold. But if we extend the set of natural numbers to a bigger one (whole, rational, integers, or real) some of these properties will now hold. If it is the case, you would state which property for which operation now holds (for example, a sample answer can be „closure for multiplication is gained“).

10. Explain what property is gained when the system of whole numbers is extended to the system of integers.

11. Explain what property is gained when the system of natural numbers is extended to the system of whole numbers.

12. Explain what property is gained when the system of integers is extended to the system of rational numbers. Does your answer change if we exclude 0 from the set of rational numbers?